

Precise measurement of the mass of Cs: Toward a new determination of the Fine Structure Constant

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Abstract. Preliminary results on a measurement of the mass of Cesium at 0.22 ppb are reported, an improvement by a factor of more than 100 over the currently accepted value. Combined with a precise measurement of the D₁ line of Cs and an ongoing measurement of the recoil velocity of Cs atoms absorbing photons, this will provide a QED-independent value of the fine structure constant, α , at about 5 ppb. Since Cs atoms are widely used in high precision experiments, this result is of general interest in atomic physics.

THE FINE STRUCTURE CONSTANT

If QED correctly predicts the g-2 anomaly of the electron, the fine structure constant, α , is known to 5 ppb [1,2] (and groups at the U. of Washington and Harvard University plan to reduce this error to about 1 ppb). In order to test QED at this accuracy, and to remove theoretical uncertainties highlighted by the recent readjustment of the theoretical value of g-2 [1], other measurements of α at this level of accuracy are needed. Unfortunately, the next most precise measurements are far from this accuracy. α has been measured to 39 ppb using a beam of neutrons [3], and two different Quantum Hall effect measurements have been made with 24 ppb precision but 42 ppb systematic difference [4,5] (See Fig. 1). There is obviously a great need for a QED-independent determination of α at the ppb level: not only could it probe QED an order of magnitude more accurately, but it could possibly reveal the source of the discrepancies in these other measurements.

A route to α that appears likely to yield a value at the ppb level is opened by the relationship (in SI units):

$$\alpha^2 = \frac{2R_\infty}{c} \frac{10^3 m_p}{M_p m_e} (N_A h). \quad (1)$$

where $N_A h$ is known as the molar Planck constant. The Rydberg constant, R_∞ has been measured to an accuracy of about than 0.008 ppb [7], m_p/m_e is known to 2 ppb [8], and we have determined M_p to 0.5 ppb [9] (Van Dyck's group at UW have preliminary results for M_p at 0.14 ppb [10]). Thus a measurement of $N_A h$, the "molar Planck constant", at the ppb level can determine α to about 1 ppb.

The molar Planck constant can be determined from the quantum relationship between de Broglie wavelength and momentum:

$$\lambda = \frac{h}{mv} \quad \Rightarrow \quad \lambda v = \frac{h}{m} = \frac{10^3 N_A h}{M} \quad (2)$$

Note that to determine $N_A h$ we need to measure the particle's atomic mass, M . This simple physics underlies the neutron-based measurement of α [3], where our value of the neutron's atomic mass leads to the value for α mentioned in Fig. 1. Chu's group at Stanford University has measured the recoil velocity v for Cs atoms absorbing photons of laser light at the D₁ line to 10^{-6} using an atom interferometer [11]. They now predict an accuracy of 10 ppb using a vertically configured apparatus like the one used to reach ppb measurements of g.

