INVITED PAPER

MAGNETIC FIELD GRADIENT EFFECTS ON MAGNETIC FLUID STABILIZATION

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The penetrating finger instability which develops when a less viscous fluid pushes a more viscous fluid can be stabilized through the use of a magnetizable fluid in the presence of a magnetic field tangential to the interface. A uniform magnetic field only stabilizes suitably short waves traveling along the field lines. Transverse waves of all wavelengths and orientations are also stabilized if the tangential magnetic field is non-uniform with field decreasing in the direction away from the magnetically permeable fluid. Confirming experiments are described using laboratory sandpacks.

1. Introduction

The moving interface between two superposed fluids in a porous medium is unstable forming penetrating fingers when the more viscous fluid is driven by the less viscous fluid [1]. Previously reported analysis has shown that the use of a magnetizable fluid layer can stabilize the interface if a uniform magnetic field is applied tangential to the interface [2,3]. A magnetic field component perpendicular to the interface is always destabilizing [4]. When the interface remains flat there are no magnetic forces. Electromechanical coupling arises when small interfacial distortions perturb the magnetic fields [5,6].

However, the uniform tangential magnetic field only stabilizes those waves propagating in the direction along the magnetic field, having no effect on transverse waves. Earlier work by others with analogous inviscid systems has shown that the non-uniform magnetic field can stabilize all waves with a lower required value of magnetic field [6]. This occurs because the interface feels a restoring perturbation force even without distorting but by simply moving through the non-uniform magnetic field. This paper examines the effect of a non-uniform magnetic field on stabilization of the Saffmann–Taylor pusher-fluid problem.

2. Equations of motion

The governing incompressible hydrodynamic and current-free magnetic field equations are unchanged from the earlier uniform magnetic field analysis [2,4].

\[ \nabla p - \eta \nabla^2 \mathbf{v} + \rho g \mathbf{i} + \mu_n (\mathbf{M} \cdot \nabla) \mathbf{H} = 0, \]  
\[ \nabla \cdot \mathbf{v} = 0, \]  
\[ \nabla \times \mathbf{H} = 0 \Rightarrow \mathbf{H} = \nabla \psi, \]  
\[ \nabla \cdot \mathbf{B} = 0; \quad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}), \]  
\[ \mathbf{M} = \chi_m \mathbf{H}; \quad \mathbf{B} = \mu_0 [1 + \chi_m] \mathbf{H} = \mu \mathbf{H}. \]

The magnetic force density in (1) has an associated stress tensor

\[ \mathbf{F} = \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} \rightarrow F_i = \mu_0 M_j \frac{\partial}{\partial x_j} H_i = \frac{\partial T_{ij}}{\partial x_j}, \]  
\[ T_{ij} = H_i B_j - \delta_{ij} \frac{3}{2} \mu_0 H_k H_k. \]
3. Analysis of a general layer

Earlier work has shown the convenience of introducing a generalized prototype layer as in fig. 1, relating interfacial variables of pressure, displacement, scalar magnetic potential and magnetic field [3]. Since these quantities appear in the interfacial boundary conditions between superposed layers, a multi-layered system can be easily spliced together avoiding the redundancy of solving the same bulk equations in each region.

We assume that when the upper and lower interfaces in fig. 1 deform all variables are slightly perturbed from their equilibrium values. Because the interfacial displacements are small, the boundary conditions are evaluated at the equilibrium positions rather than at the interfaces. Since the fluid velocity is a perturbation, the difference between evaluating it at the interface or at the equilibrium position is second order in the perturbation amplitudes. This illustrates the general approach used in linearized surface deformation problems. The boundary condition at the distorted interface is replaced by one at the equilibrium position, thus greatly simplifying the analysis.

However, as the interfaces deform, in addition to perturbing all variables the equilibrium quantities acting on the interfaces also change. Thus to compute the total first order change in all variables at the interface, linear changes of equilibrium quantities must be included. This is especially true for the case of non-uniform equilibrium magnetic fields.

