Kerr electro-optic field mapping measurements in water using parallel cylindrical electrodes

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An extensive set of Kerr electro-optic field mapping measurements are reported for highly purified water over the temperature range of 8.8–29.5 °C. The Kerr constant is independent of temperature over this range and equals \( B \approx 3.4 \times 10^{-14} \text{ m/V}^2 \). Pulsed high voltages up to 140 kV across parallel cylindrical electrodes with a 1-cm gap are applied on millisecond time scales. For early times, the measured results agree with the space-charge-free electric field distribution while for times greater than 500 \( \mu \text{sec} \), there is significant space-charge distortion due to positive charge injection.

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I. INTRODUCTION

Because of its high dielectric constant, water is being used as the electrical energy storage medium in capacitors, pulsed forming and transmission lines, and switches used in pulsed power devices for laser, fusion, and charged particle beam systems. Water's low cost, self-repairability and ease of handling and disposal offer further advantages to its use as a dielectric. Because water dielectrics are a critical component of pulsed power systems, the electrical conduction and breakdown characteristics of water and water mixtures are being actively investigated.

Recent analysis and measurements have shown that space-charge effects in water/ethylene glycol capacitors have significant effects on the open circuit voltage decay and the electric field distribution. Preliminary Kerr electro-optic field mapping measurements in highly purified water confirmed positive charge injection.

This paper reports on an extensive set of Kerr electro-optic field mapping measurements made in highly purified water using parallel cylindrical electrodes over the temperature range of 8.8–29.5 °C, confirming again significant space-charge effects on millisecond time scales.

II. PARALLEL CYLINDRICAL ELECTRODES

A useful transmission line configuration is the two wire line of parallel cylindrical electrodes. In the absence of space charge in the dielectric volume with permittivity \( \varepsilon \), the electric field distribution outside the cylindrical electrodes can be found using the method of images

\[
E(x, y) = \frac{q_i}{2\varepsilon} \left[ \frac{4a(x^2 + a^2 - x^2)}{y^2 + (x + a)^2 + (y^2 + (x - a)^2)} \right],
\]

where \( q_i \) is the charge per unit length on each electrode and \( x \) is the coordinate direction on the line connecting the image charges which are a distance \( 2a \) apart. If the cylinders have identical radius \( R \) with their centers a distance \( D \) apart, then

\[
a = \sqrt{ \left( \frac{D}{2} \right)^2 - R^2 },
\]

and the capacitance per unit length between the cylinders at a potential difference \( V \) is

\[
C' = q_i/V = \frac{\pi \varepsilon}{\ln \left[ \frac{D}{2R} + \sqrt{\left( \frac{D}{2R} \right)^2 - 1} \right]^{1/2}}
\]

\[
= \frac{\pi \varepsilon}{\cosh^{-1} \left( \frac{D}{2R} \right)}. \quad (3)
\]

The Kerr electro-optic effect produces lines of constant electric field magnitude, which from Eq. (1) requires

\[
\frac{16a^2 x^2 y^2 + 4a^2 [y^2 + a^2 - x^2]^2}{[y^2 + (x + a)^2]^2 [y^2 + (x - a)^2]^2} = \frac{4a^2}{[y^2 + (x + a)^2][y^2 + (x - a)^2]} = C,
\]

where \( C \) is a constant to be determined by specifying a coordinate \( (x_0, y_0) \) on a particular equipotential line. This can be rewritten as

\[
y^4 + 2a^2 (x^2 + a^2) + (x^2 - a^2)^2 - 4a^2/C = 0,
\]

with real solution

\[
y^2 = - (x^2 + a^2) + 2a \sqrt{x^2 + 1/C}.
\]

For equipotential lines starting on a cylinder where

\[
y_0 = R^2 - (x_0 - D/2)^2,
\]

the constant is

\[
C = \frac{a^2}{x_0^2 R^2},
\]

while lines that do not contact the cylinder but pass through the point \( (x_0, y_0 = 0) \) have constant

\[
C = \frac{4a^2}{(x_0^2 - a^2)^2}.
\]

The critical trajectory that passes through the origin \( (x = 0, y = 0) \) has

\[
C = \frac{4}{a^2}.
\]

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and starts on the cylinder at coordinate $x_0 = a^2/2R$. Figure 1 with solid lines plots these equifield magnitude lines. If the dielectric has space charge, this pattern will be changed.

