Circuits for an RF Cochlea

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Abstract—We develop a technique for approximating a WKB-type solution to a wave equation as a cascade of unidirectional filters. This allows us to design improved building-block circuits for a bio-inspired RF cochlea. By comparing properties of different cochlear filter stages using experimental results and circuit simulations, we demonstrate that our technique significantly improves the characteristics of the RF cochlea.

I. INTRODUCTION

The biological cochlea is a sophisticated signal processing system that acts as a traveling-wave spectrum analyzer. In healthy humans, it has 120dB of input-referred dynamic range and consumes only about 14\mu W of power [1]. The cochlea spatially separates frequency components in incoming sound signals, thereby performing a frequency-to-place transformation. High (low) frequencies excite peak responses towards the beginning (end) of the structure. Electrically, the cochlea can be modeled as an active, nonlinear transmission line with properties that scale exponentially with position [2].

The RF cochlea is a circuit that uses ideas from the biological cochlea, which works at audio, and extends them into RF for performing fast, broadband, low-power spectrum analysis [3]. We plan to implement the RF cochlea on silicon using a standard CMOS process. There are several reasons for building such a biologically-inspired system:

- Exponentially tapered traveling-wave architectures like the cochlea are more hardware-efficient than banks of bandpass filters for performing spectral analysis [1]. As a result, they are simpler and faster than conventional spectral analysis techniques with comparable resolution.
- The RF cochlea has inherently higher dynamic range than audio-frequency silicon cochleas, mainly because integrated passive inductors can be used at RF. Active inductors, which produce $Q^2$ times as much noise as passive inductors with the same quality factor $Q$, must be used at audio.
- The RF cochlea is a complex signal processing system that uses collective computation to reduce power consumption and improve dynamic range. It allows us to explore the design and control of large systems with many interacting components that have similar goals.

In this paper, we model the cochlea as a linear system. However, nonlinear behavior is important in the biological cochlea, particularly for spectral masking and gain control. Incorporating suitable nonlinearities into our models is a work in progress.

II. COCHLEAR ARCHITECTURES

Two basic approaches to cochlear modeling are shown in Fig. 1. Fig. 1(a) shows a model [4] that consists of a bidirectional transmission line with series inductors $L(x)$ coupling together complex shunt impedances $Z(j\omega, x)$. Since the properties of the cochlea scale exponentially with position $x$, we can simplify our equations by defining a dimensionless variable $s_n = \omega_x/\omega_c(x)$, where $\omega_c(x) = \omega_c(0) \exp(-x/l)$, and $l$ is a constant. Thus $ds_n/dx = s_n/l$.

A simplified alternative approach is to model the cochlea as a unidirectional cascade of low pass filters with exponentially tapered cutoff frequencies [1]. An important motivation is the fact that in the biological cochlea the magnitudes of backward propagating waves are small. The transfer function after the $N$-th filter in this cascade, shown in Fig. 1(b), is given by

$$TF(s_n, N) = \prod_{i=0}^{N-1} H(s_i) = \prod_{i=0}^{N-1} H(s_n 2^{-i/N_{oct}})$$

where $s_n = \omega_x/\omega_c(x)$, $H(s_n)$ is the normalized filter transfer function and $N_{oct}$ is the number of filters per octave. The characteristic frequency of the $x$-th filter is $\omega_c(x) = \omega_c(0) 2^{-x/N_{oct}}$. In the cochlea, $H(s_n)$ is selected such that $TF(s_{n}, N)$ becomes invariant with $N$ as $N \to \infty$.

