Hardware Architectures for Deep Neural Networks

MICRO Tutorial

October 16, 2016

Website: http://eyeriss.mit.edu/tutorial.html



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Outline

- Overview of Deep Neural Networks
- DNN Development Resources
- Survey of DNN Computation
- DNN Accelerators
- Network Optimizations
- Benchmarking Metrics for Evaluation
- DNN Training



Participant Takeaways

- Understand the key design considerations for DNNs
- Be able to evaluate different implementations of DNN with benchmarks and comparison metrics
- Understand the tradeoffs between various architectures and platforms
- Assess the utility of various optimization approaches
- Understand recent implementation trends and opportunities



Background of Deep Neural Networks



AI and Machine Learning

Artificial Intelligence

Machine Learning

"Field of study that gives computers the ability to learn without being explicitly programmed"

- Arthur Samuel, 1959



Brain-Inspired Machine Learning





How Does the Brain Work?



- The basic computational unit of the brain is a neuron
 → 86B neurons in the brain
- Neurons are connected with nearly **10¹⁴ 10¹⁵ synapses**
- Neurons receive input signal from dendrites and produce output signal along axon, which interact with the dendrites of other neurons via synaptic weights
- Synaptic weights learnable & control influence strength



Spiking-based Machine Learning



Spiking Architecture

- Brain-inspired
- Integrate and fire
- Example: IBM TrueNorth



[Merolla et al., Science 2014; Esser et al., PNAS 2016] http://www.research.ibm.com/articles/brain-chip.shtml

Machine Learning with Neural Networks



Neural Networks: Weighted Sum





Many Weighted Sums





Deep Learning





What is Deep Learning?





Why is Deep Learning Hot Now?





ImageNet Challenge

IM GENET

Image Classification Task:

1.2M training images • 1000 object categories

Object Detection Task:

456k training images • 200 object categories





ImageNet: Image Classification Task





GPU Usage for ImageNet Challenge





Deep Learning on Images

- Image Classification
- Object Localization
- Object Detection

- Image Segmentation
- Action Recognition
- Image Generation





Deep Learning for Speech

- Speech Recognition
- Natural Language Processing
- Speech Translation
- Audio Generation





Deep Learning on Games

Google DeepMind AlphaGo



Medical Applications of Deep Learning

Brain Cancer Detection





Image Source: [Jermyn et al., JBO 2016]

Deep Learning for Self-driving Cars







Connectomics – Finding Synapses





Image Source: MIT

Mature Applications

Image

- Classification: image to object class
- Recognition: same as classification (except for faces)
- Detection: assigning bounding boxes to objects
- Segmentation: assigning object class to every pixel

Speech & Language

- Speech Recognition: audio to text
- Translation
- Natural Language Processing: text to meaning
- Audio Generation: text to audio
- Games

Emerging Applications

- Medical (Cancer Detection, Pre-Natal)
- Finance (Trading, Energy Forecasting, Risk)
- Infrastructure (Structure Safety and Traffic)
- Weather Forecasting and Event Detection



This tutorial will focus on image classification

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Opportunities

\$500B Market over 10 Years!





Image Source: Tractica

Opportunities

From EE Times – September 27, 2016

"Today the job of training machine learning models is limited by compute, if we had faster processors we'd run bigger models...in practice we train on a reasonable subset of data that can finish in a matter of months. We could use improvements of several orders of magnitude – 100x or greater."

- Greg Diamos, Senior Researcher, SVAIL, Baidu



Overview of Deep Neural Networks



DNN Timeline

- 1940s: Neural networks were proposed
- 1960s: Deep neural networks were proposed
- 1990s: Early hardware for shallow neural nets

- Example: Intel ETANN (1992)

- 1998: LeNet for MNIST
- 2011: Speech recognition using DNN (Microsoft)
- 2012: Deep learning starts supplanting traditional ML

AlexNet for image classification

- Early 2010s: Rise of DNN accelerator research
 - Examples: Neuflow, DianNao, etc.



Publications at Architecture Conferences

• MICRO, ISCA, HPCA, ASPLOS





So Many Neural Networks!



