Design of Microfabricated Inductors

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Abstract—Possible configurations for microfabricated inductors are considered. Inductance can be set by adjusting permeability through control of anisotropy of a permalloy core or via a patterned quasi-distributed gap. A design methodology based on a simple model is proposed. A more accurate model and a numerical optimization are also developed. Design examples for 5- and 10-MHz buck converters and 2.5-MHz resonant converter applications are presented.

Index Terms—Anisotropy, application, automatic design, buck converter, code, coil fabrication process, computer program, control of permeability, design, design example, design methodology, distributed gap, eddy currents, efficiency, end turns, fabrication process, hard-baked photoresist, high-frequency power inductors, hysteresis losses, inductance adjustment, inductor geometries, inductors, loss analysis, magnetic thin films, microfabricated inductors, microfabricated inductors design, multilayer core, multiturn windings, numerical simulation, optimization, permalloy, planar inductors, power density, quasi-distributed gap, resonant converter, secondary effects, SEM pictures.

I. INTRODUCTION

RECENT advances in microfabrication of transformers, using thin-film magnetic materials, show much promise for miniaturization of power converters [1]–[10]. Microfabrication techniques can produce fine patterning and thin films, which are advantageous for the control of eddy-current losses. This allows the use of magnetic metal alloys at frequencies in the range of 2–20 MHz. These materials can have high usable flux density and low-hysteresis loss [8]. Although some inductors have been built using similar techniques [11]–[20], many have not been designed for power applications. Through design and optimization specifically for these applications, higher efficiencies and power densities can be achieved.

In this paper, various geometries and fabrication methods for inductors are considered. Design calculations and optimizations for one configuration are developed. Specific results for example designs are presented.

II. INDUCTOR CONFIGURATIONS AND GEOMETRIES

The designer of a magnetic component with a magnetic core, fabricated by deposition of metal or other films on a substrate, faces a basic choice between depositing two layers of magnetic material with a conductor in between, or depositing two layers of conductor with a magnetic core in between. A device that uses two layers of conductor requires low resistance via contacts, and does not allow optimal use of an anisotropic magnetic material. As discussed in more detail in [8] and [21], a configuration using two layers of magnetic material above and below a conductor is preferred for these reasons, and because it generally allows higher power density. This geometry has been applied in [9] and [20].

A high-frequency inductor with substantial ac current requires careful design to avoid high-ac conduction losses. When a material with appropriate permeability is not available, high-permeability materials are generally used, and most designs will require increasing the overall reluctance of the magnetic path by introducing a gap. An air gap can adversely affect the field distribution, causing eddy currents, particularly with planar conductors and multiturn windings.

A series of fine gaps could be used to form a “quasi-distributed gap” to approximate a low-permeability material [8], [22], [23]. However, the scale of patterning that would be required for a typical design, on the order of a few microns, is difficult to achieve with a multilayer core [9].

Discrete gaps would be more easily placed at the “magnetic vias” where the top and bottom core materials connect. This leads to a large vertical field in the winding space, and problems with ac losses in the conductor. Turns that are wide compared to a skin depth, especially in multiturn designs, become problematic. Designs that use single narrow turns, such as in the “meander coil,” are preferred [9], [17], [24].

Perhaps the most elegant solution to the gap problem is the use of a low-permeability magnetic material to act as a distributed gap across the top and the bottom of the conductors, as shown in Fig. 1. In this case, the field lines are nearly horizontal in the winding space, and the ac resistance effects are determined by the height of the conductor, not its width. Additionally, the number of turns does not affect ac resistance as long as the turns accumulate horizontally, rather than vertically [8], [25]. If the permeability required for a distributed gap is achievable, the distributed gap design...
is preferred. A way to control permeability in anisotropic permalloy is presented in Section III. An approximation of the distributed gap design can be fabricated as shown in Figs. 2 and 3 using a process similar to that presented in [9] and [10], but with a modified coil fabrication process. In [9] and [10], a photoresist mold is used to insulate the turns. But the thickness obtainable with such a mold is limited in practice. For thicker coils, the following process could instead be used. A thin layer of chrome (7 nm) and a seed layer of copper (200 nm) are evaporated over a 5-μm layer of insulation photoresist. The copper seed layer is patterned, but the chrome layer is not patterned. The coil is then deposited by electroplating in a copper sulfate solution. The copper does not grow over the unpatterned chrome layer and a mold is not necessary.

Schematic sections of the electroplated turns are shown in Figs. 14 and 15. Finally, the chrome layer can be removed with a sputter-etch process. Figs. 3 and 4 show the coil over the lower part of the core after the plating and the sputter-etching process.

The magnetic path could be closed by a lid applied on the top and built on a second silicon wafer [10]. The core laminations can be sputtered over bumps of hard-baked photoresist (Fig. 5). Such bumps allow complete closure of the magnetic path when the lid is applied.

Finite-element simulations [26] of the distributed gap geometry in Fig. 2 have been used to predict the value of the loss for a design example at the operating frequency $f = 5$ MHz (see Fig. 6). From the simulation, the ac resistance factor for a 5-MHz sinusoidal waveform, assuming a lossless core, was $F_r = 1.8$. From a one-dimensional (1-D) analysis as in Section IV, a factor of $F_r = 1.05$ would have been expected. The difference can be explained by the reluctance of the side
The posits that permeability can be controlled through the application of a dc magnetic field in the easy-axis direction, while the main flux path is chosen along the nonhysteretic hard-axis direction. To control the eddy-current loss, a laminated core is deposited as a multilayer film. Such loss is estimated during deposition or by other means would be even better.

Anisotropic materials such as permalloy (NiFe alloy) allow a given permeability to be achieved in several different ways. A particular material or alloy may be selected to meet the requirements of a given design. Since this might require a new magnetic material deposition process for each design, a more practical approach would be to develop processes for a limited set of materials giving a range of permeabilities, and then to adapt a design to match one of the available materials. A single material in which the permeability could be varied during deposition or by other means would be even better.

Anisotropic materials such as permalloy (NiFe alloy) allow the possibility of controlling permeability through the application of a dc magnetic field in the easy-axis direction, while the inductor operates with the main flux path in the hard-axis direction. The applied field acts to increase the anisotropy energy, decreasing the permeability while maintaining the low-hysteresis loss and high-saturation flux density characteristics of the material. Using an applied field of 1800 A/m, control of permeability down to one eighteenth of the zero-field permeability has been demonstrated, as shown in Fig. 8.

This has been proposed as a way to make devices with variable inductance, controlled by the applied field [11]. By applying a fixed field strength with a permanent magnet, it is possible, in principle, to use this as a method to set the permeability at the desired value for a given design.

IV. Design Based on a Simplified Model

A design methodology is presented for a distributed or quasi-distributed gap inductor, as in Fig. 2, to be used in a power converter circuit. A pulsedwidth modulation (PWM) buck converter [27] is chosen as an illustrative example, but the calculations could be adapted for other converter topologies as shown in Appendix III. The optimization, detailed in Appendix I, follows a procedure similar to that developed for a transformer design in [8].

A. Definition of the Simplified Model

In a first analysis the end turns, the lateral width $S_{btu}$ needed to close the core, and the lateral separation $S_t$ between turns have been neglected (see Fig. 2). A more accurate model will be presented in Section V to account for the effects of these “nonactive” spaces.