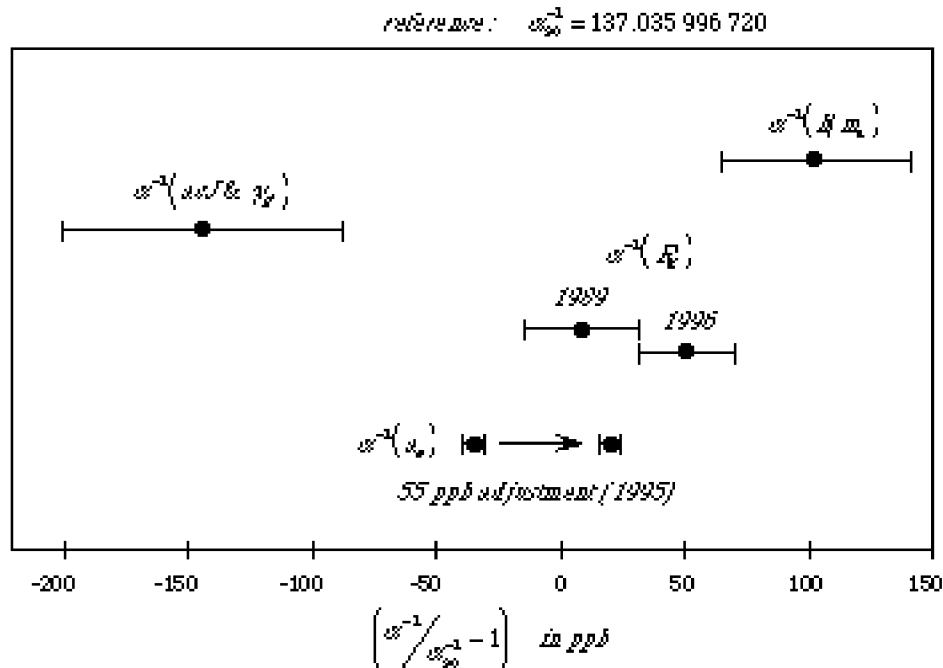


FIGURE 1. Current measurements of α below 100 ppb. $\alpha^{-1}(a_e) = 137.03599944(57)$ comes from a $g-2$ measurement of the electron and QED calculations [1,2]. The quantum Hall effect measurement $\alpha^{-1}(R_K) = 137.0359979(32)$ [4] was redone in 1996 [5] and the result was significantly different: $\alpha^{-1}(R_K) = 137.0360037(33)$. The value from AC Josephson effect and the magnetic moment of the proton is $\alpha^{-1}(acJ \& \gamma_p) = 137.0359770(77)$ [6]. Finally, $\alpha^{-1}(h/m_n) = 137.03601082(524)$ was measured using neutron beam interferometry [3]. The method discussed here should provide a value of α with a precision of about 5 ppb.

Hänsch's group in Munich has a preliminary value for the cesium's D_1 line (λ) with precision of few parts in 10^{10} [12]. Combining these results with our sub-ppb measurement of the atomic mass of Cs should give $N_A h$ to about 10 ppb. From Equation 1, this leads to a new determination of α to about 5 ppb.

The simplicity of the physics involved in this route to α recommends it as *the* preferred check of QED and the other α measurements. The most complicated physics is in the Rydberg constant, where the theory is more than adequate at the ppb level. By comparison, the quantized Hall effect may involve unknown solid state or sample geometry corrections, and ongoing programs to determine α from the fine structure of an atom involve new atomic energy level calculations with required ppt certainty, a much more dubious proposition.

EXPERIMENTAL SETUP

Our experimental apparatus and procedure for measuring ion mass ratios have been described earlier in the literature [13]. We will briefly outline here the general features of this experimental setup.

We trap a single ion in an orthogonally compensated Penning trap (of characteristic size $d = 0.549$ cm) [14,15] placed in a highly uniform magnetic field of 8.5 T. The harmonic axial oscillation of the ion in the trap is detected with a superconducting resonant circuit (niobium coil) having a stable Q of about 50000. The heart of our recently improved detector is a dc SQUID which has a technical noise floor 10 times smaller than the rf SQUID it replaced. The trapping voltage is adjusted to match the ion's axial frequency, ω_z , with the resonance frequency of the detection circuit (~ 160 kHz). Typical frequencies for the other two normal modes of motion of an Ar^+ ion in our trap are 3 MHz for the trapped cyclotron frequency, ω'_c , and 4 kHz for the magnetron motion, ω_m . The cyclotron and magnetron modes are observed and cooled indirectly by coupling them to the axial mode with a tilted quadrupole rf field applied on the split guard rings of our trap. With the proper amplitude-time product, such a coupling pulse (" π -pulse") can phase-coherently transfer all the energy

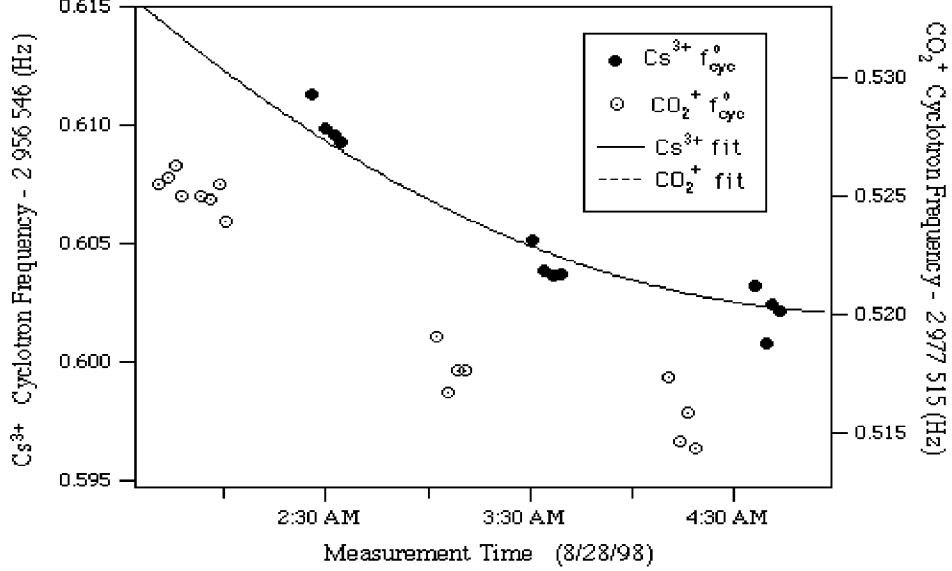


FIGURE 2. Typical night of data. In order to model the magnetic field fluctuations, a low-order polynomial is fitted simultaneously to both sets of data to determine the frequency ratio. A statistical test (F-test) is used to specify the order of the polynomial. In this case, a quadratic fit was used and the mass ratio $\text{Cs}^{3+}/\text{CO}_2^+$ is found to be 0.992 957 580 98(13).

of the cyclotron motion into the axial mode, in a manner qualitatively and formally equivalent to the Rabi two-state problem [16].