The slope of the line tangent to the electric field at an angle $\theta$ to the $y$ axis is

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{2xy}{y^2 + a^2 - x^2} = \cot \theta. \quad (11)$$

Lines of constant angle $\theta$ are then

$$y = x \tan \theta \pm \sqrt{(x \tan \theta)^2 - a^2 + x^2} \quad (12)$$

and are drawn dashed in Fig. 1.

III. KERR ELECTRO-OPTIC FIELD MAPPING MEASUREMENTS

A. Experimental apparatus

The water purification system, Kerr electro-optical system, high voltage system, triggering electronics, and recording system are schematically illustrated in Fig. 2.

1. Water purification system

The water is stored in a 55 gallon polyethylene insulated holding tank with a stirred bath cooler. Water is lifted from the holding tank to the inlet of the water deaeration column using a vacuum pump at a pressure down to 24 in. mercury which also deaerates the water. The water exits the deaeration column to a mixed resin bed deionization system which increases the water resistivity to $\approx 18 \text{ M}\Omega \text{ cm}$ at $25^\circ \text{C}$. A digital resistivity meter monitors the resistivity at the outlet of the purification system, using a thermistor compensated probe to reference the resistivity to $25^\circ \text{C}$ and an uncompensated probe to read the actual resistivity near the entrance to the test cell. The now purified water is pumped through the test cell back into the holding tank at a rate $\approx 2 \text{ liter/min}$.

2. Test cell

The test cell is a plexiglas cylinder of 3-in. diam, with 1/4-in.-thick Pyrex windows on both ends. The stainless steel parallel rod electrodes with a $l = 1 \text{ cm}$ gap spacing are $L = 44 \text{ in.}$ long (111.76 cm), $2R = 1/2 \text{ in.}$ diam (1.27 cm), and center to center spacing of $D = 2.27 \text{ cm}$. The cylindrical electrodes are aligned horizontally allowing any bubbles to rise. The entire test cell is placed under transformer oil to prevent external flashover.

3. High voltage system

A five stage Marx impulse generator having a 10-kJ, 400-kV rating is charged by a high voltage dc power supply and is connected through a protective 5 k$$\Omega$$ water resistor to limit electrical breakdown currents. Typical pulse rise times are of order $40 \mu\text{sec}$ with decay times of order 5 msec. The high voltage pulse at the Marx generator is monitored by a water resistor divider with ratio $\approx 0.17 \text{ V/kV}$, while the test cell voltage is monitored by a capacitive divider with ratio $\approx 3 \text{ mV/kV}$. 

FIG. 1. Lines of constant electric field magnitude (solid) and direction (dashed) for the electric field distribution between parallel cylindrical electrodes. The proportions of center to center spacing $D$, gap $l$, and radius $R$ correspond to the test cell used with $D = 2.27 \text{ cm}$, $l = 1 \text{ cm}$, and $R = 0.635 \text{ cm}$.
4. Triggering electronics and recording

A long pulse of order 100 msec drives the camera electronic shutter and another pulse generator whose delayed output drives the Marx trigger generator. This delayed output drives another pulse generator whose delayed output triggers the pulsed dye laser. Thus the time of the light pulse is controlled relative to the high voltage pulse, allowing us to sequentially view the electric field distribution in the test cell at various times by increasing the delay time between the laser and high voltage trigger pulses.

5. Optical system

The high voltage output of a pulsed tunable dye laser at wavelength 590 nm is expanded to 50-mm diam and passed through a polarizer, the test cell, another polarizer, lens, and a camera using Polaroid film. For all our measurements, the two polarizers had their transmission axes either aligned or crossed. Usually crossed quarter-wave plates were placed on either side of the test cell but between the polarizers in the circular polariscope configuration to remove isoclinic fringes which depend only on the electric field direction and not magnitude as discussed in Sec. II B.

