Evaluation of the mechanical deformation in incompressible linear and nonlinear magnetic materials using various electromagnetic force density methods

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Mechanical deformation in incompressible linear and nonlinear magnetic materials was evaluated using various conventional electromagnetic volume and surface force density methods. These conventional force density methods are the Maxwell stress tensor method, Korteweg–Helmholtz force density method (KH), magnetic charge method, magnetizing current method, and Kelvin force density method (KV). The total force values obtained using these different force density methods were found to be the same and equal to the total force using the principle of virtual work, but the distribution of force density values calculated using the given force density methods was found to be different from each other. Using the given five force density methods, the mechanical deformations were evaluated and compared to one another. The KH and KV in incompressible material were shown to give the same mechanical deformation by employing the finite element method (FEM), verifying the theoretical equivalence. To implement the KV, the derivative of magnetic field intensity with respect to the geometrical position was calculated using a linear shape function of FEM along with the nodal field values in each element. A magnetic systems was tested to compare the mechanical deformation in linear and nonlinear magnetic materials. © 2005 American Institute of Physics. [DOI: 10.1063/1.1859771]

In electromagnetic systems, mechanical deformation causes unwanted problems, such as vibration and noise. To evaluate the mechanical deformation, we need to know the distribution of the electromagnetic force density and not just the total force. The calculating methods for magnetic force density are generally based on the equivalent source and stress tensor methods, such as the magnetic charge method (MC), magnetizing current method (CM), Kelvin force density method (KV), Maxwell stress tensor method (MX), and Korteweg–Helmholtz force density method (KH).1,2 Most reports on electromagnetic force density and mechanical deformation are limited to linear magnetic materials.2,3 Little research has been reported for nonlinear magnetic materials, and there is no numerical verification of mechanical deformation using the KV.4 In previous research, the KV was usually used to calculate the total force.5 Here, however, we evaluate the distribution of magnetic force density and compare the mechanical deformation to other methods in incompressible linear and nonlinear magnetic materials.

To evaluate magnetic force density and mechanical deformation, we use the finite element method (FEM), which is usually used to solve electromagnetic field and mechanical problems. The given five electromagnetic force density methods have been used to calculate the mechanical deformation in linear magnetic materials, and four methods except the KH, have been used to evaluate the mechanical deformation in nonlinear magnetic materials. The KH was not treated for nonlinear materials because of its complex nature, as it will be shown in the next section. The physical nature of each force density is compared to consider the behavior of mechanical deformation. Utilizing this consideration, we show that the CM yields a different mechanical deformation compared with other methods in linear and nonlinear magnetic materials. The MX gives a significant numerical error, which strongly depends on the surface taken to calculate the force. We verify that the MC, KV, and KH are equivalent methods for calculating the mechanical deformation in incompressible linear and nonlinear magnetic materials.

The generalized Korteweg–Helmholtz force density $f_{KH}$ for a magnetic system is expressed as:

$$f_{KH} = J_i \times B + \sum_i \frac{\partial W}{\partial \alpha_i} \nabla \alpha_i - \nabla \left( \sum_i \alpha_i \frac{\partial W}{\partial \alpha_i} \right).$$

(1)
The magnetizing current force density $f_M$, the magnetic charge force density $f_C$, and the changes in the property can be expressed as the magnetization. The magnetic charge force density $f_C$ is the free current density, where

$$f_C = \frac{1}{\mu_0} B_{air} (B_{air} \cdot n) - \left( \frac{B_{air}^2}{2 \mu_0} \right) n,$$

$$f_M = H_j (B_1 \cdot n) - \left( \frac{H_1 \cdot B_1}{2} \right) n - \left[ H_2 (B_2 \cdot n) - \left( \frac{H_2 \cdot B_2}{2} \right) n \right],$$

where the subscripts air, 1, and 2, represent the region in which the fields are considered. The mechanical deformation using the MX and KH can be obtained by employing the surface traction force.

