Single Ion Mass Spectrometry and the Fine Structure Constant

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Abstract. Using a Penning trap single ion mass spectrometer, we have measured the atomic masses of 13 isotopes, many important for fundamental metrology and fundamental constants. The accuracy of the measurements, $\approx 10^{-10}$, is typically two orders of magnitude better than previously accepted values. A wide variety of self consistency checks greatly reduces the possibility of unknown systematic errors.

As part of a program to determine the Molar Planck constant $N_A h$ and the fine structure constant α, we measured the masses of ^{133}Cs, ^{87,85}Rb, and ^{23}Na. Our high accuracy atomic mass measurements can be combined with values of h/m_{atom} from atom interferometry measurements and accurate wavelength measurements for different atoms to give several independent determinations of $N_A h$ and α. This route to α through $N_A h$, the atomic mass of the proton M_p, the electron to proton mass ratio m_e/m_p, and the Rydberg constant R_∞ is based on simple physics. It can potentially achieve the several ppb accuracy needed to test the QED determination of α extracted from measurements of the electron g factor.

I INTRODUCTION

Using a Penning trap single ion mass spectrometer, we have measured the atomic masses of 13 isotopes several of which are important for fundamental metrology and fundamental constants. The accuracy of the measurements, typically 0.1 ppb or 10^{-10}, represents one to three orders of magnitude improvement over previously accepted values. A wide variety of self consistency checks greatly reduces the possibility of unknown systematic errors. Our measurements [1] have contributed to a precise recalibration of the γ-ray spectrum and provided an atomic reference for a realization of an atomic definition of the kilogram to replace the artifact standard currently in use.

Most recently, we measured [2] the masses of ^{133}Cs, ^{87,85}Rb, and ^{23}Na as part of a program to determine the Molar Planck constant $N_A h$ and fine structure constant

TABLE 1. Measured neutral alkali masses.

Species	MIT Mass (u)	ppb	1995 Mass (u) [5]	ppb	$difference/\sigma_{1993}$
^{133}Cs	132.905 451 931 (27)	0.20	132.905 446 800 (3200)	24.0	1.6
^{87}Rb	86.909 180 520 (15)	0.17	86.909 183 500 (2700)	31.0	-1.1
^{85}Rb	84.911 789 732 (14)	0.16	84.911 789 300 (2500)	29.0	0.2
^{23}Na	22.989 769 280 7 (28)	0.12	22.989 769 670 0 (2300)	9.8	-1.7

α from measurements of h/m_{atom}. A further motivation for our measurements is that Cs and Rb are used as reference masses for measurements of heavy radioactive nuclei which are important for modeling astrophysical heavy element formation [3], [4].

A Molar Planck constant $N_A h$

The Molar Planck constant $N_A h$ is an important quantity in metrology and for fundamental physics. New values of $N_A h$ at the few ppb level in combination with measurements of h (such as a recent 87 ppb measurement [6]) can yield values of N_A with ppb level accuracy. Precise values of $N_A h$ would also provide a way to check QED and test the unity of physics across disciplines by helping to accurately determine the fine structure constant α.

Avogadro's number N_A is the ratio of the SI and atomic units of mass [7]. The unified atomic mass unit is defined by setting the atomic mass of ^{12}C to be exactly 12. N_A is defined as the number of elementary entities in one mole (the amount of substance whose mass in grams equals its atomic mass) and has an approximate value of $N_A \approx 6.022 \times 10^{23}$/mole. Avogadro's number can then be written as the ratio of any atom's mass in atomic mass units denoted by M_{atom} and in SI units denoted by m_{atom}

$$N_A = \frac{M_{atom}}{m_{atom}} \times 10^{-3} \tag{1}$$

where the factor of 10^{-3} arises because of the definition of Avogadro's constant in terms of grams rather than the SI unit kilogram.

Transposing Eq. 1 shows that $1/N_A$ can be regarded as the universal mass quantum (in grams). The mass of any elementary entity is then its atomic mass (i.e. mass quantum number) times this mass quantum. (Unlike most other quantized quantities, the quantum number is not a simple rational number.)