III. THE UNIDIRECTIONAL COCHLEA

If we assume that the properties of the cochlea scale slowly relative to the wavelength of the traveling wave, analytical WKB-type solutions can be found for the transfer functions of the bidirectional cochlea model shown in Fig. 1(a). In this section, we show how these functions can be approximated by unidirectional filter cascades. If we consider only forward wave propagation, the WKB solution [3], [4] is

$$TF(s_n) \propto s_n \left| k_n(s_n) \right|^{3/2} \exp \left(-\int_{0}^{s_n} k_n(s') ds' \right)$$

where $k_n(s_n)$ is the wave number and is given by

$$k_n(s_n) = \omega_c(0) 2^{-x/N_{oct}}. $$

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The cochlear transfer function $TH(s_n)$ can be given by [3]:

$$[k_n(s_n)]^2 = \frac{N^2}{s_nZ_n(s_n)}$$

(4)

where $N$ is a constant and $Z_n$ is a dimensionless version of the shunt impedance $Z$ in Fig. 1(a). We can split up the cochlear transfer function in (3) into pre-exponential and exponential parts. However, the essence of cochlear action is collective amplification, represented by the exponential term in (3). If we therefore ignore the pre-exponential term $s_nk_n^{3/2}$, $TF(s_n)$ can be written as

$$TF(s_n) \propto \prod_{i=1}^{N} \exp \left( -\int_{s_{i-1}}^{s_i} k_n(s') ds' \right)$$

(5)

where we have split up the integral in the exponential into $N$ smaller regions of integration (with $s_0 = 0$ and $s_N = s_n$). This already looks like the transfer function of a cascade of filter stages, with that of the $i$-th stage being

$$H(s_i, s_{i-1}) = \exp \left( -\int_{s_{i-1}}^{s_i} k_n(s') ds' \right)$$

(6)

If the continuous integral is finely quantized, i.e., enough filters are used, $s_i - s_{i-1}$ becomes a small quantity. In this case each sub-integral becomes a small number $\ll 1$. Using the relation $\exp(-x) \approx 1/(1 + x)$ when $x \ll 1$, $H(s_i, s_{i-1})$ can now be approximated as

$$H(s_i, s_{i-1}) = \frac{1}{1 + \int_{s_{i-1}}^{s_i} k_n(s') ds'} \approx \frac{1}{1 + k_n(s_i)(s_i - s_{i-1})}$$

(7)

where we have assumed that $k_n$ remains approximately constant over the (small) interval $(s_{i-1}, s_i)$ at the value $k_n(s_i) = k_n(s_n)|_{s_n=s_i}$. We also have

$$\frac{s_{i-1}}{s_i} = \frac{\omega_0(i)}{\omega_0(i-1)} = 2^{-\frac{1}{8\pi\omega_0}}$$

(8)

Since $N_{oct} \gg 1$ (i.e., many filters are used per octave), we get $2^{-\frac{1}{8\pi\omega_0}} \approx 1 - \ln(2)/N_{oct}$. Thus $s_i - s_{i-1} \approx s_i \ln(2)/N_{oct}$, and the normalized transfer function $H(s_n)$ of the resultant filter may be written as

$$H(s_n) = \frac{1}{1 + \frac{\ln(2)}{N_{oct}}k_n(s_n)}$$

(9)

In addition, the bidirectional cochlear model predicts $k_n$ in terms of the normalized shunt impedance $Z_n(s_n)$. Substituting (4) in (9), we get

$$H(s_n) = \frac{1}{1 + \frac{N\ln(2)}{N_{oct}}\sqrt{Z_n(s_n)}}$$

(10)

In general, $\sqrt{s_n/Z_n(s_i)}$ is not a rational function, which means that $H(s_n)$ cannot be implemented using a lumped system. In order for $H(s_n)$ to be rational, $s_n/Z_n(s_n)$ must be a perfect square. The simplest possible version of $Z_n$ for the bidirectional model is given by [3]:

$$s_nZ_n(s_n) = \left( \frac{s_n^2 + 2ds_n + 1}{s_n^2 + 2s_n + \mu^2} \right)^2$$

(11)