The governing equations of (1)-(6) are linearized with all perturbation variables assumed of the form

$$\hat{x}^{a,\beta} = Re \xi^{a,\beta} e^{i(k_x y + k_z z)}; \quad k = \sqrt{k_x^2 + k_z^2}, \quad (7)$$

where $\hat{x}^{a,\beta}$ are the complex amplitudes of the interfacial deflections. The analytical approach is identical to our earlier work with uniform magnetic fields [3]. For simplicity, we take the magnetic susceptibility $\chi_m$, or equivalently, the magnetic permeability $\mu$ to be a constant, independent of magnetic field $H$. The resulting prototype equations relating perturbation pressures at each interface to interfacial displacements and magnetic field components are unchanged from the uniform field case [3]. The perturbation magnetic field components at each interface related to interfacial displacements and magnetic scalar potentials are augmented by a term $dH_0/dx |_{a,\beta}\xi^{a,\beta}$ due to the gradient in normal magnetic field at each interface.

4. Two superposed planar layers

We will apply the general relations with $\Delta$ approaching infinity in each region of the two-layer system shown in fig. 2, where the interface has surface tension $\gamma$. The equation of the interface is

$$F = x - \xi(y, z, t) = 0, \quad (8)$$

so that the interfacial normal vector is in the direction

$$n = \nabla F = i_x - \frac{\partial \xi}{\partial y} i_y - \frac{\partial \xi}{\partial z} i_z. \quad (9)$$

For equilibrium at the interface requires that

![Fig. 1. Planar prototype layer of incompressible, magnetizable fluid within a non-uniform magnetic field.](image)
the pressure jump across the interface balance the surface tension and magnetic stresses

\[
\begin{align*}
(p_1 - p_2 + \gamma \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \xi \right) n_i + (T_{ij2} - T_{ij1}) n_j &= 0, \\
\end{align*}
\]

(10)

where the magnetic stress terms are given by (6).

We also require that the tangential component of \(H\) and normal component of \(B\) be continuous across the interface, both in equilibrium

\[
H_{z1} = H_{z2} = H_z; \quad \mu H_{x1} = \mu_0 H_{x2}
\]

(11)

and for perturbations

\[
(h_{z2}' - h_{z1}') = -\frac{\partial \xi}{\partial y} (H_{x2} - H_{x1}).
\]

(12)

In terms of the scalar magnetic potential,

\[
\begin{align*}
\hat{\psi} &= \hat{\psi}_1 = \hat{\psi}_1(0) + H_{x1} \xi \\
\hat{\psi} &= \hat{\psi}_2 = \hat{\psi}_2(0) + H_{x2} \xi \\
\end{align*}
\]

so that (13) is just a statement of continuity of interfacial magnetic scalar potential where \(\hat{\psi}_1(0)\) and \(\hat{\psi}_2(0)\) are the potentials on each side of the interface evaluated at the flat equilibrium position of the interface while \(\hat{\psi}\) is evaluated at the current position of the interface.

The normal component of \(B\) must also be continuous across the interface which to first order yields

\[
\hat{b}_{x2} - \hat{b}_{x1} = jk \mu_0 \mathcal{M}_z \xi,
\]

(14)

The \(x\) component of (10) is

\[
\begin{align*}
\hat{p}_1 - \hat{p}_2 - \gamma k^2 \hat{\xi} + \hat{T}_{xx2} - \hat{T}_{xx1} + (T_{xy2} - T_{xy1}) \hat{h}_y + (T_{xz2} - T_{xz1}) \hat{h}_z &= 0. \\
\end{align*}
\]

(15)

Then relating all perturbation field and flow variables to the interfacial displacement \(\xi\) in (15) yields the characteristic equation as

\[
\frac{s}{k} (\eta_1 + \eta_2) = -\left[ (\eta_1 - \eta_2)V + g(\rho_1 - \rho_2) + \gamma k^2 \right] + \frac{1}{k(\mu_0 + \mu)} \left[ \frac{\mu}{\mu_0} (\mu - \mu_0)^2 k^2 H_{z1}^2 \right. \\
\left. - (\mu_0 H_z k_z)^2 \right] + \frac{1}{2} (\mu - \mu_0) \frac{d}{dt} [H_z^2 + H_{x1} H_{x2}].
\]