Thus, $N_A h$ is the ratio of h to the mass quantum, a universal h/m. It can be obtained from a particular value of h/m_{atom} by multiplying by M_{atom}

$$N_A h = \frac{h}{m_{atom}} M_{atom} \times 10^{-3} \ . \tag{2}$$

Our technique for measuring M_{atom} therefore allows measurements of h/m_{atom} using different atoms to be compared with $\approx 10^{-10}$ accuracy.

In both Schroedinger's equation for a free particle and the expression for magnetic moments of elementary entities, h and m always occur in the ratio h/m. Thus h/m is often measured in experiments involving simple quantum expressions. By equating the classical ($p = m_x v$) and quantum ($p = h/\lambda_{dB}$) expressions for the momentum of a particle, we see that measurements in SI units of the deBroglie wavelength λ_{dB} and the velocity v of a particle combine to measure h/m_X in SI units,

$$v\lambda_{dB} = \frac{h}{m_X} \ . \tag{3}$$

Comparison of the energy and wavelength of a photon would also yield a value of $N_A h$, but at accuracies of $\approx$ 100ppb [1].

Precision mass spectrometry now allows several independent determinations of $N_A h$ from measurements of h/m_{atom} using different atoms possessing very different experimental systematic errors to be compared with no reduction in accuracy at the 0.1 ppb level.

B Fine Structure constant α

An accurate value of the Molar Planck constant leads to a new determination of the fine structure constant α. Noting the definitions of $\alpha \equiv e^2/\hbar c$ and the infinite-nuclear-mass Rydberg constant $R_\infty \equiv (2\pi^2 m_e e^4)/(h^3 c) \approx 1.09 \times 10^5$ cm^{-1} (cgs units) makes it easy to see that

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_e} = \frac{2R_\infty}{c}\frac{m_p}{m_e}\frac{N_A h}{M_p}10^3 \ . \tag{4}$$

R_∞ is known with an accuracy of 0.008 ppb [8]. m_p/m_e has been measured to 2 ppb [9]. The mass of the proton in atomic units M_p has been measured by our group to 0.5 ppb [1], and Van Dyck et al. have recently reported a value of M_p accurate to 0.14 ppb [10]. The speed of light c is a defined constant. Thus an independent measurement of $N_A h$ is capable of determining α to 1 ppb.

The possibility of redundancy in the experimental determination of $N_A h$ would greatly enhance the confidence in determinations of α from Eq. 4. The mass ratio m_p/m_e would be the only quantity without more than a single direct measurement at the ppb level (a recent value of $m_e/m_{^{12}C}$ extracted from theory and boundstate electron g factor measurements in hydrogenic ^{12}C has confirmed the value to about 2 ppb [11]). This is not a trivial point since it would take a considerable weight of evidence to believe that disagreement between the QED and $N_A h$ determinations of α signifies some error in QED.

II MEASURING ATOMIC MASSES

A Experimental Technique

We obtain absolute atomic masses M with relative accuracies 0.1 ppb from mass ratios relating the unknown mass to the atomic mass standard ^{12}C. Experimentally, we make a mass comparison by measuring the cyclotron frequency (which is inversely proportional to the mass) of a single molecular or atomic ion in a large and highly-uniform magnetic field (8.5T). The ion is held in a small region of space by the magnetic field which provides radial confinement and by an additional weak dc quadrupole electric field which provides confinement along the axial direction. This combination of confining fields is known as a Penning trap. Trapping the ion allows the long observation time necessary for high precision. Using a single ion is crucial for high accuracy since this avoids the complex frequency perturbations caused by the coulomb interaction between multiple ions.