First, the losses and power handling per unit area are calculated. Appendix I-A contains details on these. The field in the window area is assumed horizontal. The ac losses in the windings can then be estimated by a 1-D analysis [28] and depend only on the ratio between the height of the conductor $h_c$ and the skin depth $\delta_c$, even for multiple turns. This is described by an ac resistance factor $F_r(h_c/\delta_c) = R_{ac}/R_{dc}$.

We calculate a Fourier representation for the current waveform, and we estimate the $F_{rk}$ factors for every significant harmonic $k$ as in [29].

If anisotropic NiFe alloy is used for the magnetic core, the main flux path can be chosen along the nonhysteretic hard-axis direction [8]. To control the eddy-current loss, a laminated core is deposited as a multilayer film. Such loss is estimated for each layer and for each significant harmonic of the flux.
density waveform and added together. For this estimation, the flux density is assumed parallel to the layers.

\section*{B. Core Optimization Based on the Simplified Model}

Design specifications referring to the buck-converter application can be chosen as: input voltage \( V_{\text{in}} \), output voltage \( V_{O} \), dc, peak-to-peak ripple output current \( I_{Kc}, r = \Delta I_{pp}/I_{Kc} \), and switching frequency \( \omega = 2\pi f \). The optimization calculations are reported in Appendix I-B. The resulting tradeoff between power density and efficiency \( \eta \) is shown here.

According to (9) in Appendix I-A, the power loss in the winding can be reduced by an increase in height of the conductor \( h_c \). The improvement, however, is negligible for conductors thicker than two skin depths. For this first-order analysis, \( h_c \) could be chosen as about one to two skin depths. Consideration of the neglected “nonactive” areas allows a more accurate optimization of \( h_c \) as shown in Section V. The power loss in the core, according to (11) in Appendix I-A, can be made almost negligible by an increasing number of laminations \( N \). Consideration of the fabrication costs would be needed to optimize \( N \). We assume here a given number of laminations. The height of the core can then be adjusted for maximum power density as shown in [8], yielding (e.g., for this buck-converter application) the expression

\[
P_{\text{out,opt}} = \frac{2\omega^{5}B_{pk}^{2}N^{4}h_{c}^{5}}{5^{5}5^{6}(1-D)^{8}K_{\text{wind}}^{2}K_{\text{core}}^{2}}[1-\eta^{2}] \tag{1}
\]

where \( A \) is the “active” device area, \( \rho_{s} \) and \( \rho_{c} \) are the respective resistivities of the core and of the conductor, \( D \) is the duty cycle of the converter, \( K_{\text{wind}} \) is a factor accounting for the ac loss in the windings as defined in (9) of Appendix I-A, \( K_{\text{core}} \) is a factor accounting for the harmonic loss in the core as defined by (11) in Appendix I-A, and \( [a_{1}] = 2 \sin(D\pi)/[\pi^{2}D(1-D)] \) is the first Fourier coefficient of the current waveform as defined by (6) in Appendix I-A. Variable \( B_{pk} \) is one half the peak-to-peak value of the ac flux density. For an optimized design, the peak of the total flux density should be close to (or at) the saturation level \( B_{\text{sat}} \). Hence, we choose \( B_{pk} = B_{\text{sat}}/(1+2/r) \) such that \( B_{\text{sat}} + B_{pk} = B_{\text{sat}} \). Expression (1) for the maximum power density as a function of the given efficiency is plotted in Fig. 9. Parameters in Table I have been assumed. End turns and the other “nonactive” areas have been neglected.

This produces a favorable field configuration, and avoids introducing the inductance constraint in the optimization process.

The effective permeability required to produce the desired inductance for the optimized design is calculated in Appendix I-C and is reported here

\[
\mu_{\text{eff}}(\eta) = \frac{2}{\mu_{\text{o}} \sigma_{\text{eff}}(\eta)} \left( \frac{B_{\text{sat}}}{1+\frac{r}{2}} \right) \tag{2}
\]

where \( \sigma_{\text{eff}}(\eta) \) is the current density per unit width of conductor at the efficiency \( \eta \) [Appendix I-C, eq. (23)]. For an optimal design, choosing the efficiency \( \eta \) completely specifies the permeability \( \mu_{\text{eff}} \).

As an example, assuming the parameters in Table I and neglecting end-turn and the other “nonactive” spaces, designs in the range 95.5\% < \( \eta < 98.5\% \) are possible for values of relative permeability in the range 100 < \( \mu_{r} < 4000 \), as shown in Fig. 10. Practical designs generally require, for a given efficiency, higher permeabilities than those shown in Fig. 10. This is because the spaces to close the core and to insulate the turns, neglected in this analysis, increase the length of the magnetic path (see Fig. 2).

\section*{V. DESIGN BASED ON A MORE ACCURATE MODEL}

In this section, a model and a numerical optimization are developed to account also for end turns and “nonactive” spaces needed to insulate turns and close the core (see \( S_t \) and \( S_{\text{tot}} \) in Figs. 2 and 15).

\subsection*{A. Height of the Conductor and Number of Turns}

The analysis and design optimization presented in Section IV cannot be used to determine the optimal height of conductor \( h_c \) and the number of turns \( n \). As \( h_c \) is increased up to two skin depths, both ac and dc resistances decrease. Beyond this point, the improvement in ac resistance is small. With sufficient thickness, the dc loss can be made negligible in relation to ac loss. For higher values of \( h_c \) there will not

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig9.png}
\caption{Power density versus power loss percentage for a fixed number of laminations \( N = 12 \). Logarithmic scales are used for both axes. Parameters in Table I have been assumed. End turns and the other “nonactive” areas have been neglected.}
\end{figure}
be significant advantages because only the dc losses, which are already negligible, will be reduced.

When “nonactive” spaces are also taken into account, as $h_c$ increases, the lateral width $S_t$ required to separate the turns and the lateral width $S_{lat}$ required to close the core, assuming fixed slopes, will eventually become substantial (Fig. 15). This effectively reduces the power density. Thus, the selection of conductor height $h_c$ is a trade off between reducing resistance with a thicker conductor, or minimizing area by reducing $S_{lat}$ and $S_t$ with a thinner conductor.

In a first-order analysis as shown in Section IV, the number of turns $n$ does not affect the performance of the device. If we consider end turns and “nonactive” spaces, when $n$ is too small much space is used to laterally close the core. When $n$ is too large, much space is wasted in the end turns. An optimal value exists between these two extremes.

### B. Refining of the Model

Unitless factors refine the model capturing the effects of the spaces $S_t$ and $S_{lat}$ as well as the effects of the end turns. The formulas for the simplified model presented in Appendix I-A are modified only by multiplicative coefficients as shown in detail in Appendix II. The power loss in the end turns is captured by the factor $R_{end}$ such that $R_{end} = R_{end}$. The factor $K_s$ is defined such that $W_{s,tot} = W_s K_s$, where $W_s$ is the length of the core (Fig. 2) and $W_{s,tot}$ is the total length of the device including end turns. The quantity $W_{click}$ represents half of the total width (Fig. 2) and is given by the expression $W_{click} = n W_t K_c$, where $n$ is the number of turns, $W_t$ is the width of each turn, and $K_c$ accounts for the nonactive width needed to close the core and to insulate the turns.

