4/30/2014

Coherence

Coherence of a single atom
- two levels
- three levels

Coherence between atoms
- Superradiance
- Phase matching
- (BEC, macroscopic WF)

Coherence exists if there is a well-defined phase between two or more amplitudes that interfere, i.e. one observes a physical quantity which is proportional to the square of the total amplitude

Coherence: measurement tool

relative phase is the time integral over the } energy difference → atomic clocks

energy → magnetic fields

energy → grav. Fields

Q: Spontaneous emission

A is a unitary time evolution of the WF of the total system ✓

B introduces a random phase into the time evolution of the qm system

Answer: the system evolves with the operator $\frac{\omega_R}{2}(\sigma^+ + \sigma^-)(a + a^+)$

unitary time evolution

Q: The "randomness" of spont. emitted photons is due to "loss of phase"

A the measurement process of photon

B performing a partial trace over the states of the atom ALWAYS

C A or B is possible ✓

Atom excited by a short pulse, $0 \ldots \pi$

$$|\psi\rangle = \left(\alpha_g |g\rangle + e^{i\phi} e^{-i\omega_0 t} \alpha_e |e\rangle\right) \otimes |0\rangle$$

$|0\rangle$: Photon

after spontaneous em

$$|\psi\rangle = |g\rangle \left(\alpha_g |0\rangle + \alpha_e e^{i\phi} e^{-i\omega_0 t} |1\rangle\right)$$

Quantum state of the atom has been perfectly mapped onto the photon field.

Phase ϕ : how can it be measured?

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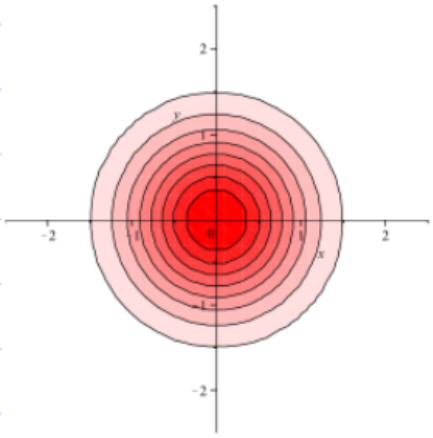
Plots of of Q(\alpha) = <\alpha|\rho|\alpha> for the pure state \rho of the light field

\sqrt{1-\epsilon}|0> + \exp(i*\pi/4)\sqrt{epsilon}|1>

$|\alpha_e|^2 = 0$



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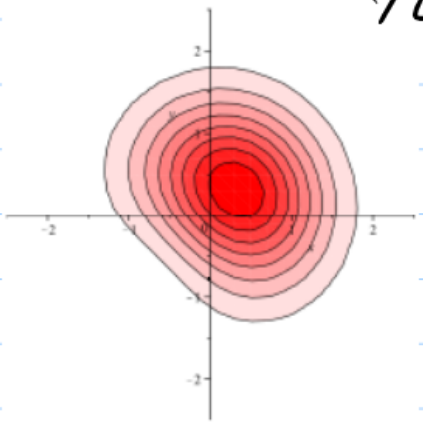
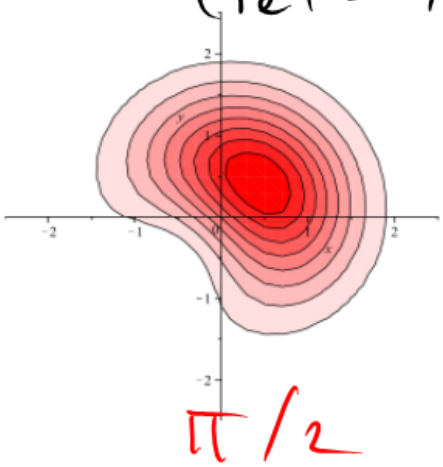
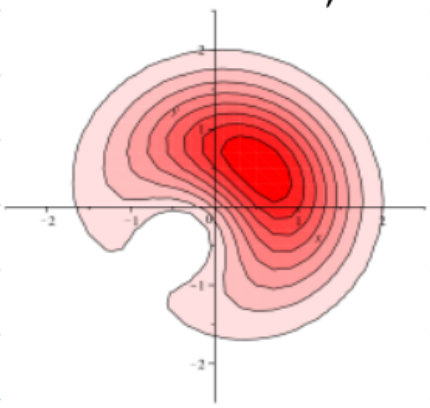


$|\alpha_e|^2 = 1/2$



$\pi/2$

3/4



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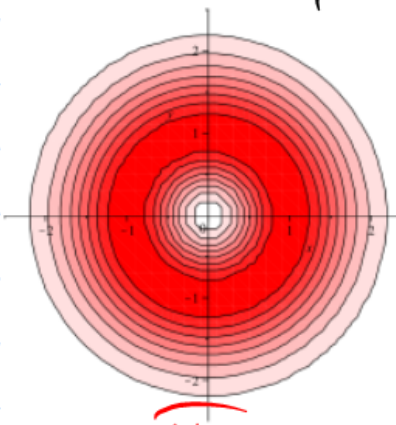


π

Phase Q: "Best" defined as the atom

when $\alpha_g = \alpha_e = 1/\sqrt{2}$

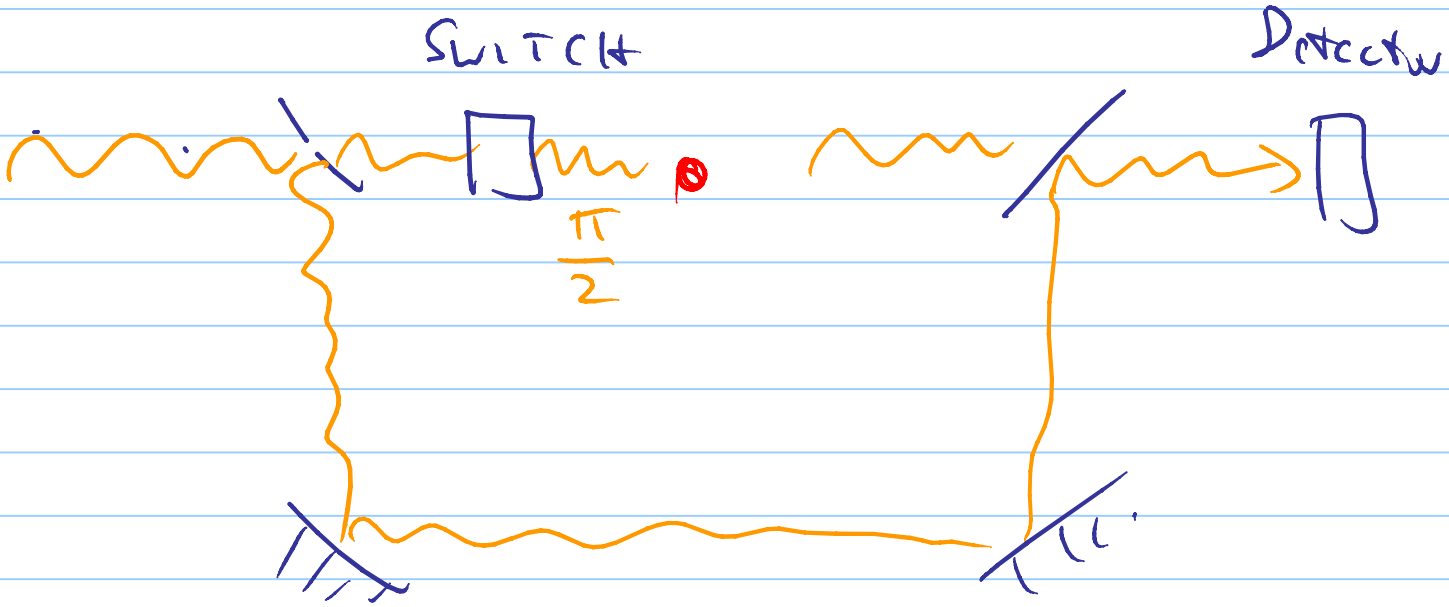
(Block vector picture)

$\Delta Q \, \Delta n \simeq 1$

Δn: # of excitations

Measure phase!

All Fluctuations come from quantum states V_0



to make measurement of the phase of the vacuum emitted by the atom

$\frac{\pi}{2}$ pulse: → retrieve Φ (but with fluctuations)

π pulse: → $|e\rangle$

at $t=0$: No coherence No phase

after 50% decay:

$$|g\rangle + e^{-i\omega_0 t + \Phi}\,|e\rangle$$

Φ is random, no coherence

[if we perform partial trace over photon field, then the atomic system is in an incoherent superposition state of wavefunctions with random phases]

ensemble average $\langle d(t) \rangle = 0$

$\langle d^2 \rangle \neq 0$

Origin of the phase uncertainty?

Vacuum fluctuations

What about two atoms:

Same vacuum fluctuations?

Correlations in the relative phase

More later: Superradiance

Coherence in a two-level system

Precession of spin in a magnetic field is effect of coherence

$$|x\rangle = \frac{1}{\sqrt{2}}\left(|z\rangle + |-z\rangle\right)$$

$|z\rangle \rightarrow e^{-i\omega_L t/2}$ $\quad$ $|-z\rangle \rightarrow e^{i\omega_L t/2}$

→ Spin precesses in x-y plan

$$\rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}e^{-i\omega_L t} \\ \frac{1}{2}e^{i\omega_L t} & \frac{1}{2} \end{pmatrix}$$

$$\langle \sigma_x \rangle = tr(\rho \hat{\sigma}_x) = \cos\omega_L t$$

no coherence: off-diagonal matrix elements are zero

$$\Rightarrow \langle \hat{\sigma}_x \rangle = 0 \qquad \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Coherent spectroscopy

Quantum beat spectroscopy

allows measurement of narrow level spacings without narrowband excitation

$|e\rangle$ ≡ ↕ Δ

short pulse excitation

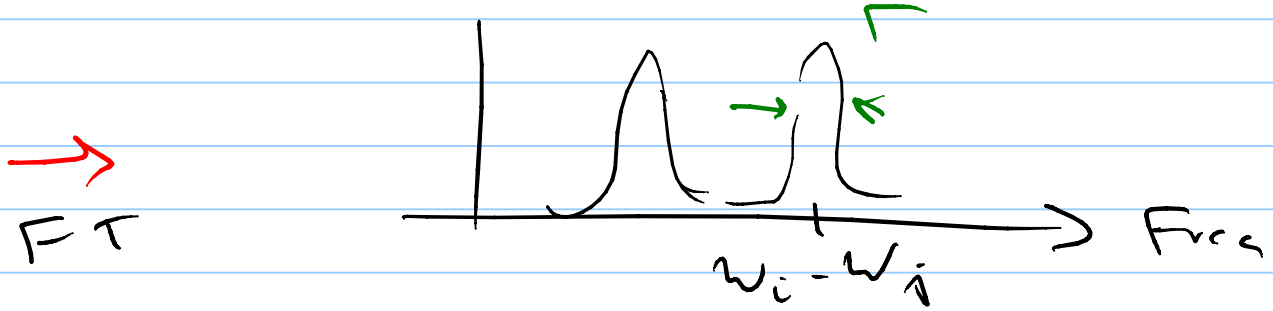
$\Delta t \ll 1/\Delta$

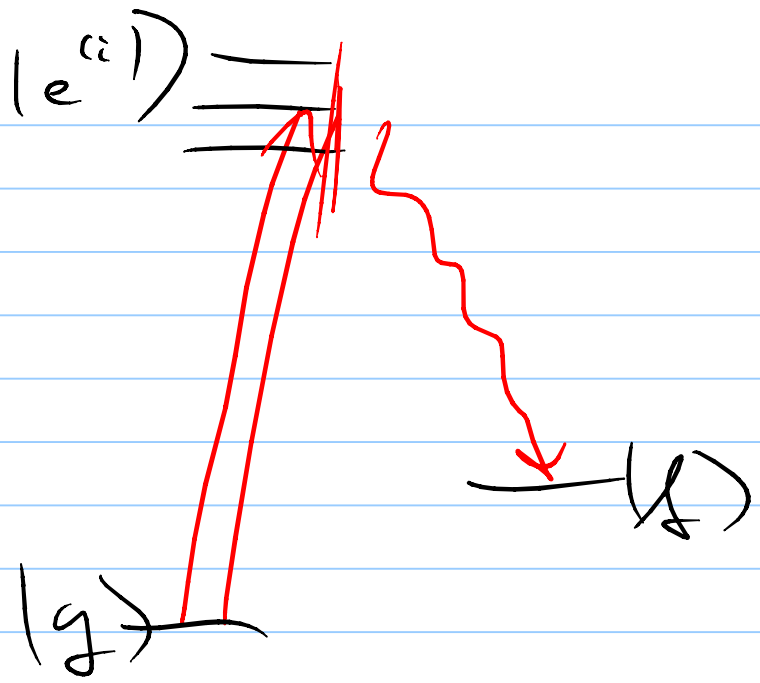
$|g\rangle$ — → coherent superposition

$$|\psi(t)\rangle = \sum c_i |e^{(i)}\rangle$$

$e^{-i\omega_i t}$

Emission





Several indistinguishable paths

Technique: Doppler free

(Doppler shift reduced by $\frac{\Delta}{\omega}$)

Delayed detection

excite system

then detect (i.e. quantum beats)

Start detection after time $t_0 \gg 1/\Gamma$

Can you obtain spectral resolution narrower than Γ?