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http://www.asimovinstitute.org/neural-network-zoo/

DNN Terminology 101



Image Source: Stanford

DNN Terminology 101



Image Source: Stanford



DNN Terminology 101

Each synapse has a weight for neuron activation





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Weight Sharing: multiple synapses use the same weight value















Image Source: Stanford



Image Source: Stanford



Popular Types of DNNs

- Fully-Connected NN
 - feed forward, a.k.a. multilayer perceptron (MLP)
- Convolutional NN (CNN)
 - feed forward, sparsely-connected w/ weight sharing
- Recurrent NN (RNN)
 - feedback
- Long Short-Term Memory (LSTM)
 - feedback + Storage



Inference vs. Training

- Training: Determine weights
 - Supervised:
 - Training set has inputs and outputs, i.e., labeled
 - Reinforcement:
 - Output assessed via rewards and punishments
 - Unsupervised:
 - Training set is unlabeled
 - Semi-supervised:
 - Training set is partially labeled
- Inference: Apply weights to determine output















Optional layers in between CONV and/or FC layers









Convolutions account for more than 90% of overall computation, dominating **runtime** and **energy consumption**



a plane of input activations a.k.a. **input feature map (fmap)**



← S -







Element-wise Multiplication









Sliding Window Processing





Many Input Channels (C)









CONV Layer Implementation



 $0 \leq n < N, 0 \leq m < M, 0 \leq y < E, 0 \leq x < F,$

$$E = (H - R + U)/U, F = (W - S + U)/U.$$

Shape Parameter	Description
N	fmap batch size
M	# of filters / # of output fmap channels
C	# of input fmap/filter channels
H/W	input fmap height/width
R/S	filter height/width
E/F	output fmap height/width
U	convolution stride



CONV Layer Implementation

Naïve 7-layer for-loop implementation:



1411

Traditional Activation Functions



 Image Source: Caffe Tutorial

Modern Activation Functions





Image Source: Caffe Tutorial

Fully-Connected (FC) Layer

- Height and width of output fmaps are 1 (E = F = 1)
- Filters as large as input fmaps (R = H, S = W)
- Implementation: Matrix Multiplication



FC Layer – from CONV Layer POV



Pooling (POOL) Layer

- Reduce resolution of each channel independently
- Increase translation-invariance and noise-resilience
- Overlapping or non-overlapping \rightarrow depending on stride •



Max pooling



POOL Layer Implementation

Naïve 6-layer for-loop max-pooling implementation:



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Normalization (NORM) Layer

- Batch Normalization (BN)
 - Normalize activations towards mean=0 and std.
 dev.=1 based on the statistics of the training dataset
 - put in between CONV/FC and Activation function



Believed to be key to getting high accuracy and faster training on very deep neural networks.



BN Layer Implementation

• The normalized value is further scaled and shifted, the parameters of which are learned from training





Normalization (NORM) Layer

- Local Response Normalization (LRN)
 - Tries to mimic the inhibition scheme in the brain



Now deprecated!



Relevant Components for Tutorial

- Typical operations that we will discuss:
 - Convolution (CONV)
 - Fully-Connected (FC)
 - Max Pooling
 - ReLU



Survey of DNN Development Resources

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html



Joel Emer, Vivienne Sze, Yu-Hsin Chen

Popular DNNs

- LeNet (1998)
- AlexNet (2012)
- OverFeat (2013)
- VGGNet (2014)
- GoogleNet (2014)
- ResNet (2015)

ImageNet: Large Scale Visual Recognition Challenge (ILSVRC)





[O. Russakovsky et al., IJCV 2015]

LeNet-5

CONV Layers: 2 Fully Connected Layers: 2 Weights: 431k MACs: 2.3M

Digit Classification!





AlexNet

ILSCVR12 Winner

CONV Layers: 5 Fully Connected Layers: 3 Weights: 61M MACs: 724M

Uses Local Response Normalization (LRN)



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[Krizhevsky et al., NIPS, 2012]

Large Sizes with Varying Shapes

AlexNet Convolutional Layer Configurations

Layer	Filter Size (RxS)	# Filters (M)	# Channels (C)	Stride
1	11x11	96	3	4
2	5x5	256	48	1
3	3x3	384	256	1
4	3x3	384	192	1
5	3x3	256	192	1

Layer 1



34k Params 105M MACs Layer 2





307k Params

224M MACs



885k Params 150M MACs

[Krizhevsky et al., NIPS, 2012]
OverFeat (fast model)