The absence of direct damping of the radial modes avoids the possibility of frequency shifts due to such coupling and means that the cyclotron resonance has practically zero linewidth. In addition, this procedure allows us to work with nearly ideal fields (pure quadrupole electric field and uniform magnetic field) which is helping us to keep systematic errors below few parts in 10^{11} .

The trapped cyclotron frequency ω'_c is obtained by directly exciting the ion's cyclotron mode, allowing it to evolve “in the dark” for a delay time T , and applying a “ π -pulse”. The axial ring down is then observed to extract the accumulated phase. Since phases are only defined between 0 and 2π , a series of measurements is made with different delay times T to determine the proper phase unwrapping. With a phase error of about 10 degrees, a measurement of one minute gives a precision of 10^{-10} . From this, we can extract the free space cyclotron frequency $\omega_c = qB/mc$ (where q is the ion charge, B is the magnetic field strength, m is the mass, and c is the speed of light) using the invariance relationship (valid even if the trap is tilted with respect to the B field or not perfectly cylindrical): $\omega_c = \sqrt{(\omega'_c)^2 + (\omega_z)^2 + (\omega_m)^2}$. We don't directly measure the magnetron frequency, ω_m , when taking data, but we use the relation

$$\omega_m \approx \frac{\omega_z^2}{2\omega_c} \left(1 + \frac{9}{4} \sin^2(\theta_m)\right) \quad (3)$$

where θ_m is the (measured) trap tilt. Finally, a mass ratio is determined by measuring alternately the cyclotron frequencies of two ions during one night (when the magnetic field fluctuations are reduced). A typical night of data is shown in Fig. 2. The data are fitted with a low-order polynomial to model the slow magnetic field drift and the ratio of cyclotron frequencies is determined from the fit. Because our magnetic field is fluctuating by few ppb per hour, the final precision on the mass ratio is currently limited by the number of data points we can take during a night, i.e., the time it takes to make and isolate a new ion. In this setup, the ions are produced in the trap, by ionizing neutral gas with an electron beam. Obviously this method is not selective and unwanted ions are also produced and trapped in the process. A crucial point for us is to be able to isolate the ion of interest as quickly as possible. The cases of Cs^{3+} and CO_2^+ are both particularly bad since Cs produces singly and doubly charged states (≥ 100 and 10 respectively for each Cs^{3+}) and CO_2 breaks into many fragments (CO^+ , C^+ , O^+ , etc.). Typically, 30 minutes was needed to make a new single ion of these two species, about twice the time needed to make simpler ions.

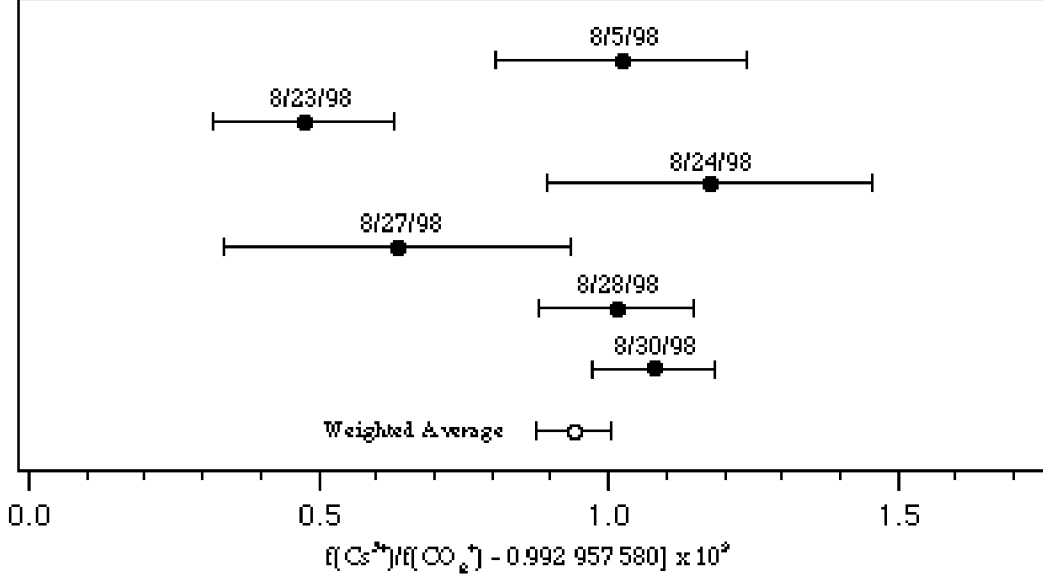


FIGURE 3. Results of the six nights of data taken in August 1998 for the $\text{Cs}^{3+}/\text{CO}_2^+$ frequency ratio. The weighed average of the points (also shown) is 0.992 957 580 937(065). Each ratio was adjusted to account for the image charge shift.