Y: you look at long-lived atoms

N: exp. decay is self-similar

$$S(t) = \left[1 + \alpha \cos \omega_0 t\right] e^{-\Gamma t}$$

$$x = (\omega - \omega_0)/\Gamma$$

$$L(x) = \frac{1}{1+x^2} \quad \text{Lorentzian}$$

FT: $\int_{t_0}^{\infty} S(t)\, e^{-i\omega t}\, dt$ $\qquad T = t_0 \Gamma$

$e^{-i\omega t}$ / $\cos\omega t - i \sin\omega t$

Real: $F(x) = L(x)\, e^{-T}\left(\cos(xT) - x \sin(xT)\right)$

Im: $G(x) = L(x)\, e^{-T}\left(\sin(xT) + x\cos(xT)\right)$

$$|F + iG|^2 = F^2 + G^2 = e^{-2T} L(x)$$

Lorentzian in shape of T

However, $F(x)$ for large x oscillations, central peak, narrow width $\propto \frac{1}{T}$

If $S(t) = \left(1 + \alpha \cos(\omega_0 t + \phi)\right) e^{-\Gamma t}$

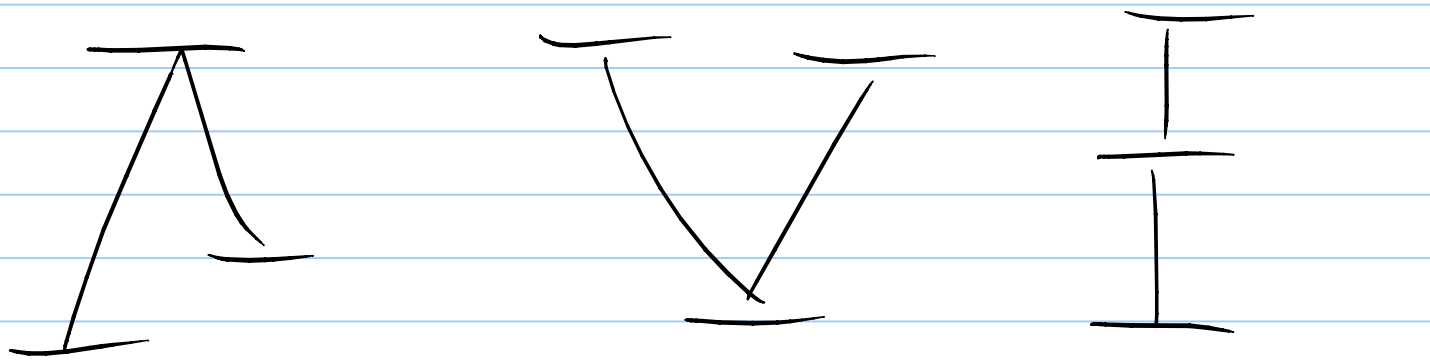
ϕ random $\Rightarrow$ You only measure $|F+iG|^2$

Crucial: knowledge (or at least reproducibility) of the phase ϕ

examples · quantum beats
· Ramsey spectroscopy
· heterodyne techniques

Coherence in three-level systems

Two states connected through a third state



Λ-type V-type ladder type

Many new effects:

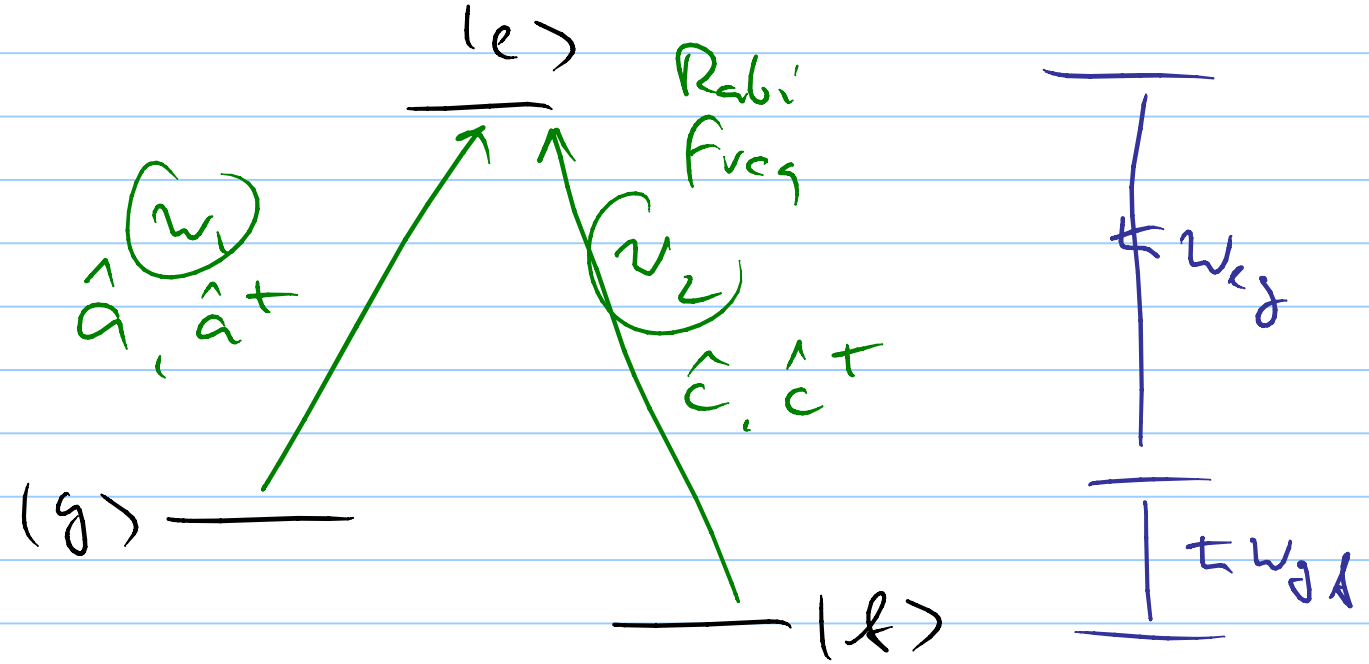
Atoms can interact with 2 em. fields which can affect each other through coherent or incoherent mechanism

* most important

2 ground states → long-living superposition state

Lasing without inversion
EIT
Slowing and stopping light
qm memory

Ex: Optical pumping



only one laser beam ω_1, ω_2

⇒ eq. population is $|\ell\rangle, |g\rangle$ by optical pumping

Q: Can we have a similar situation with BOTH lasers on, i.e. pump

the atoms into a state which does not scatter light?

Dipole Hamiltonian in the RWA:

$$H = \left[\frac{\hbar\omega}{2}\left(\sigma_{gg}\hat{a} + \sigma_{ge}\hat{a}^\dagger\right) + \frac{\hbar\nu_1}{2}\left(\hat{\sigma}_{eg}\hat{c} + \hat{\sigma}_{ge}\hat{c}^\dagger\right)\right]$$
$$+ \hbar\omega_a\left(a^\dagger a + \tfrac{1}{2}\right) + \hbar\omega_c\left(c^\dagger c + \tfrac{1}{2}\right)$$
$$+ \hbar\omega_{gf}\hat{\sigma}_{gg} + \hbar\omega_{ef}\hat{\sigma}_{ee}$$

$$\hat{\sigma}_{gg} = |g\rangle\langle g| \qquad \sigma_{eA} = |e\rangle\langle A|$$

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$$|D\rangle = \frac{\nu_2}{\sqrt{\nu_1^2+\nu_2^2}}|g\rangle - \frac{\nu_1}{\sqrt{\nu_1^2+\nu_2^2}}|A\rangle$$

is a dark state since

$$\langle e|V|D\rangle = 0$$

CPT (Coherent population trapping)

Dark State also exists for detuning

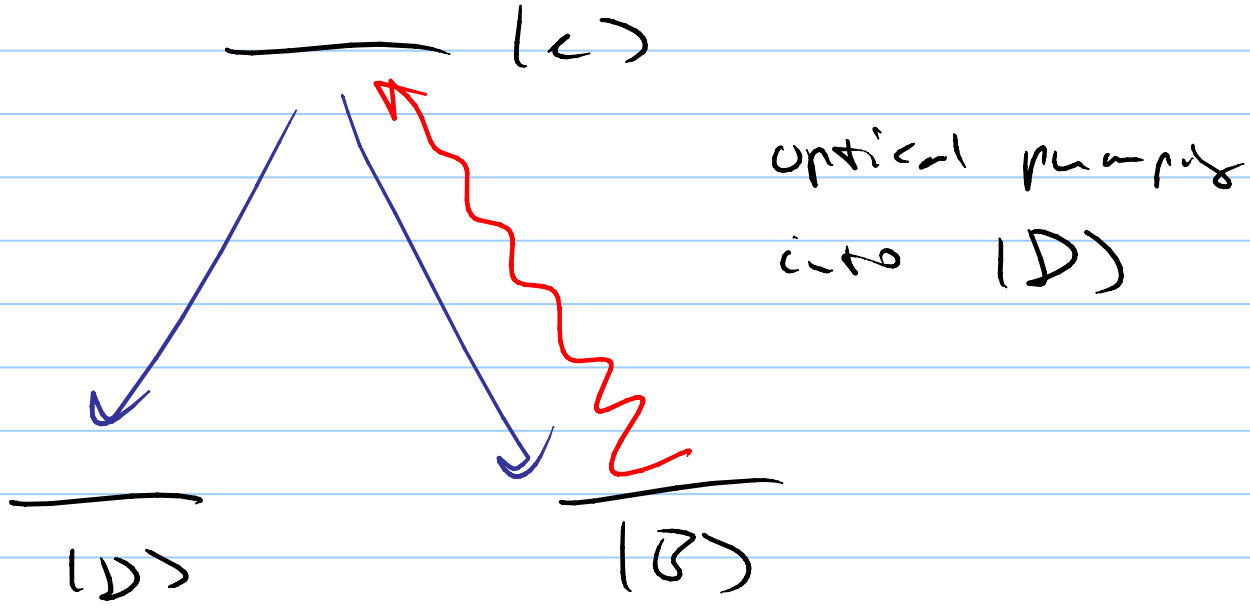
Off-resonance, as long as the two-photon resonance is maintained $\omega_a - \omega_c = \omega_{gf}$