For some designs, where $W_t$ is much larger than the height $h_c$, these factors are close to unity, reducing the model to that presented in Section IV. However, for other designs, consideration of the factors $K_{end}$, $K_s$, and $K_c$ may be necessary to achieve an optimal design. For example, for low-output-current designs, the areas needed to close the core and to separate the turns become significant compared to the active area occupied by the conductor. In these cases, the simplified model does not describe the device accurately, and optimization based on the complete model is necessary. Moreover, the same argument shows that low-current designs generally have lower power density than higher current ones.
If an efficiency of 94% is chosen for the design, a throughput power density of 10.6 W/cm² is calculated. The three main parameters characterizing this design as found by the program are: core height \( h_c = 12.6 \mu m \), number of turns \( n = 3 \), and conductor height \( h_s = 54 \mu m \). All the other parameters of the device can be calculated from these three using [Appendix II, eqs. (50)–(58), (60), (61), and (64)]. Table I collects all the specifications and parameters of this design.

VI. POSSIBLE IMPROVEMENTS OF THE DESIGN

One of the main parameters is the specified switching frequency of the converter. When the frequency is increased, the flux carrying requirement decreases. The device can then be made much smaller, while thinner core layers control the increase of the eddy-current losses. An optimization has been performed using a higher switching frequency: \( f = 10 \) MHz. The upper bound of 16 \( \mu m \) to the height of the core has also been removed. Fig. 12 shows the “power density versus efficiency” curve resulting from this optimization. A complete design is reported as an example in Table II where an efficiency of 94% allows a power density of 25.3 W/cm².

The design methodology presented in this paper can also be applied to other topologies of converters. As an example, a design procedure is detailed in Appendix III for designs of inductors to be used in resonant converters. An example design for a 2.5-MHz resonant converter is presented in Table III.

VII. CONCLUSIONS

A methodology for the design of microfabricated planar inductors to be used in power conversion circuits has been presented. Availability of low-permeability magnetic materials is desirable for high-performance designs. Permeability of anisotropic materials such as permalloy can be controlled applying a dc magnetic field in the easy-axis direction. A design tradeoff between power density and efficiency exists and a method to calculate it and plot it is given. An example design for a 5-MHz buck converter shows that a power density of 10.6 W/cm² is theoretically possible with an efficiency of 94%. If the frequency is increased to 10 MHz, a power density of 25.3 W/cm² is calculated for the same efficiency. The design methodology can be applied to other converter topologies. As an example, the design of an inductor for a resonant converter has been developed.

APPENDIX I

SIMPLIFIED MODEL ANALYSIS AND OPTIMIZATION

A. Definition of the Model and Loss Analysis

The meaning of the terminology can be found in Tables I or II and in Figs. 2, 15, and 16. The end turns, the space to insulate the conductors \( S_e \) and space to close the core \( S_{BR} \), will be neglected in this analysis. Given these assumptions, the “active” area is

\[
A = 2nW_dW_s. \tag{3}
\]

The current waveform is assumed triangular as in Fig. 13.
We represent such waveform using a Fourier series

\[ i(t) = I_{dc} + \sum_{k=1}^{\infty} I_k \sin \left( k \frac{2\pi}{T} t \right) \]

where \( I_k \) is the amplitude of the \( k \)th harmonic

\[ I_k = -\frac{\Delta I_{pp}}{2} \frac{2 \sin(D\pi k)}{(\pi k)^2 D(1-D)} \left( \frac{r I_{dc}}{2} \right) a_k \]

which we refer to the dc component \( I_{dc} \), introducing \( r = \Delta I_{pp}/I_{dc} \) the ripple factor, and the Fourier coefficients

\[ a_k = -\frac{2 \sin(D\pi k)}{(\pi k)^2 D(1-D)}. \]

We neglect harmonics higher then \( K_{max} = 6 \) to approximate the band-limited waveform of an actual converter.

![Fig. 12. Maximum power density and required permeability versus efficiency when the switching frequency is increased to \( f \approx 10 \) MHz. Specifications and technology parameters have been assumed from the example design in Table II. The number of laminations is fixed \( N = 12 \). The optimal number of turns \( n \), heights of the conductor \( h_c \), and core \( h_b \) are also shown.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>Frequency</td>
</tr>
<tr>
<td>( I_{DC} )</td>
<td>Output current</td>
</tr>
<tr>
<td>( \Delta I_{pp} )</td>
<td>Current ripple</td>
</tr>
<tr>
<td>( V_{in} )</td>
<td>Input voltage</td>
</tr>
<tr>
<td>( V_o )</td>
<td>Output voltage</td>
</tr>
</tbody>
</table>

**MATERIAL DATA AND TECHNOLOGY PARAM.**

\| Symbol | Value |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>Number of core laminations</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>Conductor (Cu) resistivity</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>Core (80% NiFe) resistivity</td>
</tr>
<tr>
<td>( h_{sep} )</td>
<td>Vertical separation core-conductor</td>
</tr>
<tr>
<td>( W_{con} )</td>
<td>Width to contact the cores</td>
</tr>
<tr>
<td>( s_{Ni,Fe} )</td>
<td>s.t. NiFe wet etch width is ( s_{Ni,Fe} )</td>
</tr>
<tr>
<td>( s_{reap} )</td>
<td>Slope of the photoresist bumps</td>
</tr>
<tr>
<td>( s_{hua} )</td>
<td>Constant such that ( s_{hua} = s_{hua} )</td>
</tr>
</tbody>
</table>

**DEVICE PARAMETERS.**

\| Symbol | Value |
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>( L )</td>
<td>Inductance required</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>permeability required</td>
</tr>
<tr>
<td>( D )</td>
<td>Duty cycle</td>
</tr>
<tr>
<td>( h )</td>
<td>Total height of core</td>
</tr>
<tr>
<td>( \delta_s )</td>
<td>Skin depth for the core at 10MHz</td>
</tr>
<tr>
<td>( W_c )</td>
<td>Length of the core (see Fig. 2)</td>
</tr>
<tr>
<td>( R_{fr} )</td>
<td>Half of the flux density ripple</td>
</tr>
<tr>
<td>( a )</td>
<td>Current density</td>
</tr>
<tr>
<td>( S_L )</td>
<td>Equivalent width to separate each turn (if rectangular section)</td>
</tr>
<tr>
<td>( W_i )</td>
<td>Equivalent width of a single turn</td>
</tr>
<tr>
<td>( h_c )</td>
<td>Height of conductor</td>
</tr>
<tr>
<td>( \delta_c )</td>
<td>Skin depth of the conductor at 10MHz</td>
</tr>
<tr>
<td>( S_{lat} )</td>
<td>Lateral width to close the core</td>
</tr>
<tr>
<td>( r )</td>
<td>Number of turns</td>
</tr>
<tr>
<td>( K_{res} )</td>
<td>Resistance factor for end turns loss</td>
</tr>
<tr>
<td>( K_L )</td>
<td>Length factor due to end turns</td>
</tr>
<tr>
<td>( K_c )</td>
<td>Width factor due to ( S_L ) and ( S_{lat} )</td>
</tr>
</tbody>
</table>