The mechanical deformation using the MC and CM can be obtained from Eqs. (4) and (5), respectively. These two equivalent source methods yield the same total force, but the mechanical deformation is different from each other because of the different distributions of magnetization sources, such as magnetic charge and magnetization current.

To utilize the MX, KH, and KV for numerical calculations, we used the total field within each FEM triangular element instead of the externally applied magnetic field. For the MC and CM numerical calculations, we subtracted the self-field as is usually done. Using the total field value, the KV can be rewritten as

$$f_{KV}^V = \mu_0 (M \cdot \nabla) H,$$

$$f_{KV}^S = \frac{\mu_0}{2} M_n^2 n,$$

where $f_{KV}^V$ and $f_{KV}^S$ denote the volume and surface force densities of the KV, respectively. The $M_n$ represents the normal component of magnetization and $n$ is the outward normal unit vector at the surface. To calculate the volume force density using the KV, the derivative of $H$ with respect to the geometry should be calculated. To obtain this derivative in Eq. (9), at first, we calculated the nodal value of the magnetic-field intensity. Using the linear shape function, $N(x, y)$, in FEM, the field inside a triangular finite element can be interpolated at each position.

To compare the magnetic force density methods and mechanical deformations obtained from utilizing each different method, we tested an electromagnetic system. The mechanical deformation using the MC and CM can be expressed as

$$W(\alpha_1, \alpha_2, \ldots, \alpha_n, B) = \int_0^B \mathbf{H}(\alpha_1, \alpha_2, \ldots, \alpha_n, B') \cdot \partial B',$$

where $J_f$ is the free current density, $B$ is the magnetic-flux density, $W$ is the magnetic-field energy density, $H$ is the magnetic-field intensity, and $\alpha_i$ represents material properties, such as density and magnetic permeability that can vary with position. The mechanical continuum, such as a fluid or a solid, is capable of undergoing the vector deformations, $\delta \xi$, and the changes in the property $\alpha_i$ are linked to the material deformations, $\delta \alpha_i = -\nabla (\alpha_i \delta \xi)$. Equation (1) is derived from the principle of energy conservation and from the constitutive laws in a continuum system. The third term in the right-hand side of Eq. (1) denotes $\nabla p_m$ where $p_m$ represents electromagnetically induced pressure. This term does not cause any mechanical deformation in incompressible media.\(^{(a)}\) For incompressible materials, in the absence of free current, Eq. (1) can be expressed as

$$f_{KH} = \sum_i \frac{\partial W}{\partial \alpha_i} \nabla \alpha_i,$$

where $\mu_0$ is the magnetic permeability of free space and $M$ is the magnetization. The magnetic charge force density $f_M$ with $J_f = 0$ can be expressed as

$$f_M = \rho_m H = -\rho_m H \nabla \cdot M,$$

where the magnetic charge density is $\rho_m = -\mu_0 \nabla \cdot M$. The magnetizing current force density $f_M$ with $J_f = 0$ can be expressed as

$$f_M = J_m \times B = (\nabla \times M) \times B,$$

where the magnetization current density is $J_m = \nabla \times M$.

The surface force density methods using the Maxwell stress tensor in free space and the Korteweg–Helmholtz stress tensor for magnetically linear material with constant magnetic permeability in regions 1 and 2, respectively, can be expressed as

FIG. 1. 2D magnetic analysis model along with a square test body, which has mechanical constraints at the four vertices. The square test body is physically elongated towards the two pole faces by the tensile stress, as expected. The excitation current densities were $J_1 = 2.5 \times 10^6 \text{ A/m}^2$ and $J_2 = 2.5 \times 10^6 \text{ A/m}^2$. The relative permeability in the linear case was chosen to be 2175. All the dimensions are given in millimeter.
deformations obtained from using the local force density methods in linear and nonlinear magnetic materials (silicon steel RM50) were examined.