The impedance function in (11) cannot be used directly for the unidirectional RF cochlea since the resultant filter transfer function, given by (10), may not be rational. However, the magnitude and phase response shapes of (11) are insensitive to the value of $Q$. In particular, we can use $Q = 0.5$ while keeping them qualitatively similar. In this case, the two poles of $s_nZ_n(s_n)$ coincide on the real axis. We can then complete a square in the denominator (i.e., $(s_n^2 + \mu s_n/Q + \mu^2) = (s_n + \mu)^2$ when $Q = 0.5$). This makes $\sqrt{s_n/Z_n}$, and thus the normalized filter transfer function $H(s_n)$ in (10), rational:

$$H(s_n) = \frac{1}{1 + \frac{N\ln(2)}{N_{oct}}\sqrt{Z_n(s_n)}}$$

(12)

where $\beta = N\ln(2)/N_{oct}$. When $(d_1, d_2) \ll 1$, the frequency response of (12) has complex pole and zero pairs at $w_i \approx \sqrt{1/\beta} < 1$ and $w_i \approx 1$, respectively, causing its magnitude to first peak, drop sharply and then gradually increase back to 1. The peak causes amplification in the cochlea transfer functions, while the near-null sharpens the roll-off slope beyond the peak and thus increases frequency resolution. As $\mu$ decreases the peak gain of the cochlea increases as the filter poles become more under-damped.

IV. THE ORIGINAL COCHLEAR FILTER

First, we describe filters that have been used for unidirectional silicon cochleas at audio frequencies and port them to RF. The cochlea described in [1] had the best overall...
performance amongst several implementations. It used all-pole second order filter transfer functions of the normalized form

$$H(s_n) = \frac{1}{s_n^2 + \frac{2\pi f_n}{f_c} + 1}$$  \hspace{1cm} (13)

where $Q$ is a constant. In order to evaluate its performance at RF, a test chip containing individual second-order filter sections was designed, fabricated using UMC’s L180 process, and tested. L180 is a standard 0.18μm CMOS technology with certain RF enhancements, such as metal-insulator-metal capacitors and thick top metal layer for inductors. For ease of implementation, the transfer function of the i-th stage, shown in (13), was modified slightly to include a zero:

$$H(s_n) = \frac{Qs_n + 1}{s_n^2 + \frac{2\pi f_n}{f_c} + 1}$$  \hspace{1cm} (14)

Each filter stage was a single-transistor common-source amplifier with shunt zero peaking [5] using on-chip circular spiral inductors. Transistors widths were increased in the later (low frequency) stages. Since the transconductance $g_m \propto \sqrt{W I_{bias}}$ (assuming square-law operation), the power consumption $V_{DD} I_{bias}$ of these stages can be lowered without affecting the $g_m$ and thus the voltage gain $g_m R_L$. High-frequency stages must use smaller transistors and burn more power so that their input capacitance (mainly $C_{gs} \propto W$) does not load the previous stage in the cascade.

Fig. 2 is a die microphotograph that shows the layout of a single amplifier. Fig. 3 shows measured frequency responses of five individual amplifier stages that were designed to have peak frequencies about an octave apart. Input and output buffers were used to match the filter to 50Ω source and load impedances. The low-frequency cutoff in the responses around 15MHz occurs because the inputs to each stage were capacitively coupled. Spurious resonances caused by cables and connectors in the test setup are also visible at low frequencies (approximately 50-500MHz). The overall match between simulation and measurement is, however, fairly good.