**CALCULATED PERFORMANCE.**

\| Symbol | Value |
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>( t_{sat} )</td>
<td>current to saturate the core</td>
</tr>
<tr>
<td>( R_{DC} )</td>
<td>DC resistance</td>
</tr>
<tr>
<td>( F_{AC} )</td>
<td>AC resist. factor at 5MHz from design</td>
</tr>
<tr>
<td>( K_{wind} )</td>
<td>Such that ( P_{wind} = K_{wind} R_{DC} P_{DC} )</td>
</tr>
<tr>
<td>( P_{wind} )</td>
<td>Power loss in the winding</td>
</tr>
<tr>
<td>( K_{core} )</td>
<td>Harmonic core loss factor</td>
</tr>
<tr>
<td>( P_{core} )</td>
<td>Power loss in the core</td>
</tr>
<tr>
<td>( W_c )</td>
<td>Total length</td>
</tr>
<tr>
<td>( \psi_{cum} )</td>
<td>Total width</td>
</tr>
<tr>
<td>( V_{out} )</td>
<td>Output power</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Power density</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Efficiency</td>
</tr>
</tbody>
</table>

The power loss in the winding is estimated using

\[ P_{wind} = R_{fr} \frac{K_{max}}{2} \sum_{k=1}^{K_{max}} F_{fr} R_{fr} \frac{P_{fr}^2}{2}. \]

Assuming a horizontal field in the winding area, the ac resistance factors \( F_{fr} \) can be estimated as in [28] and [29]

\[ F_{fr} = \psi_{fr} \left[ \frac{\sinh(2\psi_{fr}) + \sin(2\psi_{fr})}{\cosh(2\psi_{fr}) - \cos(2\psi_{fr})} + \frac{2(\psi_{fr}^2 - 1)}{3} \right]. \]

\[ \frac{\sinh(\psi_{fr}) - \sin(\psi_{fr})}{\cosh(\psi_{fr}) + \cos(\psi_{fr})} \]

\[ \psi_{fr} = \frac{\psi_{fr}}{\psi_{fr}} \]
the value \( p = 0.5 \) can be assumed. For quasi-distributed gap designs as in Fig. 7, we use the value \( p = 1 \).

Using (3), (5), and (7), the power loss in the winding per unit area is

\[
\frac{P_{\text{windless}}}{A} = \frac{R_{\text{k}} P_{\text{k}}}{A} \left[ 1 + \sum_{k=1}^{K_{\text{max}}} \frac{f_k T_k^2}{2 f_{\text{dc}}^2} \right]
\]

\[
= \frac{\rho_c 2nW_0}{W' h_c} \left[ \frac{f_k^2}{(2nW_0 W_s) \left[ 1 + \frac{1}{8} \sum_{k=1}^{K_{\text{max}}} F_{\text{y}} a_k^2 \right]} \right]
\]

\[
= K_{\text{windless}} \frac{\rho_c}{h_c} \sigma^2
\]

where \( \sigma = I_{\text{dc}}/W_{\text{dc}} \) is the dc current density per unit width of conductor and \( K_{\text{windless}} = [1 + (q^2/8) \sum_{k=1}^{K_{\text{max}}} F_{\text{y}} a_k^2] \) is the total ac factor defined such that \( \frac{P_{\text{windless}}}{K_{\text{windless}} R_{\text{k}} P_{\text{k}}} \).

To control eddy-current loss in the core, we divide the total height \( h_s \) into \( N \) laminations. The thickness of each lamination \( h_s/N \) is smaller than two skin depths. In this case, for a sinusoidal flux density of amplitude \( B \) and frequency \( \omega = 2\pi f \), the loss in one lamination due to eddy currents can be approximated as in [32]

\[
P_{\text{one lam}} = \frac{\omega^2 B^2 (2nW_0) W_s \left( h_s/N \right)^3}{24 \rho_s}.
\]

Assuming for the flux density \( B(t) \) the same waveform and Fourier representation of the current in (4) and assuming the thickness of each lamination is smaller than two skin depths for each considered harmonic, the eddy-current loss in the core per unit area is

\[
P_{\text{coreless}} \frac{A}{A} = 2N P_{\text{one lam}} \frac{A}{A} = \frac{2N}{2 n W_0 W_s} \sum_{k=1}^{K_{\text{max}}} \frac{\omega^2 B_k^2 (2nW_0) W_s \left( h_s/N \right)^3}{24 \rho_s N^3}
\]

\[
= \left( \frac{\omega^2 B_{\text{dc}}^2 13}{12 \rho_s N^2} \sum_{k=1}^{K_{\text{max}}} k^2 a_k^2 \right)
\]

\[
= \frac{\omega^2 B_{\text{dc}}^2 13}{12 \rho_s N^2} \frac{1}{12} \sum_k \frac{k^2 a_k^2}{a_k^2} K_{\text{core}}
\]

where \( B_{\text{dc}} = B_{\text{dc}} / a_k \) is the amplitude of the \( k \)th harmonic, \( B_{p k} = \Delta B_{\text{pp}} / 2 \) is half of the flux density ripple, \( a_k \) are the same coefficients defined by (6) used for the current waveform, and \( K_{\text{core}} = \sum_{k=1}^{K_{\text{max}}} k^2 a_k^2 / a_k^2 \) is a factor accounting for the loss in the core due to the harmonic contributions. If anisotropic material is used, as shown in Section III, the hysteresis losses in the hard-axis direction can be made negligible, and (11) is the total core loss per unit area.

The throughput power for a buck converter is

\[
P_{\text{out}} = \frac{V_o I_{\text{dc}}}{\Delta T_{\text{eff}}} = \frac{\Delta \lambda_{\text{pp}}}{\Delta T_{\text{eff}} I_{\text{dc}}}
\]

where the output voltage \( V_o \) has been expressed as a function of the ripple of the flux linkage \( \Delta \lambda_{\text{pp}} = 2n(2B_{\text{dc}}) W_s h_s \) and as a function of the off time \( \Delta T_{\text{eff}} = 2\pi (1 - D) / \omega \), where
\[ \frac{P_{\text{core}}}{A} = \frac{\Delta \lambda_{pp} L_c}{\Delta T_{\text{off}}(2\pi W_t W_s)} = \frac{\omega (2B_{pk})}{2\pi(1-D)} h_s \sigma. \] (13)

B. Optimization Based on Simplified Model

An efficiency objective is fixed and the throughput power density is optimized, as in [8]. The efficiency constraint, \( \eta = P_{\text{out}}/(P_{\text{out}} + P_{\text{loss}}) \), is imposed by

\[ \frac{1 - \eta}{\eta} \frac{P_{\text{out}}}{A} = \frac{P_{\text{coreless}}}{A} + \frac{P_{\text{windless}}}{A}. \] (14)

Substituting the expressions from the previous analysis

\[ \frac{1 - \eta}{\eta} \frac{\omega (2B_{pk})}{2\pi(1-D)} h_s \sigma \]
\[ = K_{\text{core}} \Delta^2 + K_{\text{wind}} \frac{P_c}{h_s} \sigma^2. \] (15)