(16)

5. Interfacial stability

If \(s\) has a negative real part, any perturbation decays with time while if \(s\) is positive, the system is unstable as any perturbation grows exponentially with time. Thus surface tension is always stabilizing. A uniform magnetization tangential to the interface stabilizes waves in the direction of the magnetization. A uniform magnetic field normal to the equilibrium flat interface is always destabilizing. A magnetic field gradient can stabilize all waves if the magnetic field decreases in the direction from more magnetically permeable fluid.
Gravity stabilizes the system if the heavier fluid is below ($\rho_1 > \rho_2$). If $\rho_1 < \rho_2$ we have the Rayleigh–Taylor instability of a heavy fluid supported by a lighter fluid. The fingering viscous instability analyzed by Saffman and Taylor [1] occurs when the more viscous fluid is being pushed ($\eta_2 > \eta_1$).

Because the magnetic field component perpendicular to the interface is destabilizing, we focus attention on the best conditions of operation where the magnetic field is purely tangential to the interface ($H_x = 0$).

The most potentially unstable situation occurs for transverse waves with $k_z = 0$. For stability of all waves, the tangential magnetic field gradient must have a large negative value so that the right hand side of (16) is negative which occurs when

$$\frac{1}{2} (\mu - \mu_0) \frac{d H_z^2}{d x} < (\eta_1 - \eta_2) V + g (\rho_1 - \rho_2). \quad (17)$$

If this condition is not met a range of long wavelengths (small wavenumbers) will be unstable. The uniform magnetic field stabilization term in (16) only stabilizes waves propagating in the same direction as the field. It has no effect on transverse waves ($k_z = 0$, $k_y \neq 0$). If the condition of (17) is not met, the transverse wavenumbers are unstable over the range

$$k_y < \left[ \frac{1}{\gamma} \left( (\eta_2 - \eta_1) V + g (\rho_2 - \rho_1) \right) + \frac{1}{2} (\mu - \mu_0) \frac{d H_z^2}{d x} \right]^{1/2}. \quad (18)$$

For longitudinal waves with $H_{x1} = 0$ and $k_y = 0$, $k_z \neq 0$, the threshold for instability occurs when the right hand side of (16) is zero. We rewrite (16) as

$$s = -\gamma \left[ k_z^2 + Wk_z^2 - k_z / \lambda_0^2 \right], \quad \text{(19)}$$

where

$$W = \frac{\mu_0 M_z^2}{\gamma [1 + \mu / \mu_0]}.$$
positive and real only if $\lambda_0^2$ is positive. If $\lambda_0^2$ is negative, the system is stable for all wavenumbers as given by (17).

The shortest unstable wavelength $\lambda_c = 2\pi/k_c$ and the fastest growing wavelength $\lambda^* = 2\pi/k^*$ are plotted in fig. 3 versus the magnetic stabilizing parameter $W\lambda_0$. Wavelengths shorter than $\lambda_c$ are stable. If $\lambda_0^2$ is negative all wavelengths are stable.

6. Sandbed measurements

6.1. Test fluids

Confirming experiments were performed using laboratory sandpacks of clean -40/50 mesh Ottawa sand with non-magnetic fluids glycerol, a hydrocarbon solvent, and Dow Corning 200 silicone oil. Magnetic fluids used were Georgia-Pacific aqueous Lignosite FML (100 gauss magnetization in 350 gauss magnetic field), a US Bureau of Mines aqueous base sample, a Ferrofluidics Corp. hydrocarbon base HO1, and a Union Carbide hydrocarbon base ferrofluid.