CONV Layers: 5 Fully Connected Layers: 3 Weights: 144M MACs: 5.4G

								Output
Layer	1	2	3	4	5	6	7	8
Stage	conv + max	conv + max	conv	conv	conv + max	full	full	full
# channels	96	256	512	1024	1024	3072	4096	1000
Filter size	11x11	5x5	3x3	3x3	3x3	-	-	-
Conv. stride	4x4	1x1	1x1	1x1	1x1	-	-	-
Pooling size	2x2	2x2	-	-	2x2	-	-	-
Pooling stride	2x2	2x2	-	-	2x2	-	-	-
Zero-Padding size	-	-	1x1x1x1	1x1x1x1	1x1x1x1	-	-	-
Spatial input size	231x231	24x24	12x12	12x12	12x12	6x6	1x1	1x1



VGG-16



Image Source: http://www.cs.toronto.edu/~frossard/post/vgg16/

[Simonyan et al., ArXiv 2014, ICLR 2015]

GoogLeNet (v1)

CONV Layers: 21 Fully Connected Layers: 1 Weights: 7.0M MACs: 1.43G Also, v2, v3 and v4 ILSVRC14 Winner



[Szegedy et al., ArXiv 2014, CVPR 2015]

ResNet-50



[He et al., ArXiv 2015, CVPR 2016]

Revolution of Depth



ImageNet Classification top-5 error (%)

Image Source: <u>http://icml.cc/2016/tutorials/icml2016_tutorial_deep_residual_networks_kaiminghe.pdf</u>



Summary of Popular DNNs

Metrics	LeNet-5	AlexNet	OverFeat (fast)	VGG-16	GoogLeNet (v1)	ResNet-50
Top-5 error	n/a	16.4	14.2	7.4	6.7	5.3
Input Size	28x28	227x227	231x231	224x224	224x224	224x224
# of CONV Layers	2	5	5	16	21	49
Filter Sizes	5	3, 5,11	3, 7	3	1, 3 , 5, 7	1, 3, 7
# of Channels	1, 6	3 - 256	3 - 1024	3 - 512	3 - 1024	3 - 2048
# of Filters	6, 16	96 - 384	96 - 1024	64 - 512	64 - 384	64 - 2048
Stride	1	1, 4	1, 4	1	1, 2	1, 2
# of Weights	26k	2.3M	16M	14.7M	6.0M	23.5M
# of MACs	1.9M	666M	2.67G	15.3G	1.43G	3.86G
# of FC layers	2	3	3	3	1	1
# of Weights	406k	58.6M	130M	124M	1M	2M
# of MACs	405k	58.6M	130M	124M	1M	2M
Total Weights	431k	61M	146M	138M	7M	25.5M
Total MACs	2.3M	724M	2.8G	15.5G	1.43G	3.9G

CONV Layers increasingly important!

Summary of Popular DNNs

- AlexNet
 - First CNN Winner of ILSVRC
 - Uses LRN (deprecated after this)
- VGG-16
 - Goes Deeper (16+ layers)
 - Uses only 3x3 filters (stack for larger filters)
- GoogLeNet (v1)
 - Reduces weights with Inception and only one FC layer
 - Inception: 1x1 and DAG (parallel connections)
 - Batch Normalization
- ResNet
 - Goes Deeper (24+ layers)
 - Shortcut connections



Frameworks



Berkeley / BVLC (C, C++, Python, MATLAB)



Google (C++, Python)

theano

U. Montreal (Python)



Facebook / NYU (C, C++, Lua)

Also, CNTK, MXNet, etc.



More at: https://developer.nvidia.com/deep-learning-frameworks

Example: Layers in Caffe

Convolution Layer

```
layer {
```

```
name: "conv1"
```

```
type: "Convolution"
```

```
bottom: "data"
```

```
top: "conv1"
```

```
• • •
```

```
convolution_param {
```

num_output: 20

```
kernel_size: 5
```

stride: 1

```
• • •
```

Non-Linearity

layer {

name: "relu1"

```
type: "ReLU"
```

```
bottom: "conv1"
```

top: "conv1"

}

Pooling Layer

```
layer {
```

```
name: "pool1"
```

```
type: "Pooling"
```

```
bottom: "conv1"
```

top: "pool1"

```
pooling_param {
```

```
pool: MAX
```

```
kernel_size: 2
```

```
stride: 2 ...
```



Image Classification Datasets

- Image Classification/Recognition
 - Given an entire image \rightarrow Select 1 of N classes
 - No localization (detection)



Datasets affect difficulty of task



MNIST

Digit Classification

28x28 pixels (B&W) 10 Classes 60,000 Training 10,000 Testing

LeNet in 1998 (0.95% error)