To facilitate calculations, this can be rewritten as

\[ a \sigma^2 - b h_s \sigma + c h_s^2 = 0 \] (16)

where \( a = K_{\text{wind}}(\frac{P_c}{h_s}) \), \( b = -(1 - \eta) / (\eta(2B_{pk})/2\pi(1-D)) \), and \( c = K_{\text{core}} \Delta^2 / (\omega (2B_{pk})/2\pi(1-D))^2 \). We solve (16) for \( \sigma \) by choosing the largest root for largest power density

\[ \sigma = \frac{b}{2a} \left(1 + \sqrt{\frac{b^2}{4a} - \frac{c}{a}}\right) \] (17)

where \( d = 1 - \frac{4ac}{b^2} \). Using (13) and (17), the power density can now be expressed as a function of the variable \( d \)

\[ \frac{P_{\text{out}}}{A} = \left( \frac{\eta}{1 - \eta} \right) \frac{b^6}{32a^3 c^2} (1 - d)^2 \left(1 + \sqrt{d}\right). \] (18)

Theoretically, one optimizes for the best height of the core \( h_s \), but in practice calculations are easier if \( d \) is calculated first. Setting to zero the derivative with respect to \( d \) of the expression above, the optimal value \( d_{\text{opt}} = 1/25 \) is found. Using this value, the maximum power density is

\[ \frac{P_{\text{core, opt}}}{A} = \frac{c h_s^3}{64 \sigma c^2} (1 - d_{\text{opt}})^3. \] (19)

The core loss per unit area is

\[ \frac{P_{\text{coreless, opt}}}{A} = c h_s^3 = \frac{b^6}{64a^2 c^2} (1 - d_{\text{opt}})^3. \] (20)

The winding loss per unit area is

\[ \frac{P_{\text{windless, opt}}}{A} = a \sigma^2 \]
\[ = \frac{b^6}{64a^2 c^2} \left(1 + \sqrt{d_{\text{opt}}^2} \right)^2 (1 - d_{\text{opt}})^2. \] (21)

In the optimal design, the losses will then be divided between core and winding such that

\[ \frac{P_{\text{coreless, opt}}}{P_{\text{windless, opt}}} = \frac{(1 - d_{\text{opt}})^3}{(1 + \sqrt{d_{\text{opt}}^2})^2 (1 - d_{\text{opt}})^2} = \frac{2}{3}. \] (22)

C. Estimation of the Permeability Required to Produce the Desired Inductance

The current density \( \sigma = I_c / W_t \) can be calculated for any efficiency substituting for \( h_s = (1 - d)b^2/[4\sigma c] \) in (17), and using the optimal value \( d_{\text{opt}} = 1/25 \)

\[ \sigma_{\text{opt}} = \frac{12 \omega B_{pk} \rho_N h_s^2}{2\pi(1-D)^3} \left( 1 + \sqrt{25(1 - d_{\text{opt}})} \right) \left( \frac{1 - d}{d_{\text{opt}}} \right)^3. \] (23)

For a closed-core structure, with low reluctance via connections, such as the high-permeability materials in Fig. 1, this current density produces a field which can be calculated using Ampère’s law

\[ \oint_{\Sigma_{\text{dc}}} \mathbf{H} \cdot d\mathbf{l} = ni \] (24)

yielding

\[ H_{\text{dc}} = \frac{n I_c}{2\pi W_t} = \frac{\sigma_{\text{opt}}}{2}. \] (25)

The field gives a flux density

\[ B_{\text{dc}} = \frac{\mu_0 h_r}{1 + \frac{r}{2}} \sigma_{\text{opt}}. \] (26)

This flux density \( B_{\text{dc}} \) should be chosen in order to have the maximum value of the flux density correspond to the saturation level [8]

\[ B_{\text{dc}} + \Delta B_{pp} = B_{\text{sat}}. \] (27)

The desired flux density level \( B_{\text{dc}} \) is then

\[ B_{\text{dc}} = \frac{B_{\text{sat}}}{1 + \frac{r}{2}} \] (28)

where \( r = \Delta B_{pp} / I_c = \Delta B_{pp} / B_{\text{dc}} \) is the ripple of the current. To attain such a level, the permeability should be adjusted to

\[ \mu_r = \frac{2}{\mu_0 \sigma_{\text{opt}}} \left( \frac{B_{\text{sat}}}{1 + \frac{r}{2}} \right). \] (29)

APPENDIX II

MORE ACCURATE PROBLEM FORMULATION AND OPTIMIZATION

A. Definition of the More Accurate Model

The analysis refers to the configuration in Figs. 2 and 14–16. Several factors have been neglected in the simplified model presented in Appendix I-A: the width \( S_t \) to insulate adjacent turns, the width \( S_{\text{int}} \) required to close the core laterally, and the space occupied by the end turns and their effects on the total dc resistance of the windings. These factors will be included, refining the simplified model by means of multiplicative unitless factors.

The width \( S_t \) to insulate adjacent turns is calculated using

\[ S_t = S_{\text{int}} h_c \] (30)

where \( S_{\text{int}} \) is a constant dependent on the process. We assume that for small conductor heights \( h_c \leq 20 \mu m \), a hard-baked
photoresist mold is used to insulate the turns. Due to the photolithographic process, the minimum width of the mold is approximately proportional to its height. For larger conductor heights, a mold is not likely to be practical. In this case, a process as described in Section II can be used. A copper “seed” layer for an electroplating process is created over a thin chrome layer. During the electroplating process, the copper does not grow on the unpatterned chrome. Anyway, we observed that the copper extends laterally from its seed layer by the same amount of the final conductor height as shown in Fig. 14. The space to separate the turns can still be assumed proportional to their height. We use for the “no-mold process” a value \( s_{N\text{NiFe}} \) larger than the one used for the “mold process.” The turn width \( W_t \) and the separation width \( S_t \) are the equivalent widths for rectangular section conductors with the same area of those shown in Figs. 14 and 15.

The space \( S_{\text{bat}} \) includes the space to close laterally the core, the width \( W_{\text{con}} \) needed for the upper core to contact with the lower core, and the width required by a lamination etching process \( s_{\text{NiFe}}h_s \) (see Figs. 15 and 16)

\[
S_{\text{bat}} = s_{\text{res}}(h_c + h_{\text{sep}}) + W_{\text{con}} + s_{\text{NiFe}}h_s. \tag{31}
\]

As presented in Section II and in [10], the magnetic path is closed by a core processed on another silicon substrate. Hard-baked photoresist bumps produce the needed slope \( s_{\text{res}} \) to allow the upper core on the “lid wafer” to contact the lower core on the “coil wafer.” The total height of the bumps is given by the height of the conductor \( h_c \) plus the height \( h_{\text{sep}} \) to separate and insulate vertically the two cores from the coils. For a quasi-distributed gap design \( h_{\text{sep}} \) might be chosen larger than the minimum value needed to insulate the coils from the core. This can allow a favorable configuration of the field seen by the conductors [22] (Fig. 7). The width required by the lamination etching process is approximately proportional to the height of the core \( h_s \). The symbol \( s_{\text{NiFe}} \) is used to indicate the slope of the etched NiFe core.