B. Kerr effect fringe patterns

The high voltage stressed water is a birefringent medium whereby light polarized along the applied local electric field \( E \) travels at speed \( c_0 \), while light polarized perpendicular to the local electric field travels at speed \( c_1 \). For the Kerr effect, these different speeds are related as

\[
\frac{c_0 - c_0}{c_1} = n_\perp - n_\parallel = \lambda BE^2,
\]

where \( \lambda \) is the free space wavelength, \( c_0 \) is the speed of light in free space, \( n_\perp \) and \( n_\parallel \) are the refractive indices for light polarized parallel and perpendicular to the electric field, \( B \) is the Kerr constant, and \( E \) is the applied electric field magnitude. The phase shift for light propagating perpendicular to the electric field in the \( z \) direction along a length \( L \) of this birefringent medium is

\[
\phi = \frac{2\pi}{\lambda} \int_0^L (n_\perp - n_\parallel) dz = 2\pi \int_0^L BE^2 dz \approx 2\pi BE^2 L,
\]

where the last equality assumes the electric field is uniform along the light propagation path. The slight correction due to edge effects near the electrode ends is negligible because the end effect fields extend over a distance of order equal to the electrode spacing (1 cm) which is very short compared to the electrode length (111.76 cm). The birefringent medium converts incident linearly polarized light to elliptically polarized light.
1. Polarizer and analyzer

We assume that the electric field \( E \) in the birefringent medium is at an angle \( \theta_K \) from the vertical \( y \) axis. A polarizer whose transmission axis is at an angle \( \theta_{p1} \) from the vertical is placed on one side of the test cell while another polarizer with transmission axis \( \theta_{p2} \) from the vertical is placed on the other side. The light transmitted through this system is

\[
\frac{I}{I_0} = \cos^2(\theta_{p1} - \theta_{p2}) - \sin 2(\theta_K - \theta_{p1}) \\
\times \sin 2(\theta_K - \theta_{p2}) \sin^2 \frac{\phi}{2},
\]

where \( I_0 \) is the peak light amplitude.

(a). Crossed polarizers \( (\theta_{p1} - \theta_{p2} = \pi/2) \). If the transmission axes of the polarizers are perpendicular, then Eq. (15) reduces to

\[
\frac{I}{I_0} = \sin^2 2(\theta_K - \theta_{p1}) \sin^2 \frac{\phi}{2}.
\]

Light transmission minima then occur when

\[
\frac{\phi}{2} = n\pi, \quad n = 0, 1, 2, ..., \tag{17}
\]

\[
2(\theta_K - \theta_{p1}) = m\pi, \quad m = -1, 0, +1. \tag{18}
\]

The latter condition depends only on the directions of the incident polarized light and the applied electric field. Independent of the electric field magnitude, there will be dark fringes wherever the light polarization is parallel or perpendicular to the applied electric field. These field directional lines are called isoclinic fringes. Superimposed on this pattern are light minima wherever Eq. (17) is satisfied. These field magnitude dependent lines are called isochromatic lines.

When \( E = 0, \phi = 0 \) and the light pattern is uniformly dark. As the electric field is increased some light is transmitted. It is convenient to define the electric field magnitude necessary to reach the first light maximum when \( \phi = \pi \) as

\[
\frac{\phi}{2} = \frac{\pi}{2}, \quad n = 0, 1, 2, ..., \tag{19}
\]

\[
2(\theta_K - \theta_{p1}) = \frac{\pi}{2}, \quad m = -1, 0, +1. \tag{20}
\]

The Kerr electro-optic fringe patterns using crossed and aligned polarizers without quarter-wave plates showing field direction dependent isoclinic lines that rotate with changes in angle \( \theta \) of incident light polarization and field magnitude dependent isochromatic lines that do not depend on \( \theta \). The measurements shown were taken with a water temperature of \( T = 26.4 {\degree} C \) and resistivity \( \rho = 16.4 \) M\( \Omega \) cm. The charging voltage was 120\,kV with the measurements taken near voltage crest 60\,\mu\text{sec} from start with instantaneous voltage\(\approx110\text{ kV} \), and at \( t = 1 \text{ msec} \) from start with instantaneous voltage\(\approx92\text{ kV} \).
\[ E_m = \frac{1}{\sqrt{2}BL} \] (19)

so that (16) can be rewritten as

\[ \frac{I}{I_0} = \sin^2(\theta_K - \theta_p) \sin^2 \left[ \frac{\pi}{2} \left( \frac{E}{E_m} \right)^2 \right]. \] (20)

Light maxima and minima will then occur for

\[ E = \sqrt{n}E_m, \]

\[ n = 1, 3, 5, \ldots \text{ odd maxima,} \]

\[ n = 0, 2, 4, \ldots \text{ even minima,} \] (21)

where \( n \) is odd for maxima and \( n \) is even for minima.