ICML 2013 (0.21% error)

3681796691 6757863485 2179712845 4819018894 7618641560 7592658197 1222234480 0238073857 0146460243 7128169861

CIFAR-10/CIFAR-100

Object Classification

32x32 pixels (color) 10 or 100 Classes 50,000 Training 10,000 Testing

CIFAR-10 RBM+finetuning in 2009 (35.16% error)



airplane
automobile
bird
cat
deer
dog
frog
horse
ship
truck



Image Source: http://karpathy.github.io/

Subset of 80 Tiny Images Dataset (Torrabla)



https://www.cs.toronto.edu/~kriz/cifar.html

IM GENET

Object Classification

~256x256 pixels (color) 1000 Classes 1.3M Training 100,000 Testing (50,000 Validation)

Image Source: http://karpathy.github.io/





http://www.image-net.org/challenges/LSVRC/

IM GENET



Image Source: http://karpathy.github.io/

Image Source: Krizhevsky et al., NIPS 2012

Top-5 Error Winner 2012 (16.42% error)

Winner 2016 (2.99% error)

l'lli7



http://www.image-net.org/challenges/LSVRC/

Image Classification Summary

	MNIST	CIFAR-10	CIFAR-100	IMAGENET
Year	1998	2009	2009	2012
Resolution	28x28	32x32	32x32	256x256
Classes	10	10	100	1000
Training	60k	50k	50k	1.3M
Testing	10k	10k	10k	100k
Accuracy	0.21% error (ICML 2013)	3.47% error (arXiv 2015)	24.28% error (arXiv 2015)	2.99% top-5 error (2016 winner)

http://rodrigob.github.io/are_we_there_yet/build/ classification_datasets_results.html



Next Tasks: Localization and Detection



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[Russakovsky et al., IJCV, 2015]

Others Popular Datasets

- Pascal VOC
 - 11k images
 - Object Detection
 - 20 classes
- MS COCO
 - 300k images
 - Detection, Segmentation
 - Recognition in context





http://host.robots.ox.ac.uk/pascal/VOC/

Recently Introduced Datasets

- Announced Sept 2016:
- Google Open Images (~9M images)
 - <u>https://github.com/openimages/dataset</u>
- Youtube-8M (8M videos)
 - <u>https://research.google.com/youtube8m/</u>



Survey of DNN Hardware

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html



Joel Emer, Vivienne Sze, Yu-Hsin Chen

CPUs Are Targeting Deep Learning

Intel Knights Landing (2016)



- 7 TFLOPS FP32
- 16GB MCDRAM- 400 GB/s
- 245W TDP
- 29 GFLOPS/W (FP32)
- 14nm process

Knights Mill: next gen Xeon Phi "optimized for deep learning"

Intel announced the addition of new vector instructions for deep learning (AVX512-4VNNIW and AVX512-4FMAPS), October 2016

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Image Source: Intel, Data Source: Next Platform

GPUs Are Targeting Deep Learning

Nvidia PASCAL GP100 (2016)



- 10/20 TFLOPS FP32/FP16
- 16GB HBM 750 GB/s
- 300W TDP
- 67 GFLOPS/W (FP16)
- 16nm process
- 160GB/s NV Link



Systems for Deep Learning

Nvidia DGX-1 (2016)



- 170 TFLOPS
- 8× Tesla P100, Dual Xeon
- NVLink Hybrid Cube Mesh
- Optimized DL Software
- 7 TB SSD Cache
- Dual 10GbE, Quad IB 100Gb
- 3RU 3200W



Cloud Systems for Deep Learning

Facebook's Deep Learning Machine



- Open Rack Compliant
- Powered by 8 Tesla M40 GPUs
- 2x Faster Training for Faster Deployment
- 2x Larger Networks for Higher Accuracy

SOCs for Deep Learning Inference

Nvidia Tegra - Parker



- GPU: 1.5 TeraFLOPS FP16
- 4GB LPDDR4 @ 25.6 GB/s
- 15 W TDP

(1W idle, <10W typical)

- 100 GFLOPS/W (FP16)
- 16nm process

Xavier: next gen Tegra to be an "AI supercomputer"



Source: Nvidia

Mobile SOCs for Deep Learning

Samsung Exynos (ARM Mali)