The total dc resistance, including the end turns, can be estimated by the expression

\[
R_{\text{dc,tot}} = \frac{\rho_c}{W_t h_c} \left[ 2nW_s + 2(2S_{\text{bat}} - 2S_t)n + 2\pi \sum_{i=1}^{n} r_i \right] \tag{32}
\]

where \( r_i = W_t/2 + (W_t + S_t)(i - 1) + S_t \) is the radius of the \( i \)th turn (Fig. 16)

\[
R_{\text{dc,tot}} = \frac{\rho_c}{W_t h_c} 2nW_s K_{\text{end}} \tag{33}
\]

where

\[
K_{\text{end}} = 1 + \frac{4S_{\text{bat}} + [2\pi - 4 + \pi(n - 1)]S_t + \pi W_t n}{2W_s}
\]

is a unitless factor accounting for the additional resistance of the end turns.

The total length of the device, including the end turns, can be estimated by the expression

\[
W_{s,tot} = W_s + 2(W_t + S_t)n
= W_s \left[ 1 + \frac{2(W_t + S_t)n}{W_s} \right] = W_s K_s \tag{34}
\]

which defines the unitless factor \( K_s \). The total width of the device, including the space \( S_t \) to insulate adjacent turns, and the space \( S_{\text{bat}} \) to close the core laterally can be approximated by the expression

\[
2W_{c,tot} = 2[nW_t + nS_t + 2S_{\text{bat}}]
= 2nW_t \left[ 1 + \frac{nS_t + 2S_{\text{bat}}}{nW_t} \right]
= 2nW_t K_c \tag{35}
\]

which defines the unitless factor \( K_c \).

The total area occupied by the device can now be easily expressed as a function of these factors

\[
A_{\text{tot}} = W_{s,tot} 2W_{c,tot} = (W_s K_s)(2nW_t K_c) = AK_s K_c \tag{36}
\]

where \( A \) represents the “active” area as defined in the simplified model presented in Appendix I-A.

The width of the core is \( K_c \) times larger than in the simplified model, as the lateral width \( S_{\text{bat}} \) has been included. The power loss in the core due to eddy currents scales linearly with the width of the core. Hence

\[
P_{\text{coreloss,tot}} = P_{\text{coreloss}} K_c \tag{37}
\]
where $P_{\text{coreless}}$ is the core loss neglecting "nonactive" spaces as given by (11) in Appendix I-A. Thus, the total power loss per unit area is

$$\frac{P_{\text{coreless, tot}}}{A_{\text{tot}}} = \frac{P_{\text{coreless}} K_c}{AK_c K_s} = \frac{P_{\text{coreless}}}{A} \frac{1}{K_s K_c}. \quad (38)$$

As shown in (33), the end turns increase the resistance by a factor $K_{\text{end}}$. An approximation for the total power loss in the windings can then be written as

$$P_{\text{windless, tot}} = P_{\text{windless}} K_{\text{end}}. \quad (39)$$

The power loss in the winding per unit area is

$$\frac{P_{\text{windless, tot}}}{A_{\text{tot}}} = \frac{P_{\text{windless}}}{A} \frac{K_{\text{end}}}{K_s K_c}. \quad (40)$$

Finally, the increase of the area by a factor $K_c K_s$ decreases the throughput power density

$$\frac{P_{\text{out}}}{A_{\text{tot}}} = \frac{P_{\text{out}}}{A} \frac{1}{K_s K_c}. \quad (41)$$

The equation that determines the efficiency (14) in Appendix I-A can now be refined as

$$\frac{1 - \eta}{\eta} \frac{P_{\text{out}}}{A} = \frac{P_{\text{coreless}}}{A} \frac{1}{K_c} + \frac{P_{\text{windless}}}{A} \frac{K_{\text{end}}}{K_s K_c}. \quad (42)$$

Simplifying and rearranging the terms we obtain

$$1 - \frac{\eta}{\eta} \frac{P_{\text{out}}}{A} = \frac{P_{\text{coreless}}}{A} \frac{1}{K_c} + \frac{P_{\text{windless}}}{A} \frac{K_{\text{end}}}{K_s K_c} \quad (43)$$

where $P_{\text{out}}/A$, $P_{\text{coreless}}/A$, and $P_{\text{windless}}/A$ are the power throughput and the power loss components per unit area calculated from the simplified model, respectively.

The equation above shows an example of how the factors $K_c$ and $K_{\text{end}}$ can easily refine the equations of the simplified model to capture the effects of the neglected items. Substituting (9), (11), and (13), from Appendix I-A the equation can be expanded to

$$\rho_c K_{\text{wind}} K_{\text{end}} \sigma^2 - \frac{1}{\eta} \frac{\omega (2B_{pk}) h_s}{h_c} + \frac{\omega^2 B_{pk} h_c^2 a_k^2 K_{\text{core}} K_c}{12 \rho_s N^2} = 0. \quad (44)$$

The throughput power density to be maximized is

$$\frac{P_{\text{out}}}{A_{\text{tot}}} = \frac{\omega (2B_{pk}) h_s}{2 \pi (1 - D) K_c K_s} \sigma. \quad (45)$$

Finally, the specification on the flux capability requirement $\Delta \lambda_{pp}$ adds to the problem the equation

$$\Delta \lambda_{pp} = 2n(2B_{pk}) h_s W_s. \quad (46)$$

### B. Optimization Problem

The problem of finding the maximum power density for a given efficiency can be summarized and formulated mathematically as follows:

$$\max \frac{\omega (2B_{pk}) h_s}{2 \pi (1 - D) K_c K_s} \sigma \quad (47)$$

given the equality constraints defined explicitly or implicitly by (8), (9), (30), (31), (33), (35), (44), and (46) which we collect here

$$\rho_c K_{\text{wind}} K_{\text{end}} \sigma^2 - \frac{1}{\eta} \frac{\omega (2B_{pk}) h_s}{h_c} + \frac{\omega^2 B_{pk} h_c^2 a_k^2 K_{\text{core}} K_c}{12 \rho_s N^2} = 0 \quad (48)$$

$$\Delta \lambda_{pp} = 2n(2B_{pk}) h_s W_s \quad (49)$$

$$K_{\text{end}} = 1 + \frac{4 S_{\text{lat}} + [2 \pi + 4 + \pi (n - 1)] S_t}{2 W_s} \sigma \quad (50)$$

$$K_c = 1 + \frac{2 S_{\text{lat}}}{W_t} \frac{S_t}{n W_t} \quad (51)$$

$$W_t = \frac{I_{\text{out}}}{\sigma} \quad (52)$$

$$S_t = s_{\text{stax}} h_c \quad (53)$$

$$S_{\text{lat}} = S_{\text{res}} (h_c + h_{\text{ser}}) + W_{\text{con}} + S_{\text{NIKE}} h_s \quad (54)$$

$$K_{\text{wind}} = 1 + \frac{r^2}{8} \sum_{k=1}^{K_{\text{max}}} \frac{h_c}{\delta_k} \left[ \frac{\sinh \left( \frac{2 h_c}{\delta_k} \right)}{\cosh \left( \frac{2 h_c}{\delta_k} \right) - \cos \left( \frac{2 h_c}{\delta_k} \right)} + \frac{2(y^2 - 1)}{3} \sinh \left( \frac{h_c}{\delta_k} \right) - \sin \left( \frac{h_c}{\delta_k} \right) \cosh \left( \frac{h_c}{\delta_k} \right) + \cos \left( \frac{h_c}{\delta_k} \right) \right]. \quad (55)$$

We choose as specifications for the problem, the following five parameters referred to the converter behavior: the frequency $f$, the input and output voltages $V_{\text{in}}$ and $V_{\text{out}}$, and the dc and "peak-to-peak" current output $I_{\text{out}}$ and $\Delta I_{pp}$. Directly from the specifications, we calculate the current ripple $r = \Delta I_{pp}/I_{\text{out}}$, the duty cycle $D = V_{\text{out}}/V_{\text{in}}$, the "peak-to-peak" flux linkage ripple $\Delta \lambda_{pp} = V_{\text{out}} ((1 - D)/f)$, the conductor skin depth at each significant harmonic $k$: $\delta_k = \sqrt{k \rho_c / (\pi f h_s)}$, the Fourier coefficients $a_k = 2 \sin(D \pi k)/(\pi k^2 D (1 - D))$ and the harmonic core loss factor $K_{\text{core}} = \sum_{k=1}^{K_{\text{max}}} k^2 a_k^2/\delta_k^2$.