The combination of magnetic and electric fields in our Penning trap results in three normal modes of motion: trap cyclotron, axial, and magnetron, with frequencies $\omega_c'/2\pi \approx 5$ MHz $>> \omega_z/2\pi \approx 0.2$ MHz $>> \omega_m/2\pi \approx 0.002$ MHz, respectively. The free-space cyclotron frequency ω_c is recovered from the following expression (invariant with respect to trap tilts and ellipticity) [12]:

$$\omega_c = qB/mc = \sqrt{(\omega_c')^2 + (\omega_z)^2 + (\omega_m)^2}. \tag{5}$$

We have developed ultrasensitive superconducting electronics to detect the miniscule currents ($\approx 10^{-14}$ amperes) that the ion's axial motion induces in the trap electrodes. The detector consists of a DC SQUID coupled to a low loss superconducting resonant transformer ($Q \approx 4 \times 10^4$) connected across the endcaps of the Penning trap. Our detection noise is currently dominated by the 4 K Johnson noise present in the resonant transformer. Detection damps the axial motion at a rate $\gamma_z \sim 1$ s^{-1} quickly bringing the axial motion to equilibrium at 4 K.

The trap cyclotron motion is detected phase coherently via an RF coupling to the axial motion (the same coupling is also used to "cool" both the trap cyclotron and magnetron modes). In the spirit of the separated oscillatory fields technique, RF drives and couplings are applied only briefly at the beginning and end of a measurement thus eliminating systematics and uncertainties involved with continuously observing the trap cyclotron motion. Briefly driving the trap cyclotron motion with a fixed phase and then measuring the accumulated phase versus delay time yields the trap cyclotron frequency. A typical phase accumulation time of 1 minute yields a precision of $\approx 2 \times 10^{-10}$. The precision of a single cyclotron frequency measurement is limited by the $\approx 2.5 \times 10^{-10}$ short term fluctuations of the magnetic field. The typical precision with which we can compare the cyclotron frequencies of two ions ($\approx 1 \times 10^{-10}$) is limited mainly by magnetic field drift over time scales between 1 and 15 minutes.

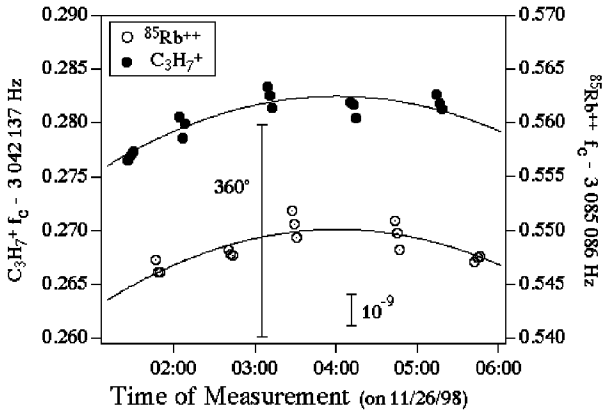

FIGURE 1. Typical night of data. The solid line is a second order polynomial fit to the data. The 360° bar shows the magnitude in Hz of a 360° error in phase unwrapping.

From Eq. 5, it is clear that the magnetron frequency does not need to be known very accurately to recover the free-space cyclotron frequency. It is accurate enough just to use the measured values of ω_c' and ω_z to calculate $\omega_m \approx (\omega_z^2/2\omega_{cyc}')(1 + 9/4\sin^2\theta_m)$ where $\theta_m = 0.16°$ is the measured angle between the B-field and trap axes obtained by actually measuring the magnetron frequency once. The effect of θ_m on a cyclotron frequency ratio is at most 0.002 ppb.

B Recent Alkali Measurements

To determine the masses of the alkali atoms ^{133}Cs, ^{87,85}Rb, and ^{23}Na, we measured the free-space cyclotron frequency ratios $r \equiv \omega_{c2}/\omega_{c1}$ listed in Table 2. The reference ions were selected because of the similar mass to charge ratios (aiding in the reduction of systematic errors) and because we have previously measured the atomic masses of each of the consituent atoms.

A cyclotron frequency ratio r of two different ions was determined by a run measuring a cluster of ω_c values for an ion of type A, then for type B, etc. In a typical 4-hour run period (from 1:30-5:30 am when the nearby electrically-powered subway was not running), we recorded about 5 alternations of ion type (Fig. 1). The measured free-space cyclotron frequencies exhibited a common slow drift. We fit a common polynomial $\Omega(t)$ plus a frequency difference to the data. From this we obtained the frequency ratio r_n and the uncertainty σ_n for a single night. The average order of $\Omega(t)$ was 3 and was chosen using the F-test criterion [13] as a guide.

