Torque measurements on ferrofluid cylinders in rotating magnetic fields

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Abstract

We study the response of magnetic nanoparticle suspensions (ferrofluids) to uniform rotating magnetic fields generated by a two-pole three-phase magnetic induction motor stator winding. Measurements of the torque required to rotate a polycarbonate spindle submerged in ferrofluid subjected to co-rotating and counter-rotating fields yield experimental observations of negative magnetoviscosity in a cylindrical Couette geometry, conceptually similar to the observations of Bacri et al. (Phys. Rev. Lett. 75 (1995) 2128) in a Poiseuille flow under an oscillating magnetic field. Further measurements are presented for the torque required to restrain a spindle when it is (i) entirely filled with ferrofluid, (ii) entirely surrounded with ferrofluid, and (iii) both entirely filled and surrounded with ferrofluid. Some of the results for the spindle either entirely filled or entirely surrounded with ferrofluid are compared to theoretical expressions obtained from the ferrohydrodynamic equations using a rigorous regular perturbation expansion in the small parameter $\Omega t$, where $\Omega$ is the applied field frequency and $t$ is the effective magnetic relaxation time of the suspension.

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1. Introduction

We report experiments using water-based (MSGW11) ferrofluid with magnetite nanoparticles from Ferrotec Corp. The ferrofluid had a mass density of $\rho = 1204 \text{ kg/m}^3$, viscosity $\eta = 0.007 \text{ Ns/m}^2$, saturation magnetization $\mu_0M_s = 0.0203 \text{ T}$, volume fraction $\phi = 0.036$, and low-field magnetic susceptibility $\chi = 0.65$. From the low- and high-field limits of the measured magnetization curve, described by the Langevin relation [1], the mean particle diameter was estimated to be in the range from 5.5 to 11.9 nm, so that the Brownian relaxation time, $\tau_B$, is in the range of $2-10 \mu$s and the Néel relaxation time, $\tau_N$, is in the range of $5 \text{ ns}-0.02 \text{ s}$, taking the anisotropy constant for magnetite nanoparticles to be $K \approx 78,000 \text{ J/m}^3$ [2].

We used a Brookfield Model LVDV-I+ viscometer as a torque meter with a measurable torque range of approximately $-10.0$ to $+67.3 \mu\text{N-m}$. When a fixed rotation speed is selected, the spindle rotates...
counter-clockwise and the viscometer applies the necessary torque in order to keep it rotating at the specified speed. When the magnetic-field-induced shear stress on the spindle is in the clockwise (negative torque) direction, i.e., in the direction opposite to spindle rotation, it is harder to turn the spindle at the specified speed; therefore the viscometer applies a higher torque, above and beyond the torque required to shear the fluid in the absence of a magnetic field, and it records an increase of effective ferrofluid viscosity. On the other hand, when the magnetic-field-induced shear stress on the spindle is in the counterclockwise direction, i.e., in the same direction as spindle rotation, it is easier to rotate the spindle at the specified speed; therefore the viscometer applies a lower torque, as compared to the torque required to shear the fluid in the absence of a field, and the viscometer records a decrease of effective ferrofluid viscosity.

2. Rotating spindle measurements

Fig. 1 shows the measured torque on a solid polycarbonate (Lexan) spindle with 25.5 mm diameter and 122 mm length. The water-based ferrofluid was placed inside a 38.5 mm inner diameter beaker giving a 6.5 mm annular gap. With the Lexan spindle in the ferrofluid, the upper free surface exactly matched the top of the 63.8 mm high stator winding and the bottom of the beaker matched the lower end of the stator winding. The ferrofluid sample was centered in the gap of a three-phase, two-pole motor stator winding with 78 mm internal bore diameter, which was excited with balanced three-phase currents to create a uniform applied clockwise (CW) or counter-clockwise (CCW) rotating magnetic field. The spindle was set to rotate at 100 rpm CCW and the torque was measured over the frequency range of 5–500 Hz with magnetic field strengths from 0 to 130 G rms. Fig. 1 shows that the magnetic-field-induced torque on the spindle was opposite to the direction of magnetic field rotation. With no magnetic field, the required torque is about 20 μN m. With magnetic field co-rotating with the CCW spindle, the torque increased with increasing magnetic field amplitude and frequency, while with magnetic field counter-rotating (CW) with the CCW spindle, the spindle torque decreases as magnetic field amplitude and frequency increased. This would correspond to a negative magnetoviscosity analogous to that reported in Ref. [3]. The spindle torque reaches zero and negative values for counter-rotating (CW) magnetic fields greater than about 41 G at 500 Hz, above about 60 G at 100 Hz, and above about 96 G at 50 Hz.