Because of magneto-viscosity, a Brookfield viscometer was modified to measure ferrofluid viscosity as a function of magnetic field parallel or perpendicular to the spindle rotation axis, as listed in table 1. Modification consisted of replacing magnetic parts of the drive with non-magnetic parts and furnishing an extension rotor to permit the drive head and read-out mechanisms to be located out of the high field region. Alternatively, a bucking field was applied to the viscosity head to reduce the resultant field to a low value. Viscosities were determined at 22°C temperature and at shear rates ranging from about 0.5 to 5 s⁻¹.

6.2. Apparatus

A cylindrical, transparent, thick-wall plastic tube having an inside diameter of 2.4 cm was used to contain a sandpacking, as shown in fig. 4. Sandpacks were prepared by pouring sand at a slow rate into the tabular vessel while the vessel is rotated with a motorized drive. The packed length was 6.3 cm. The tube was equipped with screw-on metal end caps with fittings to 1/4 inch supply and effluent lines. Bed retainer caps and feed lines are threaded to the bed vessel with a screen held pressed over the bed top with the top retainer cap. The driving head to force fluid upwards through the vertically mounted bed was supplied by either a syringe pump or a gravity feed reservoir. When using the syringe pump, the inlet gauge pressure was monitored and flow rate adjusted to run at nearly constant gauge pressure. Using gravity feed, nearly constant drive pressure was assured since the reservoir cross-section was considerably larger than that of the bed. Effluent was collected in one, or a series, of graduated receivers.

Uniform field tests with field intensity less than

Table 1

<table>
<thead>
<tr>
<th>Magnetic fluid</th>
<th>Field intensity (G)</th>
<th>Field orientation relative to spindle rotation axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>parallel</td>
<td>perpendicular</td>
</tr>
<tr>
<td>Georgia-Pacific (aqueous)</td>
<td>0</td>
<td>48</td>
</tr>
<tr>
<td>100</td>
<td>131</td>
<td>330</td>
</tr>
<tr>
<td>200</td>
<td>212</td>
<td>2000</td>
</tr>
<tr>
<td>$\rho = 1.21$ g/cm³</td>
<td>260</td>
<td>–</td>
</tr>
<tr>
<td>400</td>
<td>303</td>
<td>–</td>
</tr>
<tr>
<td>600</td>
<td>333</td>
<td>–</td>
</tr>
<tr>
<td>Bureau of Mines (aqueous)</td>
<td>0</td>
<td>5.5</td>
</tr>
<tr>
<td>100</td>
<td>5.6</td>
<td>7.4</td>
</tr>
<tr>
<td>200</td>
<td>6.0</td>
<td>12</td>
</tr>
<tr>
<td>$\rho = 1.15$ g/cm³</td>
<td>400</td>
<td>9.3</td>
</tr>
<tr>
<td>800</td>
<td>12.8</td>
<td>70</td>
</tr>
<tr>
<td>1000</td>
<td>–</td>
<td>96</td>
</tr>
<tr>
<td>Ferrofluidics (hydrocarbon)</td>
<td>0</td>
<td>5.7</td>
</tr>
<tr>
<td>100</td>
<td>5.8</td>
<td>12</td>
</tr>
<tr>
<td>200</td>
<td>5.8</td>
<td>35</td>
</tr>
<tr>
<td>$\rho = 1.04$ g/cm³</td>
<td>400</td>
<td>7.2</td>
</tr>
<tr>
<td>500</td>
<td>–</td>
<td>47</td>
</tr>
<tr>
<td>600</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Union Carbide (hydrocarbon)</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>1000</td>
<td>9.0</td>
<td>11.3</td>
</tr>
<tr>
<td>2000</td>
<td>10.1</td>
<td>13.0</td>
</tr>
<tr>
<td>$\rho = 1.23$ g/cm³</td>
<td>3000</td>
<td>10.8</td>
</tr>
<tr>
<td>4000</td>
<td>–</td>
<td>15.3</td>
</tr>
<tr>
<td>5200</td>
<td>–</td>
<td>16.0</td>
</tr>
</tbody>
</table>

* Values are viscosity in centipoise.
** Estimated viscosity is 48 cP in perpendicular field orientation of 5300 G.
500 G were done with a Helmholtz pair of air core electromagnets, both coils having 6 inch I.D. and 4 inch wound length. The field orientation is tangential to the undisturbed fluid interface in the sandpack; i.e., the orientation of field is perpendicular to the mean flow direction. Higher intensity uniform field tests were done in the central region of an iron yoke electromagnet having 24 cm diameter pole faces, producing a uniform centerline field that is horizontal.