The distribution of residuals from the polynomial fits had a Gaussian center with a standard deviation $\sigma_{resid} = 0.28$ ppb and a background ($\approx 2\%$ of the points) of non-Gaussian outliers, as in our earlier measurements [1]. As in [1] we chose to handle the non-Gaussian outliers using a robust statistical method to smoothly deweight them [14].

TABLE 2. Measured ion cyclotron frequency ratios, corrected for systematics.

A/B	$\frac{\overline{\omega}_c}{2\pi}$ (MHz)	Nights	ω_c[A]/ω_c[B]
$^{133}Cs^{+++}/CO_2^+$	2.968	5	0.992 957 580 983 (135)
$^{133}Cs^{++}/C_5H_6^+$	1.977	4	0.993 893 716 487 (427)
$^{87}Rb^{++}/C_3H_8^+$	2.994	2	1.013 992 022 591 (266)
$^{87}Rb^{++}/C_3H_7^+$	3.028	3	0.990 799 127 824 (174)
$^{85}Rb^{++}/C_3H_7^+$	3.064	2	1.014 106 122 230 (164)
$^{85}Rb^{++}/C_3H_6^+$	3.100	2	0.990 367 650 976 (285)
$^{23}Na^+/C_2^+$	5.578	2	1.043 943 669 690 (076)
$^{23}Na^{++}/C^+$	11.155	2	1.043 944 716 614 (098)

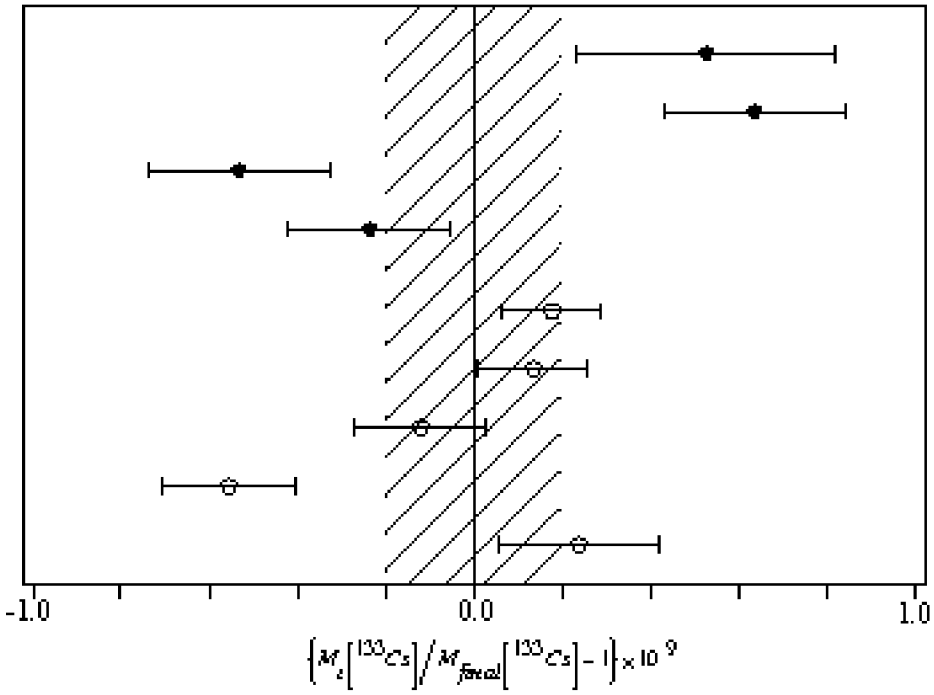

FIGURE 2. Example of Variation of Mass Ratio from Night to Night. A measurement of the neutral mass of ^{133}Cs is extracted from each night's run of cyclotron frequency ratio measurements and plotted in ppb relative to our final published value of the neutral mass of ^{133}Cs. The open and closed circles are from frequency ratio measurements of Cs^{+++}/CO_2^+ and $Cs^{++}/C_5H_6^+$ respectively. The error bars on each night's measurement are extracted from the low order polynomial fit to both ion's cyclotron frequencies and reflects the distribution of the cyclotron frequency measurements during that night. The shaded region represents the one sigma confidence interval arrived at in the final analysis.