For $\omega_1 \ll \omega_2$ the dark state is predominantly the state with the weaker coupling

e.g. $\omega_1 = 0$ $\quad |D\rangle = |g\rangle$

$$|B\rangle = \frac{\omega_1}{\sqrt{\omega_1^2+\omega_2^2}}|g\rangle + \frac{\omega_2}{\sqrt{\omega_1^2+\omega_2^2}}|f\rangle$$

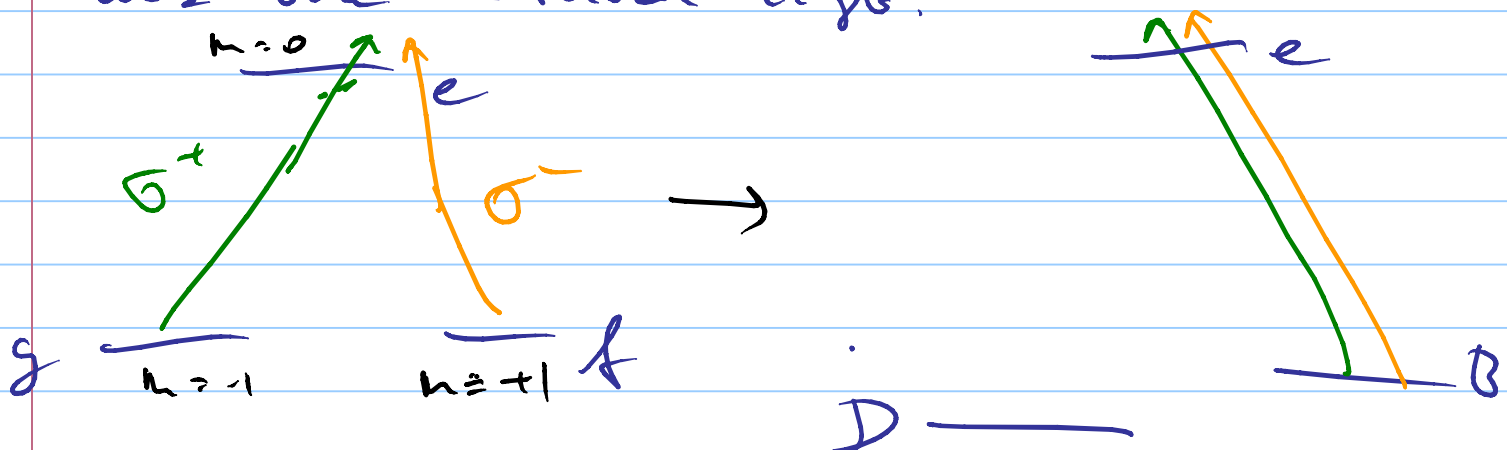
Bright state



STIRAP – Adiabatic population transfer

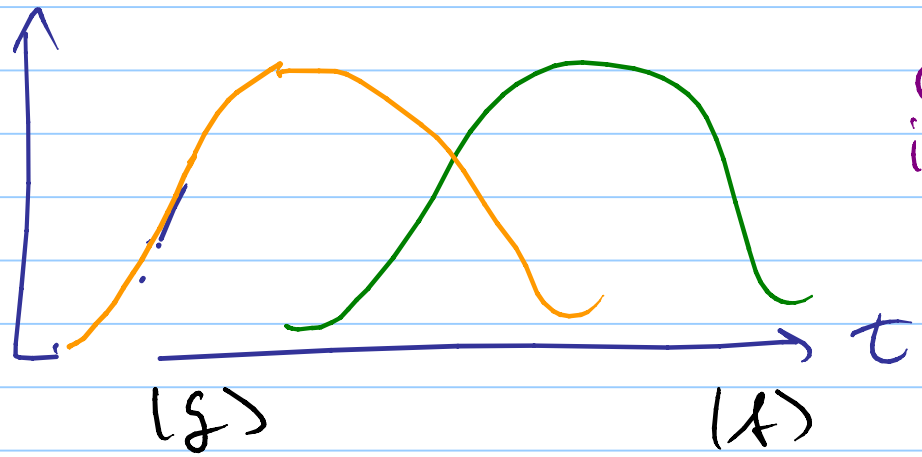
What happens when ω_1 and ω_2 change slowly?

Let's discuss degenerate states $\omega_{gf} = 0$ and blue-detuned light.



$|D\rangle$ is the lowest qu. state

this allows 100% transfer from $|g\rangle$ to $|f\rangle$



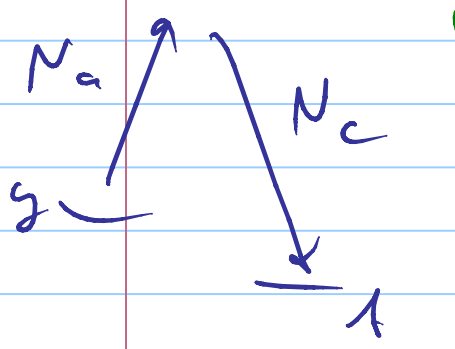
counter-intuitive sequence

is the dark state.

Notes: Same result for red detuning

Same result when $|f\rangle$, $|g\rangle$ are non-degenerate

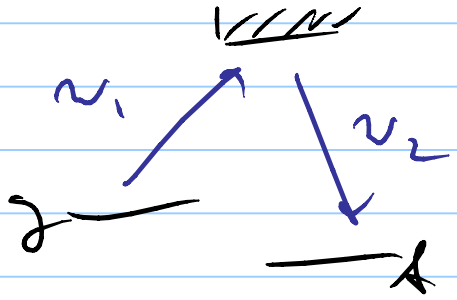
use dressed state basis



$|g, N_a, N_c\rangle$ \
$|f, N_a-1, N_c+1\rangle$ / are degenerate

STIRAP =
DARK STATE TRANSFER

Magic?



No!

$|e\rangle$ is needed as a stepping stone

$$i\begin{pmatrix} \dot{c}_g \\ \dot{c}_e \\ \dot{c}_f \end{pmatrix} = \begin{pmatrix} 0 & \frac{\omega_1}{2} & 0 \\ \frac{\omega_1}{2} & 0 & \omega_2/2 \\ 0 & \omega_2/2 & 0 \end{pmatrix} \begin{pmatrix} c_g \\ c_e \\ c_f \end{pmatrix}$$

$$\dot{c}_f = i\frac{\omega_2}{2} c_e \simeq \frac{1}{t_{transfer}} \qquad \omega_1 = \omega_2 = \omega_R$$

$$c_f(t_{transfer}) = 1$$

$$\Rightarrow c_e \simeq \frac{1}{\omega_R t_{transfer}}$$

$$P_e \sim |c_e|^2$$

Prob of spont em

$$P_{sp} = P_e \cdot \Gamma t_{transfer} = \frac{\Gamma}{\omega_R^2 t_{transfer}}$$

$\omega_R^2 \to \infty$, $t_{transfer} \to \infty$ $\longrightarrow 0$

$t_{transfer} \leq t_{coh}$

Incoherent picture

π pulse — e — π pulse

g — — s

$$P_{sp} = \frac{\Gamma}{\omega_R}$$

2-ph Raman transition

$$\omega_{2ph} = \frac{\omega_R^2}{\delta}$$

$$t_{transfer} = \frac{1}{w_{2R}} = \frac{\delta}{w_R^2}$$

$$\Rightarrow \quad \delta = t_{transfer}\, w_R^2$$

$$P_e = \left(\frac{w_R}{\delta}\right)^2 \qquad P_{sp} = \frac{w_R^2}{\delta^2}\,\Gamma\, t_{transfer}$$

$$= \frac{\Gamma}{w_R^2\, t_{transfer}}$$

Lasing / Gain } without inversion

Common belief $N_e > N_g$ for optical gain & lasing is not true

[Steve Harris PRL 62, 1033 (1989)]