Gradient magnetic field was provided in the fringe field of the iron yoke electromagnet. The sandpack bed was vertically positioned with its center point 12 cm directly above the centerline of the pole pieces. This geometry produces a magnetic field that is oriented tangential to the driving interface and which possesses a gradient of magnetic field in the direction normal to the interface with field increasing from top towards the bottom of the sandpack. All tests were conducted with a magnetic field of 5200 to 5300 G measured at the midpoint of the bed. The field profile was measured and it was determined that the field gradient was constant at 820 G/cm. over the packed bed length.

6.3. Measurements

The sand is retained on a US no. 60 screen grid (openings slightly smaller than the sand size). Reservoir fluid is introduced to fill the bed interstices using the syringe pump. About 20 min injection time is allowed to prevent trapping air in the bed. The bed feed line is filled with driver fluid and flow of driver fluid established at the desired rate. After sufficient displacement occurs, the interface between the two fluids may be observed through the vessel side wall and its appearance monitored as the test continues. The fluid produced overhead is collected in the receiver with special note taken of the cumulative flow at the point where the first drop of the driver fluid appears overhead.

Table 2 summarizes the sandpack test conditions and observed results. A key listing is “breakpoint” corresponding to cumulative collected volume of reservoir fluid at the point when the first drop of pusher fluid is collected. High value of “breakpoint” volume corresponds to a high sweep efficiency and absence of fingering.

Tests 1 through 5 were done in the absence of an applied laboratory field source in order to establish baseline behavior. In Test 1, a denser, more viscous fluid (glycerin) displaces a less dense, less viscous fluid (solvent). This provides the classical setting for stable displacement according to Saffman–Taylor theory and well-established practice. Indeed, the test produced a wavy but nonfingering interface that yielded a high percent recovery of the reservoir fluid (89% recovery at breakpoint).

The recovery is calculated from the data with a correction for excess reservoir-type fluid initially present in the delivery line.

\[
\% \text{ Recovery} = \frac{(\text{Breakpoint volume}) - (\text{Excess volume})}{(\text{Sandpack saturation volume})} \times 100.
\]

Excess volume is 1.8 cm³ and sandpack saturation volume is 11.5 cm³.

In Test 2, glycerol is used to displace a much
Table 2
Summary of magnetic fluid driver sandpack tests

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Field condition</th>
<th>Field intensity $B$ (G)</th>
<th>Magnetic pusher fluid Type</th>
<th>Magnetic pusher fluid density (g/cm$^3$)</th>
<th>Magnetic pusher fluid viscosity cp ($R = 0$)</th>
<th>Reservoir fluid Type</th>
<th>Reservoir fluid density (g/cm$^3$)</th>
<th>Reservoir fluid viscosity (cp)</th>
<th>Flow rate (cm$^3$/min)</th>
<th>% Recovery at breakpoint</th>
<th>Fingering</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>absent</td>
<td>0</td>
<td>glycerol</td>
<td>1.26</td>
<td>750</td>
<td>solvent</td>
<td>0.79</td>
<td>0.95</td>
<td>0.15</td>
<td>89</td>
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<td>glycerol</td>
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<td>750</td>
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<td>0.97</td>
<td>340</td>
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<tr>
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<td>UC hydrocarbon FF</td>
<td>1.23</td>
<td>8</td>
<td>glycerol</td>
<td>1.26</td>
<td>1100</td>
<td>0.69</td>
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<tr>
<td>4</td>
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<td>1.21</td>
<td>25</td>
<td>silicone</td>
<td>0.97</td>
<td>970</td>
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<td>8</td>
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</tr>
<tr>
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<td>Bu Min aq. FF</td>
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<tr>
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<td>uniform</td>
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<td>Bu Min aq. FF</td>
<td>1.15</td>
<td>5.5</td>
<td>silicone</td>
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<td>0.20</td>
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<td>silicone</td>
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<tr>
<td>9</td>
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<td>silicone</td>
<td>0.97</td>
<td>970</td>
<td>0.55</td>
<td>37</td>
<td>no</td>
</tr>
</tbody>
</table>