As shown in Table 2 and Figure 2, we measured each frequency ratio on more than a single night. For ratios involving Cs and Rb the measured ion mass ratios were distributed from night to night with a scatter larger than the uncertainty predicted from the statistical scatter within a single night ($\chi^2_\nu \approx 5$) . By contrast $\chi^2_\nu \approx 0.8$ for ratios involving Na. None of the earlier data taken using this apparatus [1] exhibited these excess night-to-night variations. A search for the source of these fluctations is discussed elsewhere [2] and was unsuccesful. To account for this excess scatter, the uncertainties in the weighted average of the ion mass ratios involving $Cs^{++}/C_5H_6^+$ and Cs^{+++}/CO_2^+ were increased by factors of 2.6 and 2.2 respectively so that $\chi^2_\nu = 1$. Since the Rb measurements all had similar m/q, we assumed that the night-to-night fluctuations involving the Rb ratios were drawn from a common statistical distribution. Therefore, we increased the uncertainties for the Rb ion ratios by a factor of 2.2 so that the overall Rb χ^2_ν was reduced to 1. For Na, $\chi^2_\nu \approx 0.8$ so the uncertainties were not adjusted.

By correcting for molecular binding and electron ionization energies, we obtained a set of neutral mass difference equations. We added to this the set of mass difference equations used to determine the atomic masses in [1]. Solution of this overdetermined set of linear equations gave the neutral masses of the alkali metals (see Table 1) with uncertainties σ_{od} as well as the previously published neutral masses with $\chi^2_\nu = 0.83$. The previously published masses were essentially unchanged and so are not reported here. Uncertainties in M[^{16}O] and M[H] (the only atoms other than ^{12}C in the ratios of Table 2) contributed < 0.1 ppb uncertainty to the alkali masses.

The use of two distinct reference ions gave a check on systematics by providing two independent values for each neutral mass. For Rb and Cs χ^2_ν is less than 1. However, because of the larger uncertainty on M[Cs] from $Cs^{++}/C_5H_6^+$ we quote a final uncertainty of 0.20 ppb $> (\sigma_{od}(Cs) = 0.16$ ppb). For ^{87,85}Rb we quote $\sigma_{od}(^{87,85}\mathrm{Rb})$ as the final uncertainties. For the neutral masses from Na^{++}/C^+ and Na^+/C_2^+, the statistical uncertainties are 0.09 and 0.07 ppb respectively. The 0.2 ppb disagreement of the two values may be evidence for a systematic at the 0.1 ppb level. To reflect this we assigned M[^{23}Na] a 0.12 ppb uncertainty $> (\sigma_{od}(Na) = 0.06$ ppb) which spans both independent measurements.

Table 1 quotes the final values for M[^{133}Cs], M[^{87}Rb], M[^{85}Rb] and M[^{23}Na] obtained from the solution of the overdetermined set of mass difference equations with uncertainties from the above discussion. Also included in Table 1 are the alkali masses from the 1995 mass evaluation [5]. Our values differ from the 1995 values by typically $1.5\sigma_{1995}$, which suggests that the uncertainties on the masses from the 1995 evaluation were slightly underestimated. Our value for M[^{133}Cs] lies within the uncertainty of the recent measurement of M[^{133}Cs] reported by the SMILETRAP collaboration [15].