(b) *Aligned Polarizers* \( (\theta_p = \theta_p) \). If the polarizers are aligned, then Eq. (15) reduces to

\[ \frac{I}{I_0} = 1 - \sin^2(\theta_K - \theta_p) \sin^2 \left( \frac{\pi}{2} \left( \frac{E}{E_m} \right)^2 \right). \] (22)

We then have light transmission maxima with the condition of Eq. (18) whenever the light polarization is parallel or perpendicular to the applied field. Similarly, light maxima also occur when the condition of Eq. (17) applies. Thus, the isochromatic field magnitude dependent light minima with aligned polarizers occur at the maxima for crossed polarizers and vice versa.

Figure 3 shows the Kerr electro-optic fringe patterns with crossed and aligned polarizers for various angles of incident light polarization. The isoclinic minima for crossed polarizers and maxima for aligned polarizers rotate with changes in direction \( \theta \) of incident light polarization, while the field magnitude dependent isoclinic lines remain unchanged as \( \theta \) is varied. The isoclinic minima (crossed polarizers) and maxima (aligned polarizers) which occur along lines where the applied electric field is either parallel or perpendicular to the light polarization agree with the dashed lines shown in Fig. 1 and provide a measure of the electric field direction. For instance, the crossed polarizer dark isoclinic lines in Fig. 3 for \( \theta = +15^\circ \) agree with the lines of Fig. 1 with \( \theta = 15^\circ, \theta = 105^\circ, \) and \( \theta = -165^\circ. \) The lines at \( \theta = -75^\circ \) are obscured by the electrode holders. Similarly, the isoclinic lines for \( \theta = -15^\circ \) agree with Fig. 1 for \( \theta = -15^\circ, \theta = 75^\circ, \) and \( \theta = 165^\circ \) with the \( \theta = -105^\circ \) lines obscured by the electrode holders. However, these isoclinic lines are broad and obscure the field magnitude dependent isochromatic lines, so it becomes preferable to remove them.

2. Circular polariscopes

The isoclinic lines can be removed if crossed quarter wave plates are placed on either side of the test cell but between the polarizers as shown in the optical path of Fig. 2, with the incident polarization at an angle of \( \theta = \pm 45^\circ \) to either of the quarter wave plate axes, with the second quarter wave plate axis perpendicular to the first. This configuration is called a circular polariscope. A quarter wave plate acts as a birefringent medium with phase shift \( \phi = \pi/2 \) between perpendicular field components. The incident linearly polarized light is converted to circularly polarized light by the first quarter wave plate. Then after passing through the birefringent water medium, the second quarter wave plate perpendicular to the first, and the polarizer the light is converted back into linear polarization. The light transmission intensity for either crossed or aligned polarizers with crossed quarter wave plates is then

\[ \frac{I}{I_0} = \begin{cases} 
\sin^2 \left[ \frac{\pi}{2} \left( \frac{E}{E_m} \right)^2 \right] & \text{Crossed Polarizers,} \\
\cos^2 \left[ \frac{\pi}{2} \left( \frac{E}{E_m} \right)^2 \right] & \text{Aligned Polarizers.} 
\end{cases} \] (23)

Thus there are no field directional isoclinic fringes and the minima and maxima are interchanged when the polarizers are crossed or aligned. Aligned polarizers offer a slight advantage as the first minimum when \( E = E_m \) occurs at a lower field value than for crossed polarizers where the first minimum occurs when \( E = \sqrt{2}E_m \). Thus for a given voltage, there is generally one more dark fringe with aligned polarizers.

3. Kerr constant measurements

Figure 4 shows the isochromatic fringe patterns for a temperature \( T = 19 \, ^\circ C \) with aligned and crossed polarizers with charging voltages from 60 to 150 kV. We can calculate the Kerr constant \( B \) from measurement of the voltage necessary to reach a minimum at the center \( x = 0, y = 0 \) using Eqs. (1)-(3), (10), (19), and (23).