Absorption cancellation by interference

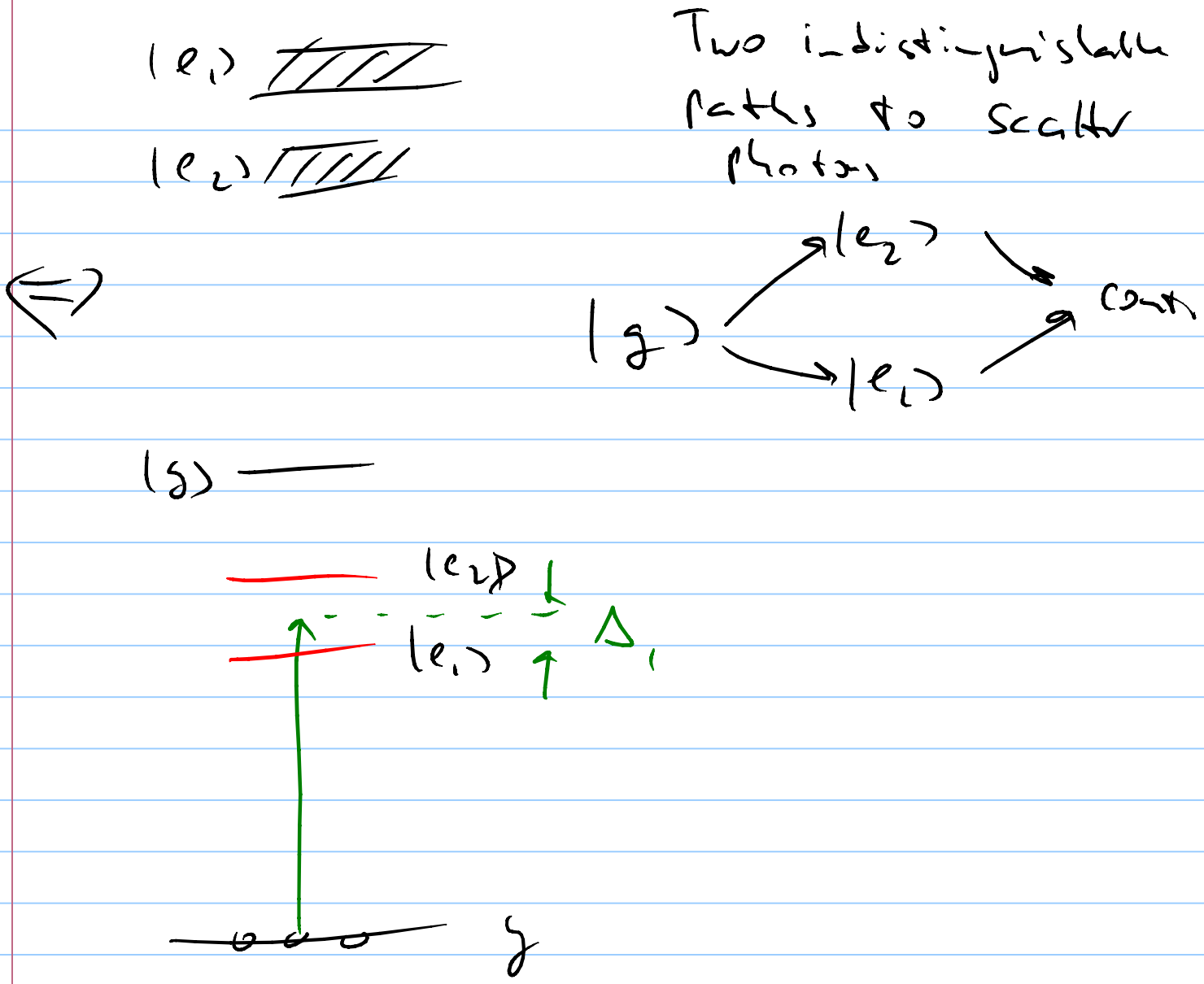
Simplest system: Λ system

$|e_2\rangle$ —

$|e_1\rangle$ —

e_1, e_2 couple to the same continuum

$|g\rangle$ —



add amplitudes and over perturbation theory

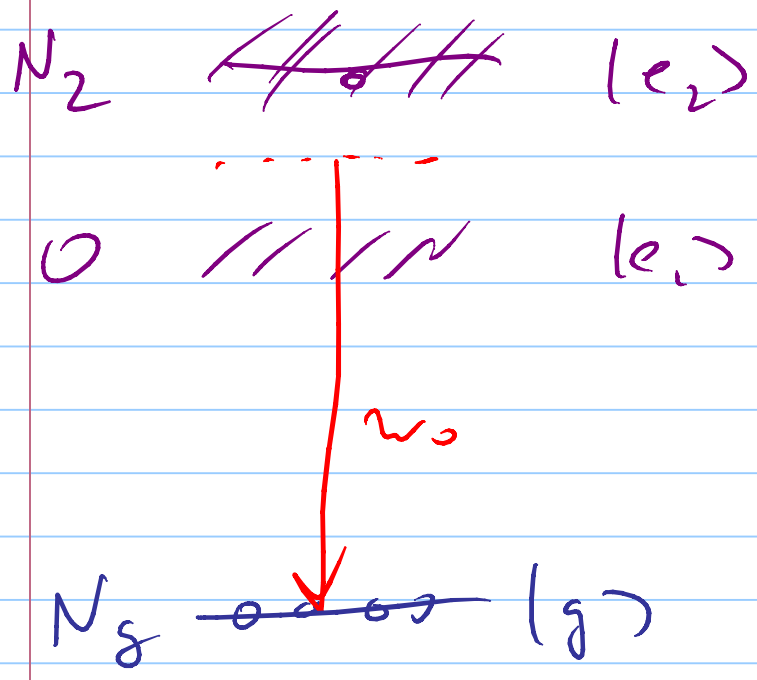
$$M = \frac{\sum_{coh} \langle f|V|i\rangle \langle e_j|V|g\rangle}{\Delta_j - i\Gamma/2}$$

$= 0$ for a certain laser freq ω_0 tuned between $|e_1\rangle$ and $|e_2\rangle$

(assuming $|\Delta_1|, |\Delta_2| \gg \Gamma_1, \Gamma_2$)

Light @ ν_0 : Not absorbed

Now: population $|e_2\rangle$ with $N_{|e_2\rangle} \ll N_{|g\rangle}$



Caving @ ν_0

GAIN

Some stimulated em @ ν_0

no absorption

Possible realization

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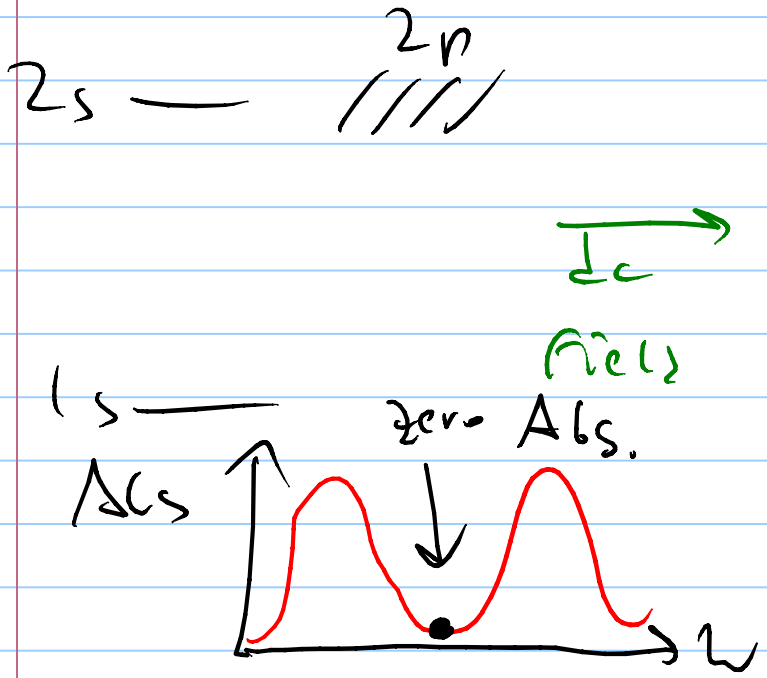
(1) It acts like electric field



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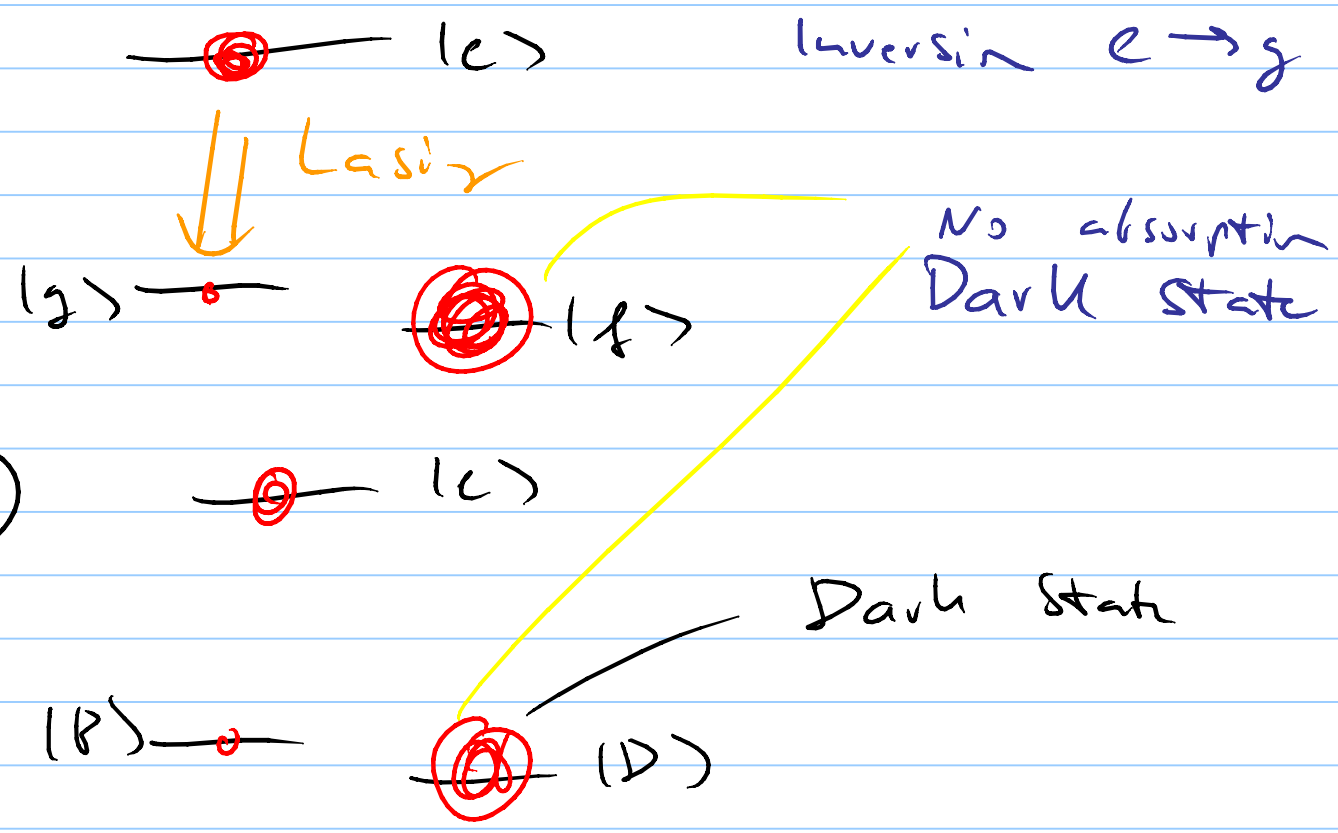
(2) Use Ac electric field to mix s and p states

(3) Trivial realization

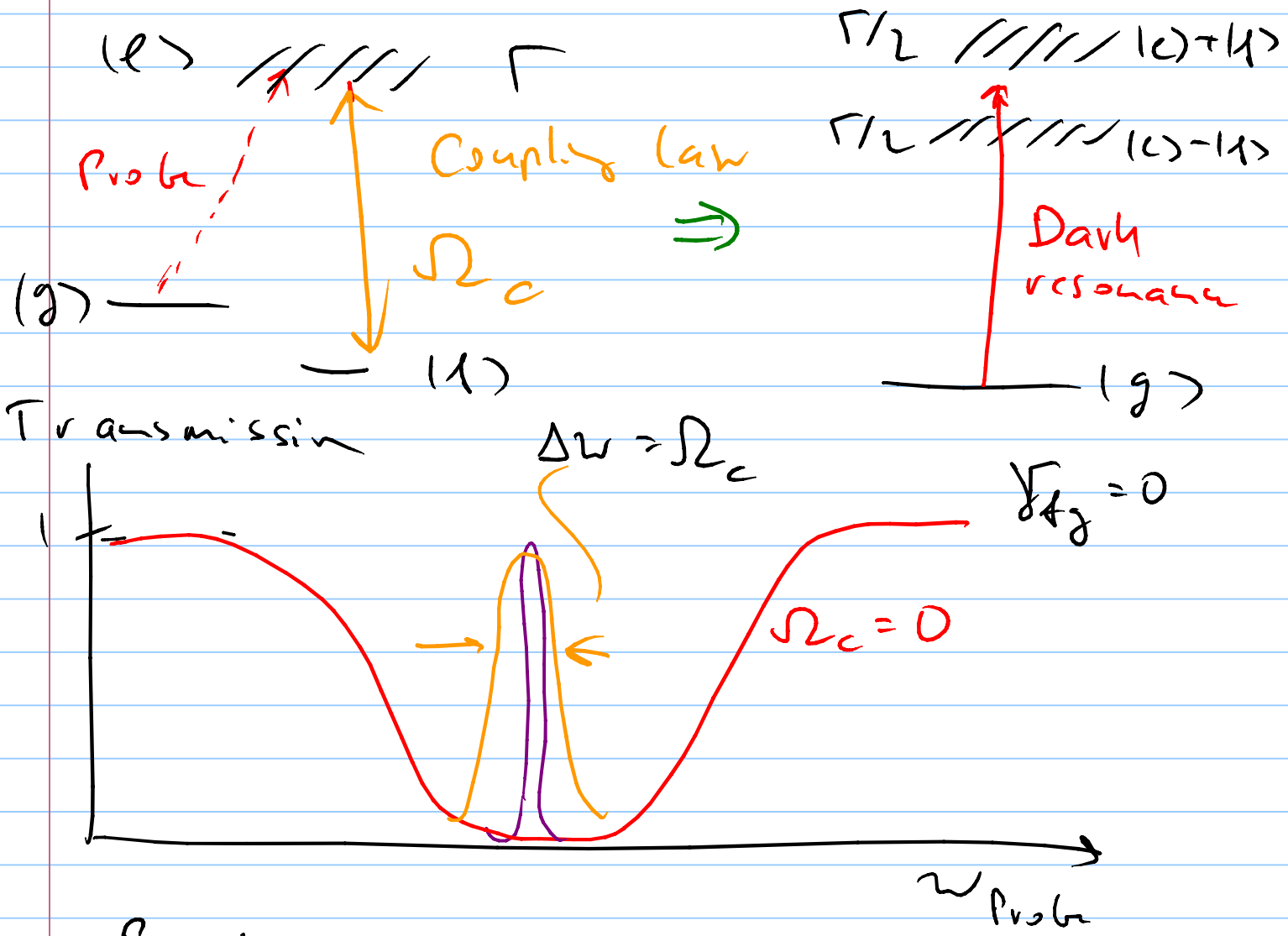


Electromagnetically induced transparency EIT

Can a laser beam penetrate an optically thick medium?