RESULTS

To determine the atomic mass of ^{133}Cs , we compared its triply charged state with CO_2^+ . Figure 3 shows the six measurements taken for that mass ratio. The weighted average of these data (also shown on the figure) gives the following result, with an error of 6.5×10^{-11} :

$$\frac{\omega_c[\text{Cs}^{3+}]}{\omega_c[\text{CO}_2^+]} = \frac{m[\text{CO}_2^+]}{m[\text{Cs}^{3+}]} = 0.992\,957\,580\,937(065). \quad (4)$$

From this ratio, the atomic mass of ^{133}Cs can be computed by adding the appropriate number of electron masses, their ionization energies, the molecular binding energy of CO_2 , and using the values previously measured by our group for the atomic mass of ^{16}O [9]. The dominant source of error in this process, the mass of ^{16}O is known to 1.3×10^{-10} and translates into a 9×10^{-11} error in the mass of ^{133}Cs . The ionization energy of the third electron in Cs is only known to 1 eV (out of 35 eV), but affects the final result by only 8×10^{-12} . Adding in quadrature all these contributions, we find a value for the atomic mass of ^{133}Cs with a precision of 0.11 ppb.

Table 1 shows the estimated systematic errors, using the measured field inhomogeneities ($B_1 \simeq 0.134$ G/cm, $B_2 \leq 0.24$ G/cm², $C_4 \leq 10^{-4}$), and assuming a systematic 1% imbalance in the cyclotron radii ($\sim 114\mu\text{m}$) of the compared ions. Here, C_4 is the unitless coefficient of the leading order correction to the potential of an “ideal” (quadratic) Penning trap:

$$\Phi_s = \frac{1}{2}C_2 \frac{z^2 - 1/2\rho^2}{d^2} + \frac{1}{2}C_4 \frac{z^4 - 3z^2\rho^2 + 3/8\rho^4}{d^4} + \dots \quad (5)$$

The dominant shift comes from the image charge that the ion sees in the ring electrode whose cyclotron motion acts back on the ion. For our trap, a calculation by numerical relaxation method determined this shift to be $54 \mu\text{Hz}/\text{charge}$, accurate to $\sim 10\%$. For Cs^{3+} , this gives a systematic shift of the cyclotron frequency of 5.6×10^{-11} which we included in the above mass ratio (Equation 4 and Fig. 3). (The overall shift on the ratio is only 3.6×10^{-11} since CO_2^+ is also shifted.)

In order to check further for systematic errors at the 10^{-10} level, we plan to measure another mass ratio with a different charge state of Cs (e.g. $\text{Cs}^{2+}/\text{C}_5\text{H}_6^+$, or $\text{Cs}^{4+}/\text{O}_2^+$). Making a measurement at a different mass to charge ratio involves changing the voltages on the trap, the frequencies used, and should therefore provide a

TABLE 1. Error budget.

Source	$\frac{\Delta\omega_c}{\omega_c}$
Bottle shift ($B_2 \sim 0.24 \text{ G/cm}^2$)	0.3×10^{-11}
Electrostatic ($C_4 \sim 10^{-4}$)	0.2×10^{-11}
Relativity	9.5×10^{-15}
Axial frequency pulling	4.4×10^{-11}
Image charge shift ^a	5.6×10^{-11}
Antisymmetric patch	0.1×10^{-11}

^a Accounted for in the quoted ratios.

strong test for systematics. Each mass in our previous mass table was obtained from at least two independent measurements [9]. Because this has not yet been done for Cs, we quote here an error of 0.22 ppb, twice the error obtained from above discussion. The result for our preliminary value for the atomic mass of ^{133}Cs is then:

$$M[^{133}\text{Cs}] = 132.905\,451\,938(28) \text{ u.} \quad (6)$$

Compared with the currently accepted value $M[^{133}\text{Cs}] = 132.905\,447(3) \text{ u}$, this is an improvement in precision by a factor of more than 100 [17]. This not only eliminates (for now) the atomic mass of Cs as a limiting quantity for the determination of a new value of the fine structure constant, but is of general interest for the atomic physics community. Indeed, Cs is widely used in high precision experiments, for example as a reference ion for mass measurements of radioactive isotopes [18].

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