From Eqs. (1)-(3), the electric field at \( x = 0, y = 0 \) is related to the geometry and voltage \( V \) as

\[ E_z(x = 0, y = 0) = \frac{V}{a \cosh \left[ \left( \frac{D}{2R} \right) \right]} \] (24)

Measurements identical to that shown in Fig. 4 were taken with crossed and aligned polarizers at temperatures \( T = 8.8-11.4 \, ^\circ C \), \( T = 12.1-13.3 \, ^\circ C \), \( T = 19 \, ^\circ C \), and \( T = 29.5 \, ^\circ C \). In the vicinity of minima at \( x = 0, y = 0 \), the applied voltage per stage was varied in increments of 500 V for a change in output voltage of 2.5 kV to provide greater precision in determining the voltage necessary for a light minimum at \( x = 0, y = 0 \).

The first light minima occurs when

\[ E_n = \sqrt{n}E_m; \]

\[ n = 1, 3, 5, \ldots \text{ odd Aligned Polarizers,} \]

\[ n = 2, 4, 6, \ldots \text{ even Crossed Polarizers,} \] (25)

where \( n \) is odd for aligned polarizers and is even for crossed polarizers. Using Eq. (24), we define the voltage necessary for a minimum at \( x = 0, y = 0 \) as
FIG. 4. Kerr electro-optic fringe patterns using crossed and aligned polarizers with crossed quarter-wave plates at ± 45° to the incident light polarization (circular polariscope). The water temperature \( T \) is 19°C and resistivity \( \rho \) is 21.0 MΩ cm. The charging voltage is varied from 60 to 150 kV, while the instantaneous voltage near the peak at a time about 60 μsec after the start of the pulse as measured by the capacitive divider is given to the lower right of each photograph.

\[
V_n = \sqrt{n} \ V_m, \tag{26}
\]

\[
V_{n,m} = a \cosh^{-1} \left( \frac{D}{2R} \right) E_{n,m}. \tag{27}
\]

Table I lists the measured values of \( V_n \) (\( n \)th minima) and the resulting calculated values of \( V_m \) (first minima for aligned polarizers, first maxima for crossed polarizers) for each of the measured temperature ranges, where the odd values of \( n \) are for aligned polarizers and even values of \( n \) are for crossed polarizers. Within our measurement accuracy, \( V_m \approx 40.6 \) kV and did not measurably vary over the temperature range 8.8–29.5°C. Using Eqs. (19) and (27) we then calculate \( B \) as

\[
B = \frac{1}{2L} \left[ \frac{a \cosh^{-1} \left( \frac{D}{2R} \right)}{V_m} \right]^2 = 3.37 \times 10^{-14} \ \text{m/V}^2, \tag{28}
\]

where we took \( R = 0.635 \) cm, \( D = 2.27 \) cm so that \( \cosh^{-1} \left( \frac{D}{2R} \right) = 1.1845 \), \( a = \sqrt{\left( \frac{D}{2} \right)^2 - R^2} = 0.94 \), \( L = 111.76 \).
cm, and $V_m = 40.6 \times 10^3$ V. This value for $B$ independent of temperature over our measured range agrees within measurement uncertainties to previously reported values. Then for this cell

$$E_m \approx \frac{1}{\sqrt{2BL}} = 36.4 \text{ kV/cm.} \quad (29)$$

### 4. Electric field distribution

The isochromatic fringes only provide information on electric field magnitude and not direction. However, along the lines of symmetry $y = 0$ and $x = 0$, the electric field is purely $x$ directed and is given from Eqs. (1)-(3) as

$$E_x(x, y = 0) = \frac{a/l}{V/I} \cosh^{-1} \left( \frac{D}{2R} \right) \left[ \left( \frac{a}{l} \right)^2 - \left( \frac{x}{l} \right)^2 \right]$$

$$E_y(x = 0, y) = \frac{a/l}{V/I} \cosh^{-1} \left( \frac{D}{2R} \right) \left[ \left( \frac{a}{l} \right)^2 + \left( \frac{y}{l} \right)^2 \right] \quad (31)$$

where $l$ is the gap spacing between cylinders equal to 1 cm for our experiments. These two field distributions are drawn in Fig. 5 together with verifying Kerr measurements using aligned polarizers at voltage 132 kV.