3. Stationary spindle measurements

For further experiments, shown in Fig. 2 for aqueous ferrofluid, the spindle was set to remain stationary, hence, with zero applied magnetic field no torque is required from the viscometer to restrain the spindle [4]. This experimental setup is similar to that of Ref. [5,6]. Under conditions for which a

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**Fig. 1.** Torque required to rotate a spindle surrounded with ferrofluid for counterclockwise (CCW) rotation of the spindle at 100 rpm.

**Fig. 2.** Torque required to restrain 10 ml stationary spindle syringe with 9.5 ml water-based ferrofluid (ff) for ferrofluid entirely inside, entirely outside, and both inside and outside the syringe.
CW magnetic-field-induced shear stress arises, the viscometer exerts a balancing CCW (positive) torque to keep the spindle stationary, whereas for a CCW shear stress a CW (negative) torque will be measured. Syringes of 10 and 20ml volume were used in these experiments as ferrofluid filled cylindrical spindles with respective inner diameters of 13.9 and 18.17 mm and respective wall thicknesses of 1.13 and 1.52 mm. The syringes were filled with ferrofluid so that the height of the ferrofluid in either syringe equaled 62.41 mm, corresponding to 9.5 ml in the 10 ml syringe and 16.2 ml in the 20 ml syringe. The syringe plunger with a flat bottom contacted the ferrofluid inside the syringe so that there was no free surface. A 200 ml beaker of 52.15 mm inner diameter was used in experiments with ferrofluid outside the spindle syringes. Torque measurements were taken for water-based and oil-based ferrofluids at frequencies from 100 to 500 Hz with applied magnetic field amplitudes from 0 to about 190 G rms for cases with the ferrofluid entirely inside the stationary syringes, entirely outside the stationary syringes, and both inside and outside the stationary syringes. A CW rotating magnetic field is applied when ferrofluid is entirely inside a syringe and CCW when ferrofluid is outside the syringe as then the viscometer torque will be counter-clockwise (positive), allowing the full range of torque measurements up to +67.3 μN m.

### 4. Torque theory

An approximate solution has been derived for the magnetic-field-induced flow and viscous torque when ferrofluid is either entirely inside or entirely outside a stationary infinitely long cylinder with no free surface, subjected to a rotating magnetic field. The approximate magnetic-field-induced torque $T$ on the cylinder with ferrofluid entirely outside is

$$T = -\frac{8\pi R^2 L^2 \mu_0 \chi_0 (1 + \chi_0) H^2 \Omega \tau}{(\chi_0 + 2)^2 + \gamma^2 \chi_0 (4 + 2\chi_0 + \chi_0^2)} + O(\Omega^2 \tau^2),$$  

where $R$ is the radius of the outer cylinder, $\gamma$ is the ratio of the inner cylinder radius to the outer radius of the ferrofluid container, $\Omega$ is the radian frequency of the applied magnetic field with rms amplitude $H$, $\tau$ is the effective magnetization relaxation time, $\chi_0$ is the ferrofluid magnetic susceptibility, and $\mu_0 = 4\pi \times 10^7$ H/m is the magnetic permeability of free space. Eq. (1) differs from that obtained in Ref. [6] because we have used a regular perturbation expansion to first order in the small parameter $\Omega \tau$ to solve the coupled ferrohydrodynamic equations with zero spin viscosity. Experiments with ferrofluid entirely outside a spindle, shown in Fig. 3, approximately agree with Eq. (1) for $\tau \approx 10^{-7}$ s, which is about the value for Brownian relaxation in our water-based ferrofluid. This indicates that for our fluid, $\tau_N \gg \tau_B$.

A similar analysis, to second order in $\Omega \tau$, for a volume $V$ of ferrofluid entirely inside the cylinder yields for the

![Fig. 3. Experimental measurements of torque required to restrain a stationary spindle surrounded with ferrofluid in a CCW rotating field compared to predictions of Eq. (1).](image)

![Fig. 4. Experimental measurements of torque required to restrain a stationary spindle filled with ferrofluid compared to predictions of Eq. (2). Positive frequencies correspond to counter-clockwise rotation of the applied magnetic field and negative frequencies correspond to clockwise rotation.](image)
magnetic field induced torque on the cylinder [7]

\[ T = -2\Omega_0 \zeta_0 \mu_0 H^2 V \left[ 1 - \frac{\zeta_0 \mu_0 H^2}{2\eta} \right] + O(\Omega^3 \tau^3), \quad (2) \]

where \( \eta \) is the fluid viscosity and \( \zeta = 1.5 \eta \phi \) is the vortex viscosity. Experiments with ferrofluid entirely inside a spindle, shown in Fig. 4, also agree with Eq. (2) for \( \tau \approx 10^{-5} \) s.

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