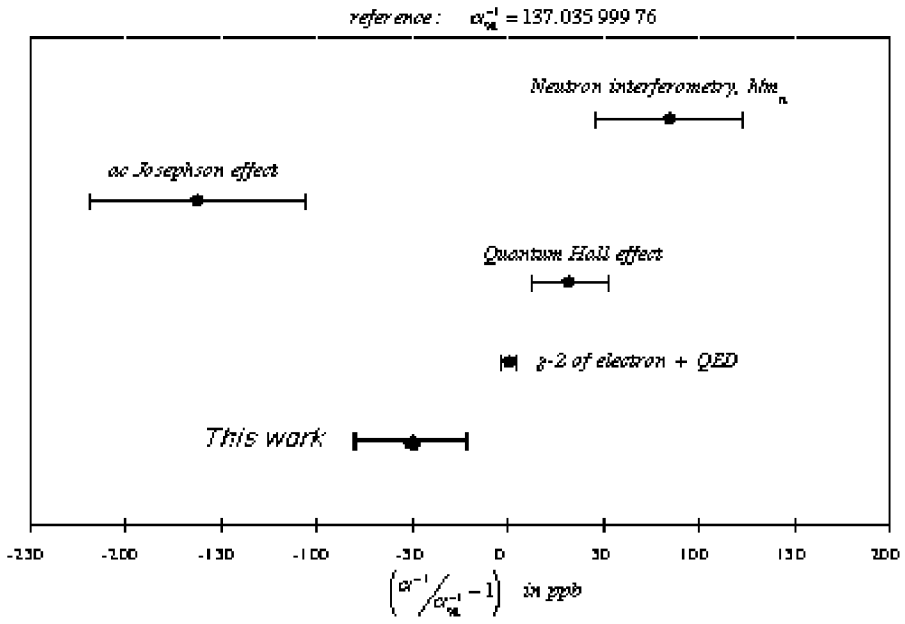

FIGURE 3. Precision Measurements of the Fine Structure Constant plotted with respect to the 1998 CODATA recommended value of α [16]. The relative uncertainties are: ac Josephson effect (56 ppb). Quantum Hall effect (24 ppb). g-2 of electron and QED (4 ppb). Neutron Interferometry, h/m_n (35 ppb). This Work yields a preliminary value with the hope of further improvement in accuracy in the future.

III PRECISE ROUTES TO α

The fine-structure constant α appears in many contexts in physics and arises in diverse physical systems because of its role as the dimensionless coupling constant describing the interaction of electrons and positrons with the electromagnetic field. This results in numerous experimental routes to measuring α involving several different disciplines. The diversity of these methods allows stringent tests across these disciplines, in principle allowing an outside check on each discipline. Figure 3 gives a compilation of some of the most accurate measurements of α which we will briefly review.

A AC Josephson Effect

The AC Josephson effect is the oscillation of frequency ν observed when a voltage V is applied across a Josephson junction. Theory predicts that ν and V are related by [17]

$$\nu = (\frac{2e}{h})V \ . \tag{6}$$

This relationship has been shown to be independent of the materials used to fabricate the Josephson junction to a level of 2 parts in 10^{16} [18]. Thus measurements of the AC Josephson effect provide a way to measure e/h to high precision.

The highest-precision route relating $2e/h$ to α is

$$\alpha^2 = (\frac{4R_\infty \gamma'_p}{c})(\frac{\mu_B}{\mu'_p})(\frac{h}{2e}) \tag{7}$$

where R_∞ is the infinite-nuclear-mass Rydberg constant, γ'_p is the proton gyromagnetic ratio (the prime refers to the fact that γ'_p is measured using nuclear magnetic resonance on an H_2O sample), μ'_p is the proton magnetic moment, and $\mu_B = e\hbar/2m_e c$ is the Bohr magneton.

R_∞ is known to 0.008 ppb [8], γ'_p is known to 110 ppb [19], (μ'_p/μ_B) is known to 10 ppb [20], and $(2e/h)$ is known to 30 ppb. Thus the overall uncertainty on α via this route is limited by γ'_p to 56 ppb.