The problem as written above, presents 8 equations and 11 unknowns: $n$, $h_c$, $h_s$, $W_s$, $\sigma$, $W_t$, $S_t$, $S_{\text{lat}}$, $K_{\text{end}}$, $K_c$, and $K_{\text{wind}}$. Hence, three independent unknowns can be used to maximize the throughput power density.

### C. Optimization Procedure

A convenient parameterization is represented by the three unknowns ($n$, $h_c$, $h_s$). We calculate the throughput power...
density as a function of only these three variables using the following steps.

- Calculate $S_t$, $S_{lut}$, $K_{wink}$, $K_s$, $a$, $b$, and $c$ using (34), (49), and (53)–(55) in Appendix II-B and (16) in Appendix I-B.
- Calculate the quantities

\[
\begin{align*}
  k_1 & = 1 + [4S_{lut} + (\pi - 4 + \pi n)S_t] \frac{2nB_{pl}h_s}{\Delta\lambda_{yp}}, \\
  k_2 & = \pi I_{ext}n \frac{2nB_{pl}h_s}{\Delta\lambda_{yp}}, \\
  k_3 & = \frac{nS_t + 2S_{lut}}{nL_{out}}.
\end{align*}
\]

defined such that $K_{end} = k_1 + k_2/\sigma$ and $K_c = 1 + k_3/\sigma$. Substituting such quantities, (49) and (52) in (48), we obtain the quadratic equation

\[
(a k_1^2)\sigma^2 + (c h_3^2 k_3 + a k_2 - bh_s)\sigma + (c h_3^2) = 0. 
\]  

- Calculate the current density $\sigma$ given by the larger solution of (59) for larger power density

\[
\sigma = \frac{-c h_3^2 k_3 + a k_2 - bh_s}{2a k_1^2} \left[ 1 + \sqrt{1 - \frac{4a k_3 c h_3^2}{(c h_3^2 k_3 + a k_2 - bh_s)^2}} \right].
\]  

- Finally, the throughput power density using (47) is

\[
\frac{P_{out}}{A_{tot}} (n, h_c, h_s) = \frac{\omega(2B_{pl})h_s}{2\pi(1 - D)(1 + k_3/\sigma)k_s} \sigma.
\]  

We summarize our optimization procedure with the following steps implemented in our Matlab program [30]. For the meaning of the symbols, refer to Table I.

1) Read specs ($V_{in}, V_{out}, I_{dc}, \Delta i_{yp}, f$).
2) Read technology parameters ($h_{mic}, p, B_{sat}, \rho_c, \rho_s, h_{sep}, W_{con}, \gamma_{m}, s_{res}, s_{in}$).
3) Calculate $(r, D_s, \Delta v_n, L, q_k, a_k, K_{core})$.
4) Fix $N$ the number of laminations.
5) Fix $\eta$ the efficiency.
6) Using a numerical function optimizer, find the optimal $(n, h_c, h_s)$ for max power density ($P_{out}/A_{tot}$).

A three-dimensional (3-D) plot of the throughput power density as a function of two of the three main parameters $h_c$ and $h_s$ can be obtained if the third one $n$ is fixed (Fig. 17). Such a plot shows a well-defined maximum. Fig. 18 shows such maxima for different number of turns $n$. The largest value of power density in Fig. 18 corresponds to the optimal design.

The entire optimization presented so far can be repeated for different values of the efficiency in order to obtain the fundamental curve in Fig. 11 showing the maximum throughput power density achievable at any chosen efficiency. Fig. 11 shows also the values of the three parameters $h_c$, $h_s$, and $n$ needed to achieve the optimal design. In that example, $h_s$ has been limited to a maximum of 16 $\mu$m.

\[\text{Fig. 17. Power density for a given efficiency } \eta = 94\% \text{ and a fixed number of laminations } N = 12. \text{ The number of turns is also fixed } n = 3. \text{ Specifications and technology parameters have been assumed from the example design in Table I.}\]

\[\text{Fig. 18. The power density maximized with respect to } h_c \text{ and } h_s \text{ is here shown for different number of turns } n. \text{ Specifications and technology parameters have been assumed from the example design in Table I. The efficiency is chosen } \eta = 94\%, \text{ and the number of turns is fixed } N = 12.\]

D. Permeability Calculations for the More Accurate Model

The permeability required to produce the desired inductance can be calculated following the procedure in Appendix I-C. The magnetic field produced in the core by the current density $\sigma$ [Appendix III-C, eq. (60)] is

\[
H_{dc} = \frac{n L_s}{2n W_l K_c} = \frac{\sigma}{2K_c}
\]  

which gives a flux density

\[
B_{dc} = \mu_0 \mu_r \frac{\sigma}{2K_c}.
\]

The permeability should then be selected so that

\[
\mu_r(\eta) = \frac{2K_c}{\mu_0} B_{dc} = \frac{2K_c}{\mu_0} \left( \frac{B_{sat}}{1 + \frac{1}{2}} \right).
\]

It can be observed that with respect to the simplified model a permeability $K_c$ times higher is required when also “nonactive” spaces are included in the calculations.
APPENDIX III

INDUCTOR DESIGN FOR A RESONANT CONVERTER

The design methodology can be applied to inductors used in other circuit topologies. As an example an optimization procedure is presented for an inductor to be used in a resonant converter. The frequency $\omega$, the desired inductance $L$, and the rms value of the current $I_{\text{rms}}$ are assumed as specification parameters. The resulting trade off existing between quality factor $Q$ and power density is shown.