The electric field direction is also known to be perpendicular to each of the cylindrical electrodes. Focussing attention on the right cylinder whose center is at $(x = D/2, y = 0)$, we define the angle $\phi$ as measured from the $y$ axis located at

| TABLE I. Measured values of voltages necessary for $n$th minima at $x = 0, y = 0$ with aligned ($n$ odd) and crossed ($n$ even) polarizers at various temperatures. The calibration voltage $V_m$ necessary for first minimum with aligned polarizers is then computed. |
|---|---|---|---|---|---|---|---|---|
| $T = 8.8-11.4^\circ C$ | $T = 12.1-13.3^\circ C$ | $T = 19^\circ C$ | $T = 29.5^\circ C$ |
| $n$ | $V_x = \sqrt{n} V_m$ (kV) | $V_x = \sqrt{n} V_m$ (kV) | $V_x = \sqrt{n} V_m$ (kV) | $V_x = \sqrt{n} V_m$ (kV) |
| $n$ | $V_m$ (kV) | $n$ | $V_m$ (kV) | $n$ | $V_m$ (kV) | $n$ | $V_m$ (kV) |
| 2 | 57.1 | 40.4 | 2 | 56.6 | 40.0 | 2 | 56.5 | 40.0 | 2 | 57.8 | 40.9 |
| 3 | 70.6 | 40.8 | 3 | 71.6 | 41.3 | 3 | 71.3 | 41.2 | 3 | 70.1 | 40.5 |
| 4 | 81.9 | 40.9 | 4 | 81.0 | 40.5 | 4 | 82.9 | 41.5 | 4 | 80.5 | 40.2 |
| 5 | 90.9 | 40.7 | 5 | 89.3 | 39.9 | 5 | 92.4 | 41.3 | 5 | 91.2 | 40.8 |
| 6 | 99.2 | 40.5 | 6 | 100.2 | 40.9 | 6 | 99.3 | 40.5 | 6 | 100.5 | 41.0 |
| 7 | 106.6 | 40.3 | 7 | 105.4 | 39.8 | 7 | 107.6 | 40.7 | 7 | 105.7 | 40.0 |
| 8 | 115.0 | 40.7 | 8 | 115.3 | 40.8 | 8 | 114.6 | 40.5 | Average | 40.6 |
| Average | 40.6 | Average | 40.5 | 9 | 122.7 | Average | 40.9 |
| | | | | 10 | 129.8 | Average | 41.0 |
| | | | | 11 | 132.0 | Average | 39.8 |

FIG. 5. $x$ directed space-charge-free electric field distributions along the $x = 0$ and $y = 0$ axes agrees well with the measured Kerr fringe pattern at $V = 132$ kV. The width of the measured values corresponds to each fringe thickness of order $n$. 

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\[ x = D/2 \text{ so that} \]
\[ y = R \cos \phi, \quad x = \frac{D}{2} + R \sin \phi. \]  

(32)

Then substituting Eq. (3) into Eqs. (1) and (4) lets us write the electric field magnitude on the right cylinder as a function of \( \phi \)

\[
\frac{|E|^2 R^4 \left[ \cosh^{-1} \left( \frac{D}{2R} \right) \right]^2}{V^2 a^2} = \frac{1}{\left[ \cos^2 \phi + \left( \sin \phi + \frac{a}{R} + \frac{D}{2R} \right)^2 \right] \left[ \cos^2 \phi + \left( \sin \phi - \frac{a}{R} + \frac{D}{2R} \right)^2 \right]}.
\]

(33)

This function is plotted in Fig. 6, together with agreeing data obtained from the Kerr electro-optic measurement shown in Fig. 5.