Yes, with the help of another laser.

Realization in a Λ System

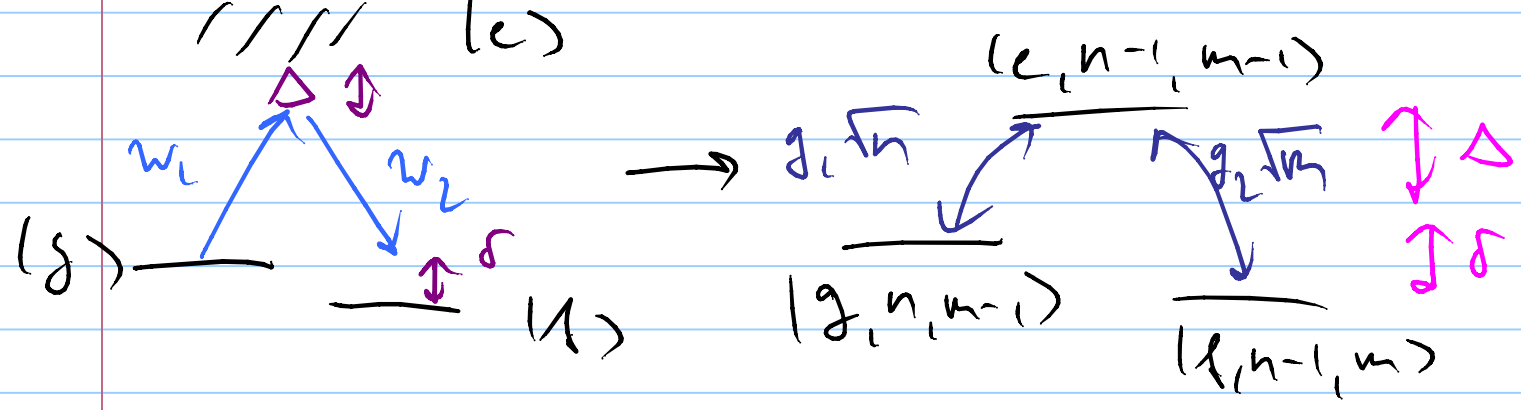


Possible applications:

- Design non-linear materials near resonance without absorption
- Sensitive detection of trace elements

EIT: eigenstate picture

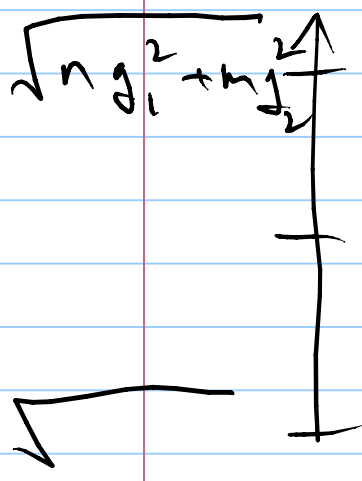
Use field quantization to easily include energy conservation



$$\Omega_1/2 = g_1\sqrt{n}$$
$$\Omega_2/2 = g_2\sqrt{m}$$

$$H = \hbar \begin{pmatrix} 0 & g_1\sqrt{n} & 0 \\ g_1\sqrt{n} & \Delta & g_2\sqrt{m} \\ 0 & g_2\sqrt{m} & -\delta \end{pmatrix}$$

On resonance: $\Delta = \delta = 0$

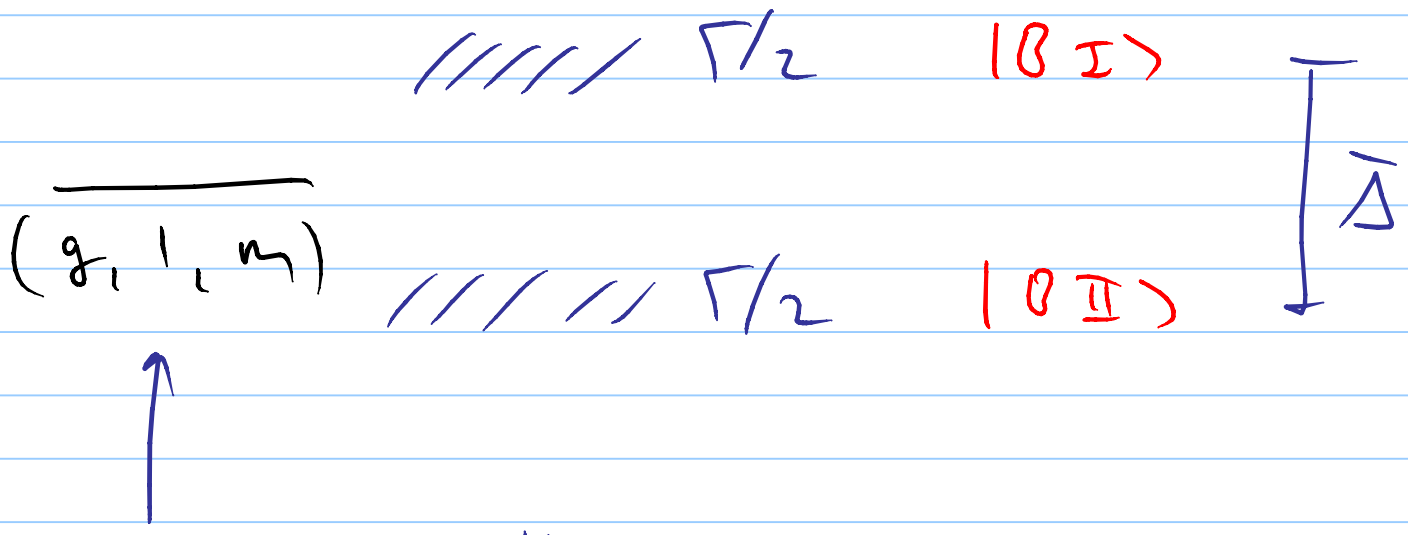


$$\frac{\Gamma}{2}\quad c_1'|g\rangle + c_2'|e\rangle + c_3'|1\rangle = |B\rangle$$

$$\quad c_1|g\rangle + c_3|1\rangle = |D\rangle$$

$$\frac{\Gamma}{2}\quad c_1''|g\rangle + c_2''|e\rangle + c_3''|1\rangle = |B_{II}\rangle$$

$n \to 1 \ll m$ Weak probe field



Scattering problem

other modes

$|g,1\rangle \otimes |0,0 \ldots 0\rangle$

$\to |g,0\rangle \otimes |0,\ldots,1,0\rangle$

two-photon process

Matrix element needs intermediate state

$$M = \frac{\langle g,0|V'|BI\rangle\langle BI|V|g,1\rangle}{-\bar{\Delta}/2} + \frac{\langle 0\,II\rangle\langle BII\rangle}{\bar{\Delta}/2}$$

vanishes (EIT)

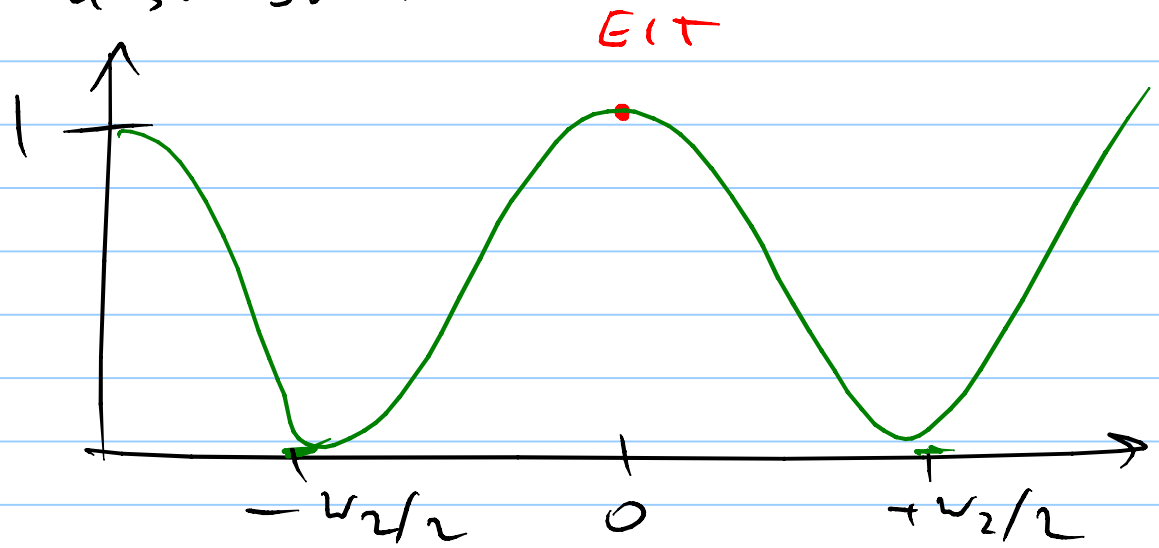
Detuning probe by δ → detunings become $\pm(\bar{\Delta} \pm \delta)/2$

→ Scattering

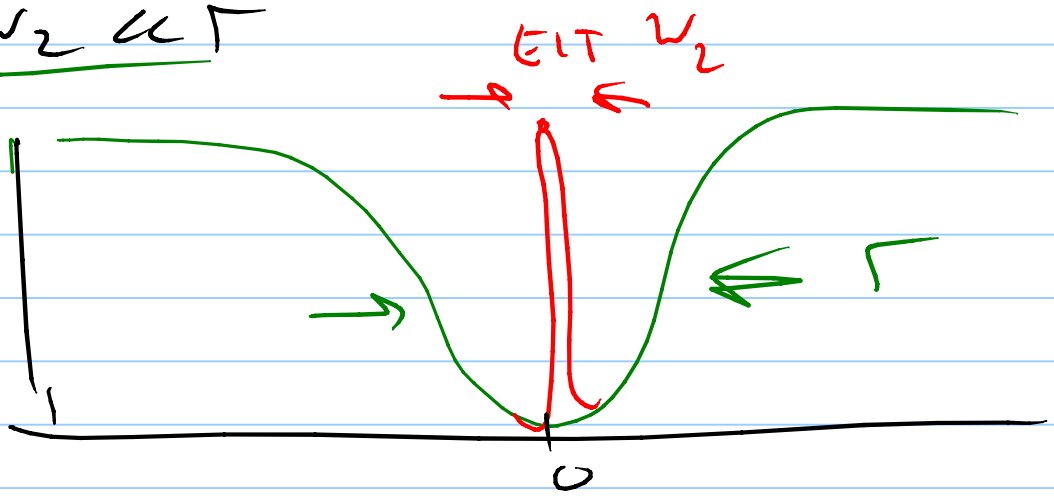
near one-photon resonance

$\nu_2 \gg \Gamma$

Transmission



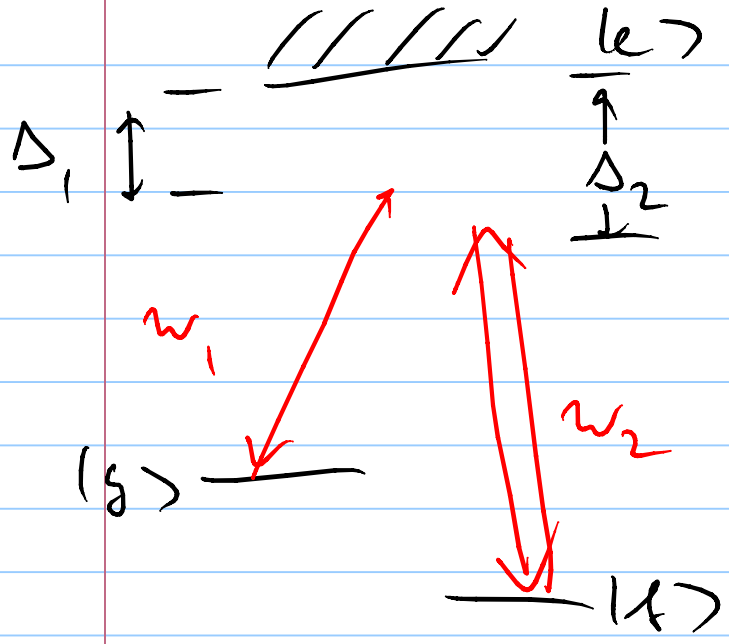
$\nu_2 \ll \Gamma$



Fano profiles, two-photon absorption

Now: laser one-photon detuning Δ_2

→ separates the EIT window from absorption features



ν_1 weak probe

$\delta = \Delta_2 - \Delta_1$

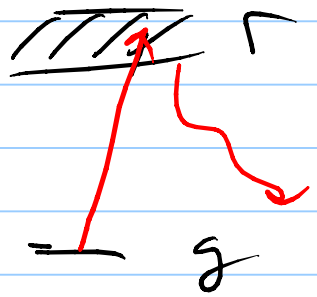
Analytic expressions

→ Lounis & Cohen-T, (1992)