![Fig. 2. Die photograph of a single cochlear amplifier stage.](image)

**V. THE IMPROVED COCHLEAR FILTER**

In this section, we describe how to implement our new filter transfer function (Eqn. 12) at RF. We again choose a single transistor common-source amplifier. The load on the drain is however modified as shown in Fig. 4. The normalized transfer function of this circuit is of the form

$$\frac{v_{out}}{v_{in}} = \left( \frac{s_n^2 + 2ds_n + 1}{s_n^2 + 2(2d + \mu\beta)s_n + 1} \right) \times \left( \frac{\tau_1 s_n + 1}{\tau_1 s_n + 1} \right) \left( \frac{\tau_2 s_n + 1}{\tau_2 s_n + 1} \right)$$  \hspace{1cm} (15)

where the first term is the ideal transfer function, given by (12). The second term is due to the high-pass filter required for DC isolation between stages. The third and final term in (15) represents additional high frequency poles and zeros. These extra terms do not significantly affect the cochlea transfer functions as long as $\tau_1 \gg 1$ and $(\tau_2, \tau_3) \ll 1$. We must now choose component values in Fig. 4 that provide suitable values of $d, \mu, \beta, \tau_1, \tau_2, \tau_3$ and $Q_3$. This network synthesis problem is too difficult to solve analytically; a computerized optimization routine was developed instead. The program uses Mathematica™ to find a symbolic representation of the transfer function in terms of $R_1, L_1, C_1, L_2, C_2, R_3, C_3, M$ and $C_C$. An optimization routine then synthesizes the target $H(s_n)$ using a suitable error function.

Given the target parameter values $d = 0.1, \mu = 0.2, \beta = \ln(2), \tau_1 = 500, \tau_2 = 0.25, \tau_3 = 0.33$ and $Q_3 = 0.5$, the following normalized component values were found by the synthesis routine: $R_1 = 1\Omega, L_1 = L_2 = 0.455H, C_1 = 0.350F, C_2 = 2.33F, R_3 = 39.8\Omega, C_3 = 12.6F, M = -0.347H$ and $C_C = 0.597F$. The circuit is now impedance and frequency scaled by $Z_r = 1/gm_1$ and $\omega_r = \omega_c(x)$ respectively (where $gm_1$ is the transconductance of M1) so that it can be implemented. $R_L$ and $C$ values transform as follows: $R \rightarrow RZ_r, L \rightarrow LZ_r/\omega_r$ and $C \rightarrow 1/(Z_r\omega_r)$. Simulated
transfer functions of the filter as implemented in the UMC 0.18μm CMOS process are shown in Fig. 5.

![Cochlear Filter Diagram](image)

**Fig. 4.** Single-ended cochlear filter stage. In practice, a differential implementation will probably be used.

![Simulated Transfer Function](image)

**Fig. 5.** Simulated transfer function of the improved cochlear filter stage. Responses of six stages spaced an octave apart are shown.

Noise accumulation is an important problem with cascaded filter stages. In the cochlea, exponential scaling eventually makes the noise saturate to a fixed value [1]. This value and the noise from each stage are both proportional to the impedance scaling ratio \( Z_r \), i.e., \( \propto 1/g_m \). A noise-power consumption trade-off is evident: In order to lower the noise, \( Z_r \), which defines the characteristic impedance of the system, must be reduced. However, to keep the voltage gain constant, \( g_m \) must be increased, thereby increasing the power consumption.

Fig. 6 compares simulated frequency responses of unidirectional cochleas using all-pole filters [1] (Eqn. 13, with \( Q = 1.3 \)) and our modified filter (Eqn. 15, with the parameters above)². The responses in Fig. 6(b) are significantly sharper than those in Fig. 6(a). The new filter improves the cochlea’s frequency resolution.

**VI. CONCLUSION**

One of the most interesting features of the RF cochlea is its distributed nature. Gain is obtained through the collective action of many filter stages. As a result, quality requirements from individual circuit components (particularly inductors) are low and the system is extremely robust to parametric fluctuations in individual component values. This makes the RF cochlea particularly suitable for implementation in IC form. In this paper, we have described a building-block filter circuit suitable for such an implementation. Our new filter improves the overall performance of the system, thereby marking an important step towards building an RF cochlea.

**REFERENCES**


²Cochleas using zero-peaked low-pass filters (Eqn. 14) have responses with slightly lower resolution than the all-pole responses shown in Fig. 6(a).