To calculate the mechanical deformation for small and large current densities, including B-H curve operating points in the linear and saturation regimes, the magnetic system containing a test body was tested, as shown in Fig. 1. The four vertices of the square test body were fixed along their length in the z direction as a mechanical constraint. The magnetic force density was calculated as a mechanical load at each FEM node. The MX, KH, MC, and KV yielded the same mechanical deformation, but the mechanical deformation using CM was different from the others, as shown in Fig. 2. In the nonlinear case, the KH was not used to calculate the mechanical deformation because of its complicated nature. The relative errors of the maximum mechanical deformation values fell within 1.7% in linear and nonlinear magnetic materials when the MX, KH, MC, and KV were used, as shown in Table I, for small and large current densities so that the operating point for maximum magnetic field is on the B-H curve shown in Fig. 1 in the linear (OP1) and highly saturated regimes (OP2).

The distinct five force density methods were compared to one another for incompressible linear and nonlinear magnetic materials by utilizing the mechanical deformation obtained from FEM. The KV using total field was successfully applied to calculate the mechanical deformation for linear and nonlinear magnetic materials. Taking the given force density methods to be theoretically equivalent in incompressible magnetic materials, the numerical result of mechanical deformation found using the KV also yielded the same result as the KH and MC. From Fig. 2, we have verified that the CM produces an incorrect mechanical deformation when compared to other force density methods. When the current density was high, the magnetic material operated in the saturation region and the deformations were much smaller than when the magnetic material remained in the linear region. According to the numerical results of the mechanical deformation, the MC, KV, and KH are alternative ways of calculating the mechanical deformation in incompressible solid materials. For the future work utilizing the given force density expressions, the behavior of motion for compressible materials and magnetic fluids when subjected to magnetic fields must be evaluated. For example, the rise of the ferrofluid over an infinitely long current carrying wire results in a magnetic field purely tangential to the ferrofluid interface. Under this condition, there is no magnetic charge or the magnetic charge force density.1(b)

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<table>
<thead>
<tr>
<th>Input Material</th>
<th>MX</th>
<th>KH</th>
<th>MC</th>
<th>KV</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_1=2.5 \times 10^6$</td>
<td>66.66</td>
<td>66.65</td>
<td>66.62</td>
<td>65.78</td>
<td>84.21</td>
</tr>
<tr>
<td>A/m$^2$</td>
<td>Nonlinear</td>
<td>0.666</td>
<td>0.665</td>
<td>0.662</td>
<td>0.657</td>
</tr>
<tr>
<td>$J_2=25 \times 10^6$</td>
<td>66.63</td>
<td>66.54</td>
<td>66.21</td>
<td>65.78</td>
<td>84.21</td>
</tr>
<tr>
<td>A/m$^2$</td>
<td>Nonlinear</td>
<td>8.20</td>
<td>8.15</td>
<td>8.06</td>
<td>10.92</td>
</tr>
</tbody>
</table>


FIG. 2. Mechanical deformation in linear and nonlinear magnetic materials by using each different method is shown after the test body is enlarged 400 times of its original size. In linear and nonlinear magnetic materials, the deformations obtained from using the local force density methods in linear and nonlinear materials (unit: μm); whereas, using the CM it was different from others as shown in (a); whereas, using the CM it was different from others as shown in (b). These four figures were obtained by using $J_1=2.5 \times 10^6$ A/m$^2$ and $J_2=25 \times 10^6$ A/m$^2$ in the KV and CM, respectively. Here, the nonlinear magnetic material (silicon steel) was employed in these four figures, and units are in the micron scale.

TABLE I. Maximum mechanical deformation values for linear ($\mu_s=2175$) and nonlinear magnetic materials (unit: μm). Refer to Fig. 1, silicon steel RM50 $B$-$H$ curves for operating point of maximum $H$ field at $J_1=2.5 \times 10^6$ (A/m$^2$) in the linear region (OP1) and $J_2=25 \times 10^6$ (A/m$^2$) in the saturation region (OP2).