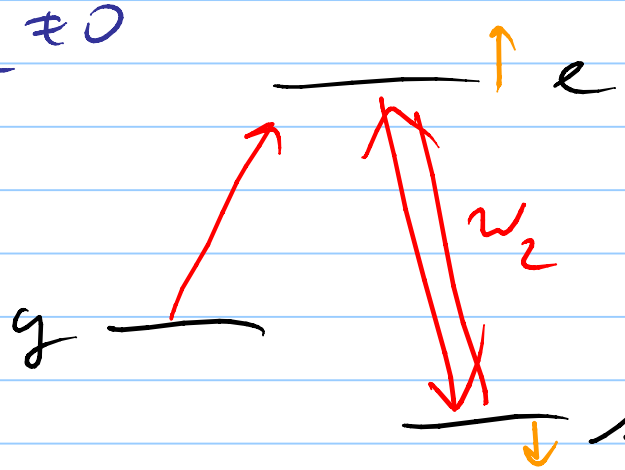
Features: $\delta = 0$ EIT

$\omega_2 = 0$

$\Delta_1 = 0$ → one photon abs.



$\omega_2 \neq 0$

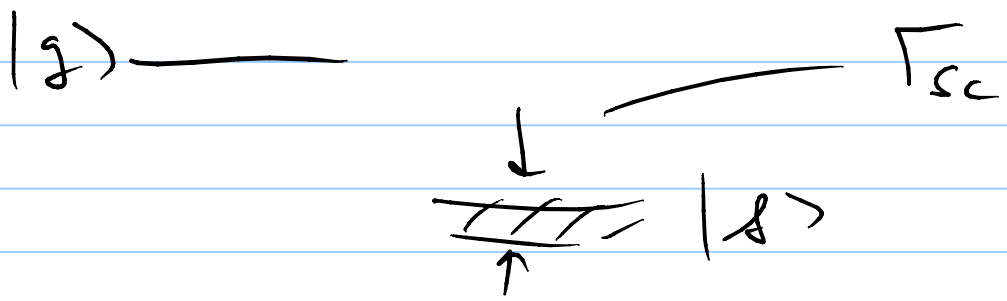


AC Stark shift

$-\frac{\omega_2^2}{\Delta_2}$

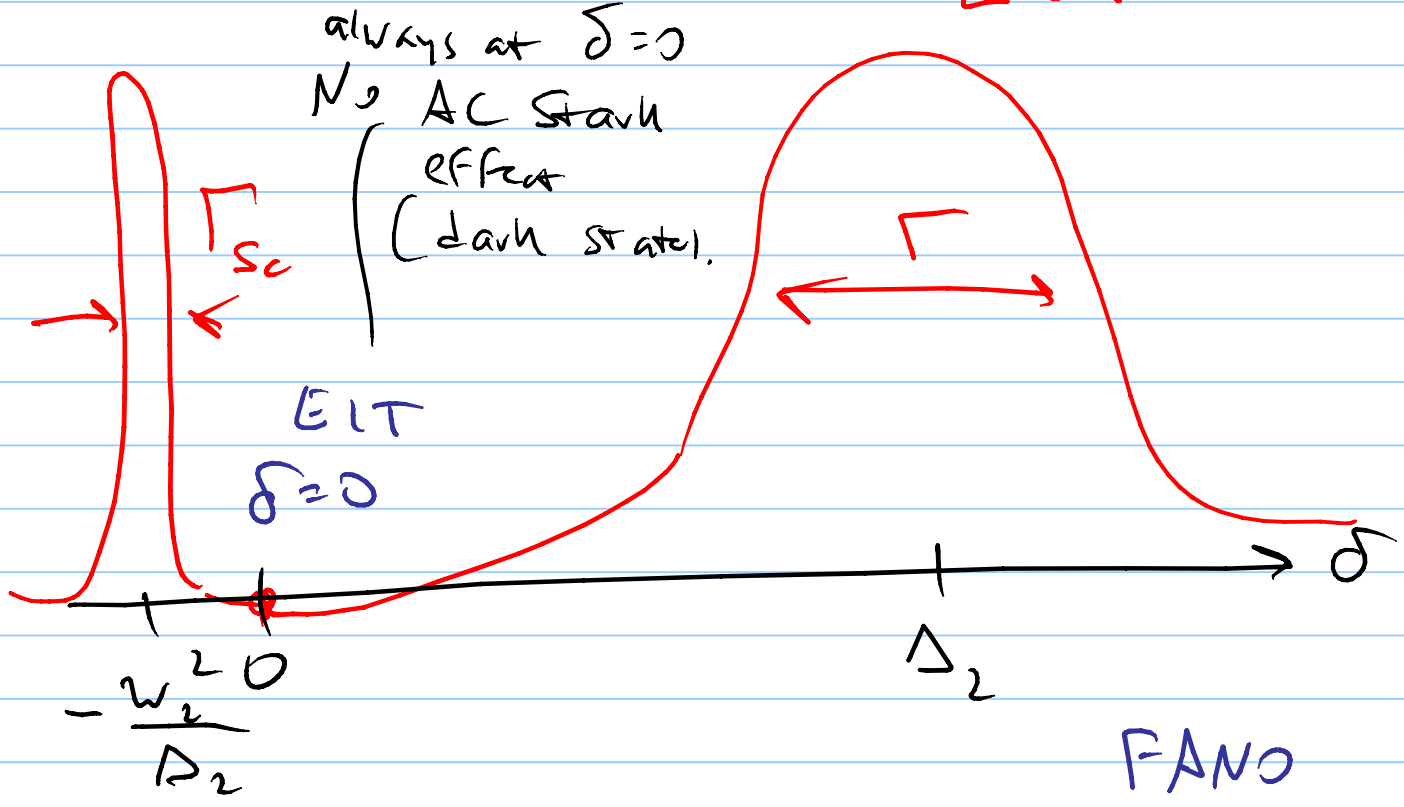
Scattering rate

$\Gamma_{sc} = \left(\frac{\omega_2}{\Delta_1}\right)^2 \frac{\Gamma}{2}$



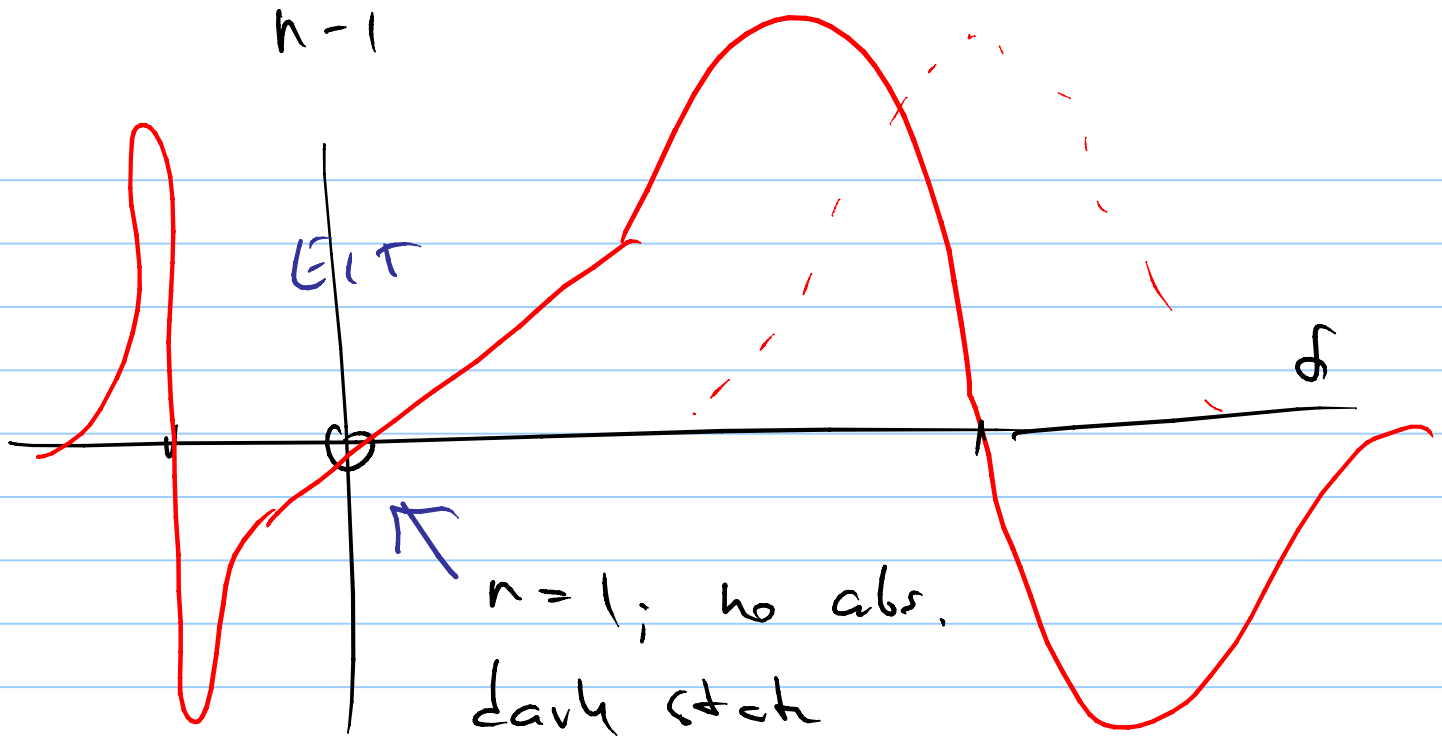
|g⟩ couples now to a narrow and a wide "excited" state

Both couple to the same continuum → INTERFERENCE
EIT



FANO PROFILE

2-ph Resonances
+ AC Stark shifts
} 4 photon process



but: (aux $\frac{dn}{d\nu}$

Slow Light

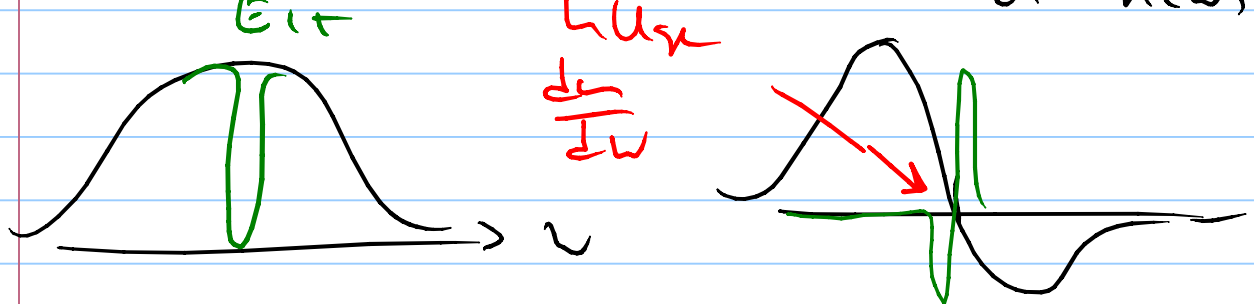
$$v_g = \frac{c}{1 + \nu \frac{dn}{d\nu}}$$