The analysis refers to the same model described in Appendix II-A and the configuration in Fig. 2. The resistance of the windings at the operative frequency, including the end turns, can be estimated by the expression

$$R_{\text{wink}} = \frac{\rho L_{\text{in}}}{W_t h_c} k_{e\text{nd}}.$$  \hspace{1cm} (65)

The power loss in a laminated core for a sinusoidal flux density is given by

$$P_{\text{coreloss}} = \frac{\omega^2 B_{\text{sat}}^2 h_s^3}{24 \rho_s N^2} 2\pi W_t W_s K_c.$$  \hspace{1cm} (66)

The total power loss in the core, assuming negligible hysteresis loss, can be modeled by the equivalent resistance

$$R_{\text{core}} = \frac{\omega^2 B_{\text{sat}}^2 h_s^3}{24 \rho_s N^2} \frac{2\pi W_t W_s K_c}{P_{\text{rms}}}.$$  \hspace{1cm} (67)

The value of the quality factor $Q$ is determined by the total resistance $R_{\text{tot}} = R_{\text{wink}} + R_{\text{core}}$

$$Q = \frac{\omega L}{R_{\text{tot}}}.$$  \hspace{1cm} (68)

The peak flux $\lambda_{pk}$ is set to saturation level imposing

$$\lambda_{pk} = LI_{pk} = 2\pi W_s h_s B_{\text{sat}}.$$  \hspace{1cm} (69)

where $I_{pk} = \sqrt{2} I_{\text{rms}}$ is the current peak. The inductance requirement, similarly to Appendix II-D, is satisfied by adjusting the permeability of the core $\mu_r$ such that

$$B_{\text{sat}} = \mu_0 \mu_r \frac{I_{pk}}{2 W_t K_c}.$$  \hspace{1cm} (70)

Finally, the goal is maximizing the power density. We use as a parameter for the power handling of the device the “volt–ampere” product $V_{\text{rms}} I_{\text{rms}}$, where $V_{\text{rms}} = \omega L I_{\text{rms}}$ is the rms voltage. The objective of the optimization procedure is then

$$\max_{\text{Area}} \frac{\text{Power}}{\text{Area}} \Rightarrow \max \frac{V_{\text{rms}} I_{\text{rms}}}{(2\pi W_t K_c)(W_s h_s)}.$$  \hspace{1cm} (71)

A. Optimization Problem for the Resonant Inductor

The problem of finding the maximum power density for a given quality factor $Q$ can be summarized and formulated mathematically as follows:

$$\max \frac{V_{\text{rms}} I_{\text{rms}}}{(2\pi W_t K_c)(W_s h_s)}.$$  \hspace{1cm} (72)

The design methodology can be applied to inductors used in other circuit topologies. As an example an optimization procedure is presented for an inductor to be used in a resonant converter. The frequency $\omega$, the desired inductance $L$, and the rms value of the current $I_{\text{rms}}$ are assumed as specification parameters. The resulting trade off existing between quality factor $Q$ and power density is shown.

The analysis refers to the same model described in Appendix II-A and the configuration in Fig. 2. The resistance of the windings at the operative frequency, including the end turns, can be estimated by the expression

$$R_{\text{wink}} = \frac{\rho L_{\text{in}}}{W_t h_c} k_{e\text{nd}}.$$  \hspace{1cm} (65)

The power loss in a laminated core for a sinusoidal flux density is given by

$$P_{\text{coreloss}} = \frac{\omega^2 B_{\text{sat}}^2 h_s^3}{24 \rho_s N^2} 2\pi W_t W_s K_c.$$  \hspace{1cm} (66)

The total power loss in the core, assuming negligible hysteresis loss, can be modeled by the equivalent resistance

$$R_{\text{core}} = \frac{\omega^2 B_{\text{sat}}^2 h_s^3}{24 \rho_s N^2} \frac{2\pi W_t W_s K_c}{P_{\text{rms}}}.$$  \hspace{1cm} (67)

The value of the quality factor $Q$ is determined by the total resistance $R_{\text{tot}} = R_{\text{wink}} + R_{\text{core}}$

$$Q = \frac{\omega L}{R_{\text{tot}}}.$$  \hspace{1cm} (68)

The peak flux $\lambda_{pk}$ is set to saturation level imposing

$$\lambda_{pk} = LI_{pk} = 2\pi W_s h_s B_{\text{sat}}.$$  \hspace{1cm} (69)

where $I_{pk} = \sqrt{2} I_{\text{rms}}$ is the current peak. The inductance requirement, similarly to Appendix II-D, is satisfied by adjusting the permeability of the core $\mu_r$ such that

$$B_{\text{sat}} = \mu_0 \mu_r \frac{I_{pk}}{2 W_t K_c}.$$  \hspace{1cm} (70)

Finally, the goal is maximizing the power density. We use as a parameter for the power handling of the device the “volt–ampere” product $V_{\text{rms}} I_{\text{rms}}$, where $V_{\text{rms}} = \omega L I_{\text{rms}}$ is the rms voltage. The objective of the optimization procedure is then

$$\max_{\text{Area}} \frac{\text{Power}}{\text{Area}} \Rightarrow \max \frac{V_{\text{rms}} I_{\text{rms}}}{(2\pi W_t K_c)(W_s h_s)}.$$  \hspace{1cm} (71)

A. Optimization Problem for the Resonant Inductor

The problem of finding the maximum power density for a given quality factor $Q$ can be summarized and formulated mathematically as follows:

$$\max \frac{V_{\text{rms}} I_{\text{rms}}}{(2\pi W_t K_c)(W_s h_s)}.$$  \hspace{1cm} (72)

Fig. 19. Maximum power density and required permeability versus quality factor. Specifications and technology parameters have been assumed from the example design in Table III. The height of conductor $h_c$, height of the core $h_s$, and number of turns $n$ are also shown. In this example, $h_s$ has been limited to values not larger then 16 $\mu$m.
unknowns \((n, h_c, \text{and } h_s)\). Given these three variables the power density can be calculated in the following ways.

- \(S_t\) and \(S_{\text{bat}}\) are first evaluated using (30) and (31) in Appendix II-A.
- Using the saturation equation (74), we can calculate \(W_s\)

\[
W_s = \frac{L_i^{pk}}{2B_{\text{sat}}n h_s}.
\]  

(75)

- Rearranging the quality factor equation (73), we see a quadratic form \(a W^2 - \beta W + c = 0\), where

\[
a' = \frac{\omega^2 B_{\text{sat}} L}{12 \rho_s I_{pk}} \left( \frac{h_s}{N} \right)^2,
\]

\[
\beta' = \frac{\omega L}{2 \rho_c F_r} n^2 n - a \frac{(n - 1) S_t + 2 S_{\text{bat}}}{n},
\]

\[
c' = 2 \frac{\rho_c F_r}{h_c} n (2 W_s + 4 S_{\text{bat}} + \pi n S_t),
\]

(76)

(77)

(78)

- Solving for \(W_t\) and using the smaller solution for larger power density

\[
W_t = \frac{2c'}{\beta' + \sqrt{\beta'^2 - 4 a' c'}}.
\]

(79)

- \(K_s\) and \(K_c\) can then be easily calculated using (34) and (35) in Appendix II.
- Finally, the area (36) and the power density (71) can be evaluated.

We summarize the design procedure used in our Matlab program [30] with the following steps. Refer to Table III for the meaning of the symbols.

1. Read specs \((L, I_{\text{ms}}, \omega)\).
2. Read technology parameters \((h_s, \text{max}, \rho_s, B_{\text{sat}}, \rho_c, \text{thickness})\).
3. Fix \(N\) the number of laminations.
4. Fix \(Q\) the quality factor.
5. Using a numerical function optimizer, find the optimal \((h_c, h_s, n)\) for max power density \((P_{\text{out}}/A_{\text{Ind}})\).

As an example, we used the specifications and the material data in Table III, obtaining the curves in Fig. 19. Once a point on the tradeoff curve “power-density versus quality factor” is specified, the physical parameters for the fabrication are derived using (30), (31), and (75)–(79). An example design for \(Q = 50\) is shown in Table III.

REFERENCES


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