C. Space-charge effects

The Kerr measurement photos shown thus far were all taken just after the peak of a high voltage pulse approximately 60 \( \mu \text{sec} \) from the start of the pulse. On this short time scale injected space charge does not have time to propagate into the dielectric volume so that the Kerr measurements agree with the space charge free solution. However, for longer times space charge does significantly accumulate in the dielectric volume and distorts the electric field distribution. As shown in Fig. 7 at \( T = 29.5^\circ \text{C} \) for aligned polarizers, the Kerr pattern becomes significantly nonsymmetric with the \( x \) coordinate for times longer than 500 \( \mu \text{sec} \) after the start of the high voltage pulse. The electric field near the positive electrode drops relative to the electric field at the negative electrode indicating positive charge injection. To verify that this positive injection is not unique to the particular characteristics of each electrode, Fig. 7 also shows the reverse polarity still with positive charge injection but now from the opposite electrode. Figure 8 shows similar positive charge injection for \( T = 19^\circ \text{C} \) with crossed and aligned polarizers while Fig. 9 shows the Kerr measurements at \( T = 10.5-10.9^\circ \text{C} \). The instantaneous voltage values at given times are larger at lower temperatures because the increased water resistivity decreases the decay time of the voltage pulse.

In Fig. 10 we remove the quarter-wave plates to show both the field magnitude dependent isochromatic and field direction dependent isoclinic lines with crossed polarizers at times \( t \approx 60 \mu \text{sec} \) and \( t \approx 1 \text{ msec} \) after the start of the high voltage pulse with charging voltage 120 kV. The space charge at \( t \approx 1 \text{ ms} \) greatly changes the isochromatic lines but hardly disturbs the isoclinic lines.

In this parallel cylindrical geometry, it is difficult to calculate the space charge density \( q \), related to the electric field as

\[
q = \varepsilon \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right)
\]

(34)

as the charge density depends on spatial derivatives of both field components. Since the Kerr measurements cannot accurately separate electric field components, the spatial derivatives required in Eq. (34) cannot be accurately performed. Even along the \( y = 0 \) line, where symmetry requires that \( E_x = 0, \partial E_y / \partial y \) is not zero. However, a partial measure of the electric field distortion due to space charge is given by the difference between measured electric field values \( E_m \) (x, \( y = 0 \)) along \( y = 0 \) and calculated theoretical space-charge-free field values \( E_0 \) (x, \( y = 0 \)).

Such a plot of this difference along \( y = 0 \) versus normalized gap distance \( x/l \) is given in Fig. 11 for various voltages and temperatures at time \( t \approx 500 \mu \text{sec} \) and 1 msec after the start of the high voltage pulse. Because of the temperature dependence of the water resistivity affecting the decay time of the voltage pulse, the difference in field values is normalized to the instantaneous nominal field value \( V/l \). Figure 11 shows that generally, the deviation in field values is larger at lower temperatures indicating stronger space-charge effects. This is most likely due to the increased resistivity resulting in a longer dielectric charge relaxation time so that injected space charge can propagate further before being neutralized by ohmic relaxation.

If we assume that the space charge along \( y = 0 \) has
T=29.5°C
ρ=12.7 MΩ-cm

Aligned Polarizers
Charging Voltage
122.5 kV

FIG. 7. Kerr effect measurements with aligned polarizers and crossed quarter wave plates for both polarities at various times after the start of the high voltage pulse. The water temperature is 29.5°C with resistivity 12.7 MΩ-cm. All measurements had a charging voltage of 122.5 kV, with the instantaneous voltages given to the lower right of each photo. The high voltage waveform as measured by the capacitive divider (3.04 mV/kV) and light pulse are also shown. Positive space-charge field distortion begins after 500 μsec.
FIG. 8. Kerr effect measurements with crossed and aligned polarizers showing space-charge distortion at $T = 19^\circ$C, $\rho = 21.0$ M$\Omega$ cm.
FIG. 9. Kerr effect measurements with aligned polarizers showing space-charge distortion at $T = 10.5-10.9^\circ C, \rho = 38.6 \, M\Omega \cdot cm$. 
FIG. 10. Kerr effect measurements using crossed polarizers without quarter-wave plates for various angles $\theta$ of incident polarization. The space-charge distortion at $t \approx 1$ msec has a large effect on the field magnitude dependent isochromatic lines, but a slight effect on the field direction dependent isoclinic lines.
**III. CONCLUDING REMARKS**

With the parallel cylindrical electrode geometry, it is difficult to be quantitative about the charge density distribution because the electric field distribution is nonuniform and two dimensional, even in the absence of space charge. Future work will repeat these measurements in coaxial cylindrical and parallel plane electrode geometries where the electric field is predominantly in one direction and only depends on one spatial coordinate, allowing easy computation of the charge density from electric field mapping measurements.

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