Harris, Han PRL 82, 4611 (1999)

EIT → very slow light

Hau et al., Nature 397 594 (1999)

Note: For $\Delta_2 = 0$, even strongly feature in $n(\nu)$

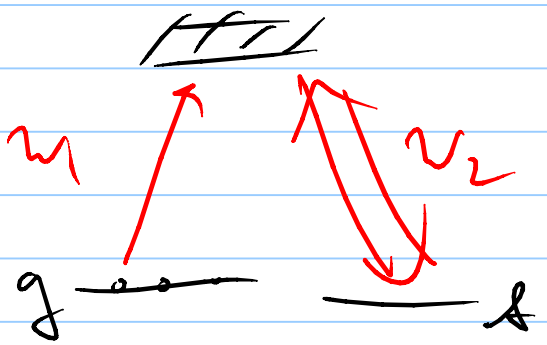


adiabatic change of control field Ω_2 while the probe pulse is in the medium

$\Omega_2 \to 0$

"stopped light"

"frozen light"



$\Omega_2 \to 0$

$|D\rangle \to |e\rangle$

Light is coherently converted into atomic population $|e\rangle$

"excitation of a spin wave"

Readout: reverse process (ramp up Ω_2)

Course evaluation form:
http://web.mit.edu/subjectevaluation/evaluate.html

Please address these additional questions:

1. What improvements would you recommend for the online problem sets?

2. What did you like most about the online problem sets?

3. What did you like least about the online problem sets?

4. For future 8.421 courses, would you prefer (a) traditional format paper-only, hand-graded problem sets, (b) a 50/50 mixture of traditional hand-graded questions and online questions, (c) all problem set questions online, with instant grading?

5/14/2014 LAST CLASS!

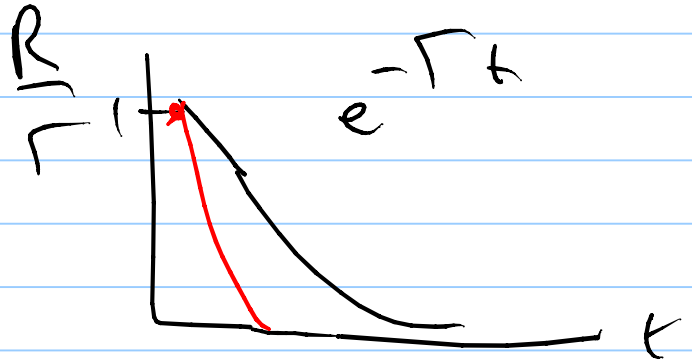
SUPERRADIANCE

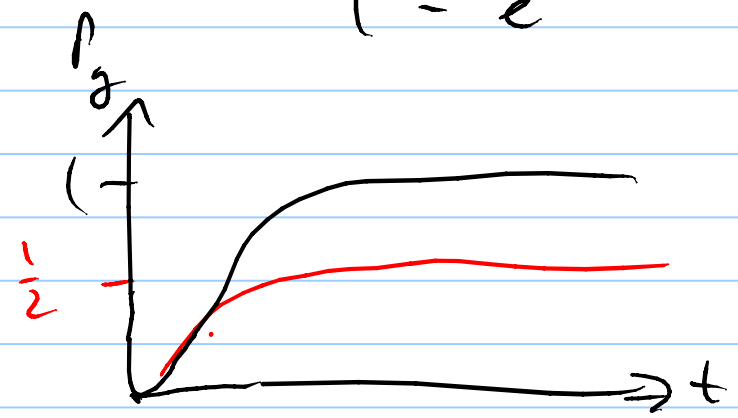
Basic idea: Classic paper by Dicke in 1954

Single atom $|e\rangle \to |g\rangle$

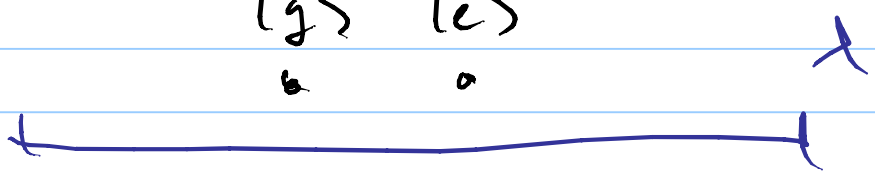
emission rate $R(t) = \Gamma e^{-\Gamma t}$

$$P_g(t) = \int_0^t R(t')\,dt' = 1 - e^{-\Gamma t}$$





What about two atoms, one excited



Same rate, same decay? NO!

Correct answer: $R(t) = \Gamma e^{-2\Gamma t}$

$P_g(t) = \frac{1}{2}\left(1 - e^{-2\Gamma t}\right)$

Same initial rate, but only probability of $1/2$ to emit at all.

$|eg\rangle$ is superposition of symm and anti-symmetric WF

$\frac{1}{\sqrt{2}}\left(|gc\rangle + |cg\rangle\right) = |\text{SUPER-RADIANT}\rangle$

$\frac{1}{\sqrt{2}}\left(\quad - \quad\right) = |\text{SUB RADIANT}\rangle$

the interaction Hamiltonian is

$V = -\vec{d}_1 \vec{E}(r,t) - \vec{d}_2 \vec{E}(r,t)$

$= -\vec{D}\,\vec{E}(r,t)$ Classical

$\vec{D} = \vec{d}_1 + \vec{d}_2$

$V = \hbar g\left(\sigma_1^+ + \sigma_1^-\right)\left(a + a^\dagger\right) + (1 \leftrightarrow 2)$ QED

$= \hbar g\left(\Sigma^+ + \Sigma^-\right)\left(a + a^\dagger\right)$

with $\Sigma^{+-} = \sum_{i=1}^{2} \sigma_i^{+-}$

$\langle gg | V | \text{SUPER RAD.} \rangle = \sqrt{2}\, t_g$

$\langle gg | V | \text{SUPRADIANTS} \rangle = 0$

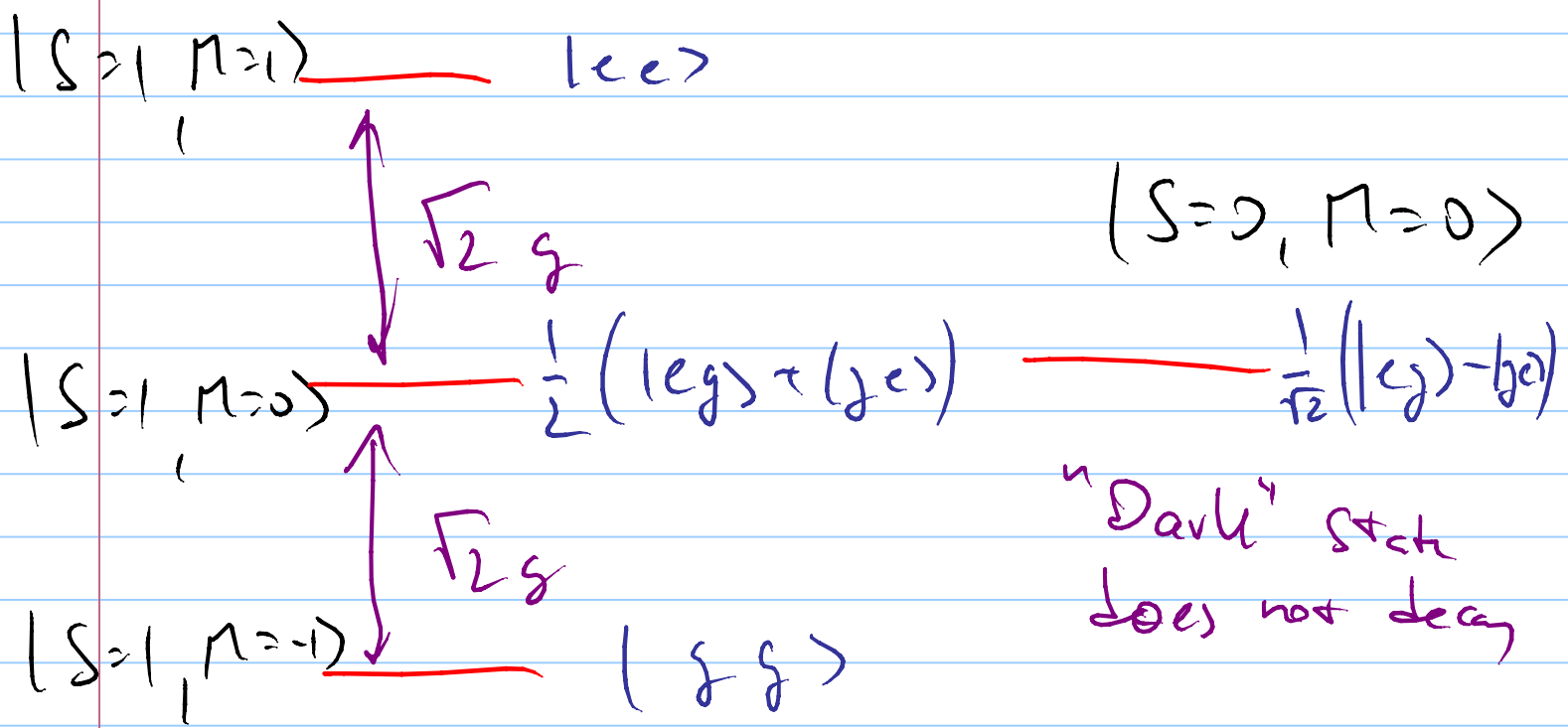
organize 4 states $|gg\rangle, |ge\rangle, |eg\rangle, |ee\rangle$

$\downarrow\downarrow \quad \downarrow\uparrow \quad \uparrow\downarrow \quad \uparrow\uparrow$