Exynos 8 Octa 8890



- GPU: 0.26 TFLOPS
- LPDDR4 @ 28.7 GB/s
- 14nm process



FPGAs for Deep Learning





Intel/Altera Stratix 10

- 10 TFLOPS FP32
- HBM2 integrated
- Up to 1 GHz
- 14nm process
- 80 GFLOPS/W

Xilinx Virtex UltraSCALE+

- DSP: up to 21.2 TMACS
- DSP: up to 890 MHz
- Up to 500Mb On-Chip Memory
- 16nm process



Kernel Computation



Fully-Connected (FC) Layer

- Matrix–Vector Multiply:
 - Multiply all inputs in all channels by a weight and sum



Fully-Connected (FC) Layer

• Batching (N) turns operation into a Matrix-Matrix multiply





Fully-Connected (FC) Layer

- Implementation: Matrix Multiplication (GEMM)
 - CPU: OpenBLAS, Intel MKL, etc
 - GPU: cuBLAS, cuDNN, etc
- Optimized by tiling to storage hierarchy



• Convert to matrix mult. using the Toeplitz Matrix





• Convert to matrix mult. using the Toeplitz Matrix



Data is repeated



• Multiple Channels and Filters





• Multiple Channels and Filters



Computational Transforms



Computation Transformations

- Goal: Bitwise same result, but reduce number of operations
- Focuses mostly on compute


Gauss's Multiplication Algorithm

$$(a+bi)(c+di) = (ac-bd) + (bc+ad)i.$$

4 multiplications + 3 additions

 $k_{1} = c \cdot (a + b)$ $k_{2} = a \cdot (d - c)$ $k_{3} = b \cdot (c + d)$ Real part = $k_{1} - k_{3}$ Imaginary part = $k_{1} + k_{2}$.

3 multiplications + 5 additions



Strassen



8 multiplications + 4 additions

$$\begin{array}{ll} \mathsf{P1} = \mathsf{a}(\mathsf{f} - \mathsf{h}) & \mathsf{P5} = (\mathsf{a} + \mathsf{d})(\mathsf{e} + \mathsf{h}) \\ \mathsf{P2} = (\mathsf{a} + \mathsf{b})\mathsf{h} & \mathsf{P6} = (\mathsf{b} - \mathsf{d})(\mathsf{g} + \mathsf{h}) & \mathsf{AB} = \\ \mathsf{P3} = (\mathsf{c} + \mathsf{d})\mathsf{e} & \mathsf{P7} = (\mathsf{a} - \mathsf{c})(\mathsf{e} + \mathsf{f}) \\ \mathsf{P4} = \mathsf{d}(\mathsf{g} - \mathsf{e}) & \end{array} \left[\begin{array}{c} \mathsf{P5} + \mathsf{P4} - \mathsf{P2} + \mathsf{P6} & \mathsf{P1} + \mathsf{P2} \\ \mathsf{P3} + \mathsf{P4} & \mathsf{P1} + \mathsf{P5} - \mathsf{P3} - \mathsf{P7} \end{array} \right]$$

7 multiplications + 18 additions

7 multiplications + 13 additions (for constant B matrix – weights)

[Cong et al., ICANN, 2014]

Strassen

- Reduce the complexity of matrix multiplication from Θ(N³) to Θ(N^{2.807}) by reducing multiplications
- Comes at the price of reduced numerical stability and requires significantly more memory



Complexity

Winograd 1D – F(2,3)

e 1 4

- Targeting convolutions instead of matrix multiply
- Notation: F(size of output, filter size)

input filter
$$F(2,3) = \begin{bmatrix} d_0 & d_1 & d_2 \\ d_1 & d_2 & d_3 \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix}$$

6 multiplications + 4 additions

$$egin{aligned} m_1 &= (d_0 - d_2)g_0 & m_2 &= (d_1 + d_2)rac{g_0 + g_1 + g_2}{2} \ m_4 &= (d_1 - d_3)g_2 & m_3 &= (d_2 - d_1)rac{g_0 - g_1 + g_2}{2} \end{aligned}$$

4 multiplications + 12 additions + 2 shifts4 multiplications + 8 additions (for constant weights)

[Lavin et al., ArXiv 2015]

Winograd 2D - F(2x2, 3x3)

• 1D Winograd is nested to make 2D Winograd



Original:36 multiplicationsWinograd:16 multiplications \rightarrow 2.25 times reduction



Winograd Halos

 Winograd works on a small region of output at a