I. SQUEEZING OF THE WIGNER DISTRIBUTION

In this section, we provide a mathematical description of squeezing by analyzing the evolution of a nuclear spin state characterized by a Gaussian Wigner distribution. As discussed in the main text, for a large spin initially oriented in the x direction, and for short times before the Wigner distribution extends significantly around the Bloch sphere, the Wigner distribution in a locally flat patch of Bloch sphere evolves as $f_t(I_y, I_z) = A e^{-\frac{1}{2}v^T Q v}$, see Eq.(10), with

$$v = \begin{pmatrix} I_y \\ I_z \end{pmatrix}, \quad Q = \frac{1}{\Delta I^2} \begin{pmatrix} 1 & \lambda I t \\ \lambda I t & 1 + (\lambda I t)^2 \end{pmatrix}. \quad (S1)$$

Here $\Delta I = \Delta I_{0}^{y,z}$ characterizes the transverse fluctuations in the initial nuclear spin state.

For times $t > 0$, the circular Wigner distribution is deformed to an ellipse, with major and minor axes determined by the quadratic form $Q$ in Eq.(S1). As shown in Fig.3 of the main text, stretching in one direction ($y'$) is accompanied by squeezing in the perpendicular direction ($z'$), such that the phase space volume of the Wigner distribution is preserved. The major and minor axes $y'$ and $z'$, which lie parallel to the eigenvectors of $Q$, are rotated relative to $y$ and $z$ by an angle $\theta$:

$$\begin{pmatrix} I'_y \\ I'_z \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} I_y \\ I_z \end{pmatrix}. \quad (S2)$$

The angle $\theta$ can be found by extremizing the quantity

$$W = w_\theta^T Q w_\theta, \quad w_\theta = \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix}. \quad (S3)$$

with respect to $\theta$. Using the identity $[1 - \tan^2 \theta]/2 \tan \theta = \cot 2\theta$, we find

$$\cot 2\theta = \lambda I t / 2. \quad (S4)$$

Note that Eq.(S4) has two solutions $\theta_{1,2}$ separated by $90^\circ$, as expected for a symmetric form.

In the eigenbasis, we write

$$f_t(I'_y, I'_z) = A \exp \left[ -\frac{1}{2} \left( \frac{I'_y}{\Delta I_+(t)} \right)^2 - \frac{1}{2} \left( \frac{I'_z}{\Delta I_-(t)} \right)^2 \right], \quad (S5)$$
with
\[ \Delta I^2_\pm(t) = \Delta I^2 \left( 1 + \frac{(\lambda t)^2}{2} \left[ 1 \mp \sqrt{1 + \frac{4}{(\lambda t)^2}} \right] \right)^{-1}. \] (S6)

In the long time limit \( \lambda t \gg 1 \), the width \( \tilde{\Delta} I(t) \equiv \Delta I_-(t) \) of the squeezed component reduces to Eq.(11).

II. PHASE DIFFUSION

The effect of time-dependent fluctuations of electron spin polarization about the mean field value can be analyzed within the rate equation model by introducing a time-dependent quantity
\[ \tilde{S}^z(t) = S^z + \delta S^z(t). \] (S7)

The fluctuating part \( \delta S^z \) can be modeled as delta-correlated noise \( \langle \delta S^z(t') \delta S^z(t'') \rangle \propto \delta(t' - t'') \), with an intensity determined by the rate process, Eq.(5). As shown in Supplementary Section III, such noise generates phase diffusion,
\[ \langle \delta \theta^2(t) \rangle = \kappa t, \quad \kappa = 2A^2 \frac{(W + \Gamma_1)W}{(2W + \Gamma_1)^3}, \] (S8)

where \( \delta \theta \) is the fluctuating part of the Larmor precession angle, \( I^x + iI^y \propto e^{i(\theta + \delta \theta)} \). The phase diffusion can be accounted for by adding a diffusion term with diffusivity \( \tilde{\kappa} = I^2\kappa \) to the equation describing the time evolution of the Wigner distribution.

An important consequence of phase diffusion is non-conservation of phase volume, which can be illustrated by the evolution of a Gaussian Wigner distribution. Similar to the mean-field case, Eq.(10), such a distribution evolves in time as
\[ f_\lambda(I^y, I^z) = A'(t) \exp \left[ -\frac{(I^z)^2}{2\Delta I^2} - \frac{(I^y + I\lambda t I^z)^2}{2(\Delta I^2 + \tilde{\kappa} t)} \right], \quad t > 0. \] (S9)

Initially, phase diffusion leads to a broadening of the Wigner distribution, characterized by the factor \( \sqrt{1 + \tilde{\kappa} t/\Delta I^2} \), which grows like \( t^{1/2} \) for \( \tilde{\kappa} t < \Delta I^2 \). At later times, \( \Delta I\lambda t \gg \sqrt{\tilde{\kappa} t} \), the behavior is dominated by the linear in \( t \) twisting/stretching dynamics. Therefore for times satisfying \( t > t_{\text{noise}} = \frac{2A^2}{\lambda^2} \), the coherent stretching overwhelms the effect of phase diffusion.