B Quantum Hall Effect

For any effectively two-dimensional electron gas system in a magnetic field and cooled to cryogenic temperatures, the Hall resistance $R_H = V_T/I$ (where V_T is the voltage generated across the sample transverse to the direction in which current I flows) is quantized:

$$R_H = \frac{h}{e^2 n} \qquad n = 1, 2, 3 \ldots \tag{8}$$

Measurement of R_H by comparison to a reference impedance based on a calculable capacitor can provide a high precision measurement of $h/e^2 \approx 25\ 813\ \Omega$; this is essentially a direct measurement of α. Two measurements of α by this route have been made by the same group at NIST. Both measurements have 24 ppb precision, but the values differ by 42 ppb. It is felt that the 1997 measurement is the correct value and is the value shown in Fig. 3 [21], [22], [23].

C Electron & Positron Anomalous Magnetic Moment

The spin magnetic moment of the electron μ_e may be written as $\mu_e = g(1/2)\mu_B$ where $\mu_B = e\hbar/2mc$ is the Bohr magneton, 1/2 is the electron spin in units of $\hbar$ and g is the electron "g-factor". Simple classical models of the electron predict $g = 1$ while the Dirac's relativistic electron equation predicts $g = 2$. The development of QED was necessary to explain why careful measurements revealed that in fact g was not exactly equal to 2. Precisely predicting the so-called "anomaly" $a_e \equiv (g-2)/2 \approx 0.001\ldots$ is one of the great successes of QED.

The electron anomaly a_e can be measured to high precision by comparing the electron spin-flip frequency $\omega_s = g\mu_B B/\hbar$ to the cyclotron frequency $\omega_c = eB/mc$

(cgs units). Simple algebra gives $\omega_s = (g/2)\omega_c$ so that $a_e = (g/2)-1 = (\omega_s/\omega_c)-1 = (\omega_s - \omega_c)/\omega_c$. Using an electron confined in a Penning trap, $(\omega_s - \omega_c)$ can be measured directly. VanDyck et al. at the University of Washington pioneered the trapping of single electrons for the measurement of $(g-2)$. The most precise results from the UW group are [24]:

$$a_{e^-} = 1\ 159\ 652\ 188.4\ (4.3) \times 10^{-12} \quad \text{electron anomaly} \tag{9}$$

$$a_{e^+} = 1\ 159\ 652\ 187.9\ (4.3) \times 10^{-12} \quad \text{positron anomaly} \tag{10}$$

Complementary to high-precision measurements of a_e are high-order QED calculations of a_e. These calculations have been contributed to by a number of authors since Schwinger and Feynman. The dean of this field is Tochiro Kinoshita of Cornell University. Using the VEGAS Monte-Carlo integration routines [25], [26], [27] running on a supercomputer he has calculated QED corrections up to and including fourth-order in α. The QED contribution to a_e is

$$A_1 = A_1^{(2)}(\alpha/\pi) + A_1^{(4)}(\alpha/\pi)^2 + A_1^{(6)}(\alpha/\pi)^3 + A_1^{(8)}(\alpha/\pi)^4 + A_1^{(10)}(\alpha/\pi)^5 + \ldots \tag{11}$$

This is the dominant contribution to the total expression:

$$a_e = A_1 + A_2(m_e/m_p) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau) \tag{12}$$

A_2 and A_3 are small mass-dependent QCD terms. The current best values for the QED terms are [27]:

$$\begin{aligned}
A_1^{(2)} &= \ 0.5 \quad \text{exact} \\
A_1^{(4)} &= -0.328\ 478\ 965\ldots \quad \text{purely analytic} \\
A_1^{(6)} &= \ 1.181\ 259\ (40) \quad \text{numerical} \\
A_1^{(6)} &= \ 1.181\ 241\ 456\ldots \quad \text{analytic} \\
A_1^{(8)} &= -1.409\ 2\ (384) \quad \text{numerical} \\
A_1^{(10)} &= \ \text{unknown}
\end{aligned} \tag{13}$$

As discussed above, $A_1^{(2)} = 0.5$ is an exact analytic result. $A_1^{(4)}$ has contributions from 7 Feynman diagrams, all obtained analytically. $A_1^{(6)}$ has contributions from 72 Feynamn diagrams. Until recently this term was calculated as a hybrid of analytical results for 57 diagrams and numerical results for 15 diagrams. More precise numerical work by Kinoshita [25] led to the discovery of an error in one of the analytical integrals which has since been corrected. In addtion, the remaining diagrams have also since been calculated analytically [28], [29], giving a result which agrees well with the precise numerical result. $A_1^{(8)}$ is the current challenge. It consists of 891 Feynman diagrams which can be reduced to 86 integrals using the Ward-Takahashi identity. Each integrand has about 20000 terms. Thus the

computations are extensive. A point-by-point cancellation of singularities is used to renormalize the integrals and make them integrable [27].