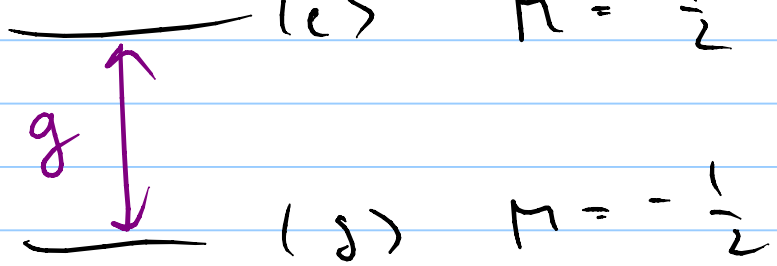
$\Rightarrow$ 2 spin $1/2$ states

Can couple to $S=1$

$S=0$



Single atom $S = 1/2$



Coupling involves $S_+ S_-$

Conserves S

$\Delta m = 1$

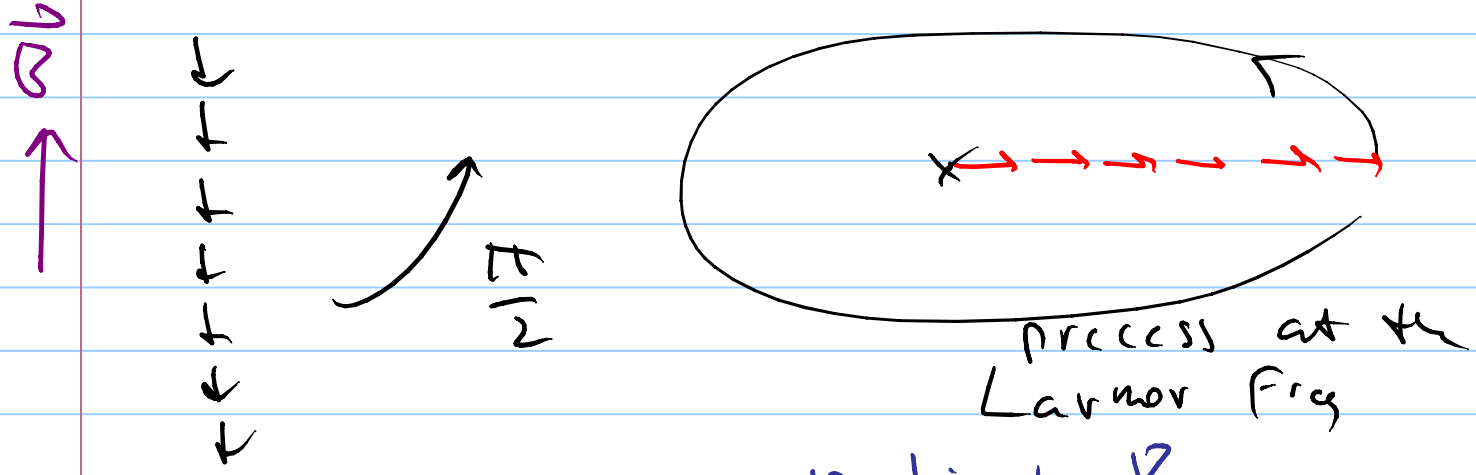
Ex: Na_2 molecule

has states with $\Gamma_{mol} \simeq 2\Gamma_{Na}$

$\simeq 0$

NOW: N particle SR

First: N spins in a magnetic field



dipole moment N times the single atom dipole moment

$\Rightarrow$ power radiated $\propto N^2$

$N\times$ higher than

N individual particles

N spins (NMR): natural picture

N excited atoms: usually regarded as independent

BUT: all two-level systems are equivalent

important difference:

Sample size $\gg \lambda$ $\ll$

Optical NMR

Radiation: ALL atoms interact with a COMMON radiation field emitted photons CANNOT be treated independently.

Coherence: in spontaneous emission for N (localized) atoms

ASSUMPTIONS: • localization $< \lambda$

• no interactions between atoms

- particle symmetry:
  does not matter
  photons have to be indistinguishable

N particles

$\vec{S} = \sum \vec{s}_i$ $\qquad S \le N/2$

$M = \frac{1}{2}(N_+ - N_-) \le S$

eigenstates $|S, M\rangle$ z comp. of $\vec{S}$

$S = \frac{N}{2}$ $\qquad S = \frac{N}{2} - 1$ $\qquad S = \frac{1}{2}$

$M = \frac{N}{2}$ —

—

—

—

—

—

—

—

—

$M = -\frac{N}{2}$ —

interact. with em field, S_+, S_-

matrix element

$\langle S, M-1 | S_- | S, M \rangle = \sqrt{(S-M+1)(S+M}$

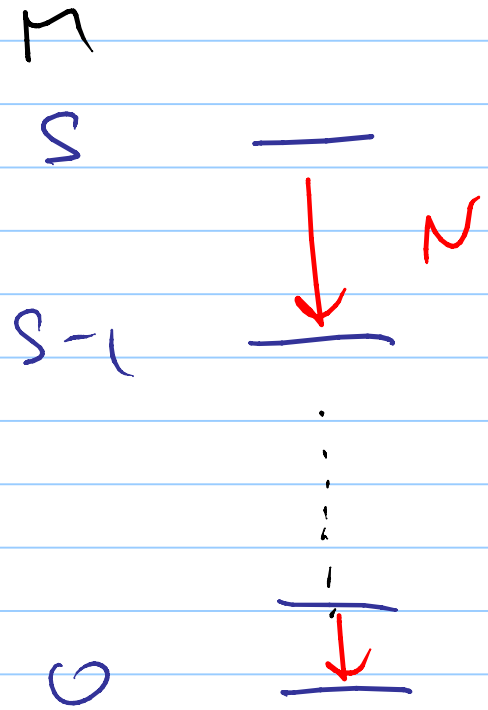
$\sqrt{\ } = 1$

single particle
$S = M = 1/2$

Rate (or intensity) of radiation relative to a single particle

$$I = (S - M + 1)(S + M)$$

let's look at $S = N/2$

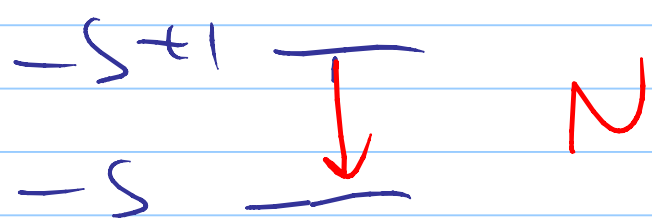


Same as for N independent atoms

$$I = \frac{N}{2}\left(\frac{N}{2} - 1\right) \simeq \frac{N^2}{4} \propto N^2$$

characteristic for SR

$N\times$ enhancement



only 1 particle excited

$N\times$ enhancement

initial state

$$\frac{1}{\sqrt{N}}\left(|e\,g\,g\,g\,g\,g\rangle + |g\,e\,g\,g\,g\ldots\rangle + |g\,g\,e\ldots\rangle\right)$$

N contributions

Final state $|g\,g\ldots\rangle$

$\Rightarrow$ Matrix element $\sqrt{N}$ times larger than for a individual atom

Vacuum Rabi splitting

$$\omega_{R,1}\sqrt{n+1}\sqrt{N}$$

$\sqrt{n+1}$: photons; $\sqrt{N}$: atoms

Enhanced induced em or absorption

initial state $|g\rangle^N$

$$|\psi(t)\rangle_N = |\underline{\phi(t)}\rangle^N$$

single particle

It takes half a Rabi period to completely invert the population,

as for single atom

However: Dicke states

matrix element of N

N steps in the Dicke ladder

$N \cdot \frac{1}{N} = 1$

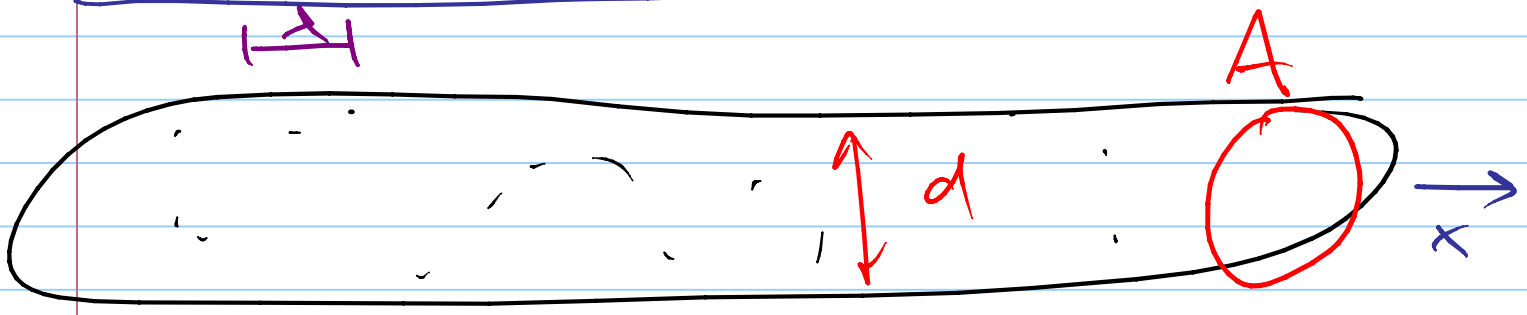
Spont. em:

each step is $\propto (\text{Matrix el})^2$

$\propto N^2$

N steps speed up $\frac{N^2}{N} = N$

Dicke states of extended samples



$L \gg d$

preferential modes along x

emission into a solid angle of $\frac{\lambda^2}{A}$ can be coherent

$\Rightarrow$ SR enhancement by $N \frac{\lambda^2}{A}$

$= n \lambda^2 \ell$ — gain cross section

analagous to optical amplification in an inverted media

Formally: define Dicke states w.r.t. preferred mode

$$|S, M=-S\rangle = |g \ldots g\rangle$$

$$|S, M=-S+1\rangle = \frac{1}{\sqrt{N}}\left(e^{ikx_1}|eg\ldots g\rangle + e^{ikx_2}|geg\ldots\rangle\right.$$

$$|S, M=-S+2\rangle = \frac{\sqrt{2}}{\sqrt{N(N-1)}}\left(e^{ikx_1}e^{ikx_2}|eeg\ldots\rangle + \ldots\right)$$

Phase factors are chosen that for the interaction Hamiltonian

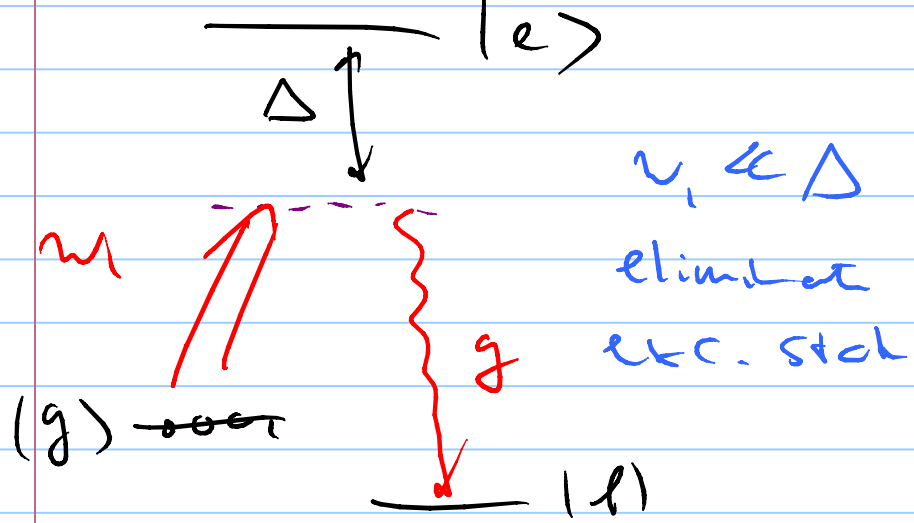
$$V = \sum \vec{d}_i \vec{E}(x_i)$$

$$= \sum \vec{d}_i E_0 e^{\pm ikx_i}$$

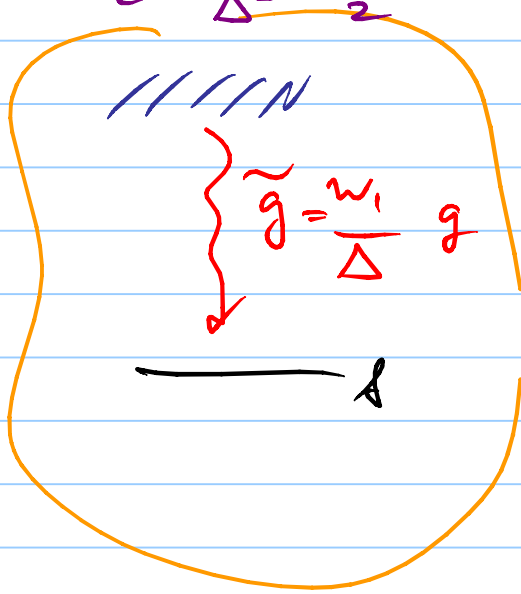
all the phase factors cancel $\Rightarrow$ SR constructive interference

$\Rightarrow$ enhancement $N\frac{\lambda^2}{A} = n\lambda^2 l = OD$

RAMAN SR



$$\Gamma_{sc} = \frac{\omega_1^2}{\Delta^2}\frac{\Gamma}{2}$$



Exp: PEC Wy
Cold atoms VV

Extended sample SR

Coupling $g \rightarrow g \times \sqrt{OD}$

Cavity QED Cavity $g \times \sqrt{OD}$