more viscous oil than in Test 1. The displaced oil, which is silicone oil, is less viscous than the glycerol so the expectation is that fingering will not occur. Indeed, the experimentally determined recovery percentage of 71% is a high value.

Test 3 deliberately creates a dynamically unstable arrangement with low viscosity ferrofluid displacing glycerol from the sandpack in the absence of magnetic field. The opaque black coloring of the ferrofluid made it well-suited for flow visualization. The run resulted in the visual detection of rapid fingering and low recovery of 11%.

Test 4 is similar to Test 3 in that a ferrofluid of low viscosity displaces a less dense, higher viscosity immiscible reservoir fluid. However, the roles of organic and aqueous-like phases are interchanged relative to Test 3. This has the effect of interchanging the phase which preferentially wets the sand surface since the sand was established to be wetted preferentially by the aqueous-like phases in side tests. Overall fingering takes place and again a low recovery is obtained, in this case 8%.

Test 5 is another dynamically unstable combination, this time employing the Bureau of Mines ferrofluid. The recovery of 3% was the lowest in this series of unmagnetized drivers, consistent with the low viscosity 5.5 cp. of the pusher fluid.

In summary, Tests 1–5 establish in the absence of magnetic field that fingering occurs when expected and is prevented when that is predicted. In addition, the tests indicate levels of recovery to be expected under fingering and nonfingering conditions.

Tests 6–9 were conducted with uniform magnetic field directed tangential to the undisturbed interface existing between the fluids. In each test the pusher fluid was chosen to be more dense than the displaced fluid so that gravitational instability would not be a factor. Calculations of stabilization from our earlier uniform field analysis [2,3] indicated that magnetization was sufficient to prevent fingering if two-dimensional flow was
achieved. However, as Tests 6 and 7 illustrate, the recovery was very low; and, in fact, fingering was not prevented. Observation of the flow pattern through the transparent tube wall, as well as removal and dissection of the sandpack, showed that two-dimensional flow is not obtained and that the fingering proceeds three-dimensionally. Accordingly, while uniform field stabilization had been effective in preventing fingering in studies in a two-dimensional Hele–Shaw cell [2], it is not as effective in realistic three dimensional formations. The result and conditions of Test 8 at first inspection seems to contradict this conclusion with a high recovery in a uniform field. However, it is found that the Georgia-Pacific ferrofluid utilized in these tests exhibits a large increase in viscosity in the presence of the field (see table 1). The viscosity increase was so large that the pusher viscosity exceeded that of the reservoir fluid so that the flow is hydrodynamically stable.

Most importantly, as illustrated in Tests 10–12, gradient magnetic field was employed to test its influence on prevention of the fingering instability. Test 11 versus Test 7 gives clear evidence for the favorable influence of using gradient field stabilization; recovery improved from less than 2% to the value of 54%. Visual observations of the interface confirmed absence of any apparent-fingering. Test 10 versus Test 9 also illustrates the benefit of using gradient field (62% recovery at the breakpoint) versus 25% using uniform field. Because the mean field intensity and flow rate were the same in these two tests, there is no question that the gradient field stabilization mechanism was brought into play. Test 3 in the absence of field using the same fluids and comparable flow rates yielded only 11% recovery at the breakpoint. Fig. 5 presents the history of exit concentration versus cumulative flow for these Tests 10, 9 and 3, and illustrates that ultimate recovery, as well as the breakpoint, of the displaced fluid is increased using the gradient field mode of operation. Test 12 gave enhanced recovery although the field increase of viscosity in the Georgia-Pacific ferrofluid complicates the interpretation.

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References