An extremely precise value of α can be obtained from the average of the experimental measurements of a_{e^-} and a_{e^+} and the QED calculations described above. This value is:

$$\alpha^{-1} = 137.035\ 999\ 93\ (52) \tag{14}$$

This value has a precision of 3.8 ppb and is the most precise available by a factor of 5. It can be seen from Fig. 3 that the $(g-2)$/QED measurement is in reasonable agreement with the 1997 Quantum Hall effect measurement, and disagrees with the 1998 h/m_n measurement (to be discussed below). The interest in the $N_A h$ route to α using atoms stems from the fact that it is an independent method with an accuracy which is potentially comparable to that of the $(g-2)$/QED measurement, and thus it may help to shed light on the apparent discrepancy in the measured values of α.

D $N_A h$: Neutron Interferometry and Photon Recoil

As discussed earlier, equating the classical ($p = mv$) and quantum ($p = h/\lambda_{dB}$) expressions for particle momentum we see that measurements of the deBroglie wavelength λ_{dB} and the velocity v of a particle provides a way to measure h/m_X. Thus measurements of λ_{dB}, v, and M_X can provide a (nearly) QED-independent measurement of α, like the AC Josephson and Quantum Hall effect methods. It is true that QED calculations of the $2P_{1/2}$, $2P_{3/2}$ and 8D Lamb shifts are needed to allow R_∞ to be obtained from measurements of the 1S-2S and 2S-8D transitions in hydrogen [8], but these corrections are small and do not need to be known to high accuracy (the largest calculational uncertainty contribution to R_∞ is 0.00026 ppb due to the 8D Lamb shift). Thus even in the event that QED is approximate, this method for measuring α is robust and reliable. If a ppb-level measurement of α can be made by this method QED could be tested for the first time at the ppb-level allowed by the QED/(g-2) measurement.

Kruger et al. have used neutron interferometry to precisely measure λ_{dB} and v for a beam of neutrons, resulting in a measurement of h/m_n with an accuracy of 73 ppb [30]. Combining this with a precise measurement of M_n (from Penning trap measurements of $M[^2\mathrm{H}]$, $M[^1\mathrm{H}]$ and γ-ray measurements of the nuclear binding energy of ^{2}H [1], [31], [32]) results in a value of α with a precision of 37 ppb (see Fig. 3). This measurement illustrated the promise of the method but was not accurate enough to test QED at the ppb level.

The photon recoil of an atom provides another way to measure h/m. After a photon absorption/emission process an atom recoils with a velocity $v_r = h/m\lambda$ where λ is the wavelength of the photon (which is equal to λ_{dB} of the recoiling atom). The resultant Doppler shift $\Delta\omega = (4\pi^2\hbar)/(m\lambda^2)$ of the atomic absorption and emission frequencies with respect to the laboratory frame provides a way to

measure h/m in terms of the resonant wavelength λ. For Cs, the group of S. Chu at Stanford University is using atom interferometry to measure $\Delta\omega$ and the group of T. Hansch at MPI in Garching has developed optical frequency measurement techniques [33] to measure λ. Both of these elegant experiments are described in papers adjacent to this one.

It seems reasonable to expect that atom interferometry experiments exploiting the properties of Bose Einstein Condensation will lead to future measurements of h/m_{atom} with increased accuracy. It is therefore likely that measurements of h/m_{atom} will be performed in ^{23}Na and the two isotopes ^{87,85}Rb [34]. This work lowers the error in M for all of these systems sufficiently that it will not be significant for 1 ppb measurements of $N_A h$.

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