The efficiency of squeezing in the presence of phase diffusion can be estimated as follows.
At long times \( t \gg t_{\text{noise}} \), the factor \( \sqrt{1 + \tilde{\kappa} t/\Delta I^2} \) describes an increase of the width of the
Wigner distribution compared to its ideal squeezed value $\tilde{\Delta}I$ in Eq.(11). Combining the $t^{1/2}$ smearing due to phase diffusion with the $t^{-1}$ squeezing, we find that the width of the Wigner distribution decreases as $t^{-1/2}$ at long times:

$$\tilde{\Delta}I_{\text{noise}} = \tilde{\Delta}I(t)\sqrt{1 + \tilde{\kappa}t/\Delta I^2} \approx \frac{\tilde{\kappa}^{1/2}}{\lambda_I} t^{-1/2}$$  \hspace{1cm} (S10)

This expression describes the slowing of squeezing due to phase diffusion.

### III. CALCULATION OF THE PHASE DIFFUSION CONSTANT

To analyze phase diffusion, we need to calculate the generating function for spin fluctuations driven by up-down and down-up switching. Denoting the two switching rates as $W$ and $W'$, we can obtain the generating function for spin fluctuations during the time interval $0 < t' < t$ by approximating a continuous Poisson process by a discrete Markov process with a small time step $\Delta \ll W^{-1}, (W')^{-1}, t$. We have

$$\chi(\lambda) = \left( \begin{array}{c} 1 \\ 1 \end{array} \right)^T e^{\lambda R_{\Delta}} \left( \begin{array}{c} 1/2 \\ 1/2 \end{array} \right)$$ \hspace{1cm} (S11)

$$R_{\Delta} = \left( \begin{array}{cc} 1 - W\Delta & W\Delta \\ W'\Delta & 1 - W'\Delta \end{array} \right), \quad N = \frac{t}{\Delta},$$

where $W' = W + \Gamma_1$. Taking the limit $\Delta \to 0, N \to \infty$ we obtain an expression

$$\chi(\lambda) = \left( \begin{array}{c} 1 \\ 1 \end{array} \right)^T e^M \left( \begin{array}{c} 1/2 \\ 1/2 \end{array} \right),$$ \hspace{1cm} (S12)

$$M = t \left( \begin{array}{cc} i\lambda/2 - W & W \\ W' & -i\lambda/2 - W' \end{array} \right).$$ \hspace{1cm} (S13)

The generating function (S12) provides a full description of the statistics of phase fluctuations, $\theta_t = \int_0^t S_Z(t) dt$, by encoding all its cumulants:

$$\ln \chi(\lambda) = \sum_{k=1}^{\infty} \frac{m_k (i\lambda)^k}{k!},$$ \hspace{1cm} (S14)

with $m_1$ and $m_2$ giving the expectation value $\langle S_z \rangle t$ and the variance $\langle (\theta_t - \langle \theta_t \rangle)^2 \rangle$, respectively. The latter quantity yields the phase diffusion constant via $m_2 = \kappa t$. 

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Matrix exponential $e^M$ can be evaluated by writing it in terms of Pauli matrices, $M = x_0 + x_i\sigma_i$, where
\begin{equation}
x_0 = -W_+, \quad x_1 = W_+, \quad x_2 = iW_-, \quad x_3 = i\lambda/2 - W_-,
\end{equation}
and we defined $W_\pm = (W \pm W')/2$. We have
\begin{equation}
e^M = e^{x_0 t} \left( \cosh(X t) + \frac{\sinh(X t)}{X} x_i \sigma_i \right)
\end{equation}
where $X^2 = x_1^2 + x_2^2 + x_3^2 = W_+^2 - \lambda^2/4 - i\lambda W_-$. Plugging this expression for $e^M$ in Eq.(S12), we find
\begin{equation}
\chi(\lambda) = 2e^{x_0 t} \left( \cosh(X t) + \frac{\sinh(X t)}{X} x_1 \right),
\end{equation}
an exact expression which is valid both at short times and at long times.

To analyze fluctuations in the steady state, we focus on the long times $t \gg W^{-1}, (W')^{-1}$. In this limit, the behavior of $\chi(\lambda)$ can be understood by replacing $\cosh X t$ and $\sinh X t$ by $e^{X t}$, giving
\begin{equation}
\ln \chi(\lambda) \approx (X - W_+)t = -\lambda^2/4 + i\lambda W_- X + O(\lambda^3)
\end{equation}
Taylor expanding this expression up to order $\lambda^2$ we find the first and second cumulants of phase fluctuations:
\begin{equation}
\ln \chi(\lambda) = -i\lambda \frac{W_- t}{2W_+} + \frac{(i\lambda)^2}{2} \frac{(W_+^2 - W_-^2)t}{4W_+^3} + O(\lambda^3)
\end{equation}
Substituting $W' = W + \Gamma_1$, we obtain the time-averaged polarization and the phase diffusion constant
\begin{equation}
\langle S_z \rangle = \frac{1}{2} \frac{\Gamma_1}{2W + \Gamma_1}, \quad \kappa = 2\frac{(W + \Gamma_1)W}{(2W + \Gamma_1)^3}
\end{equation}
Crucially, the phase diffusion slows down when the switching rates $W$ and $W'$ grow, which justifies our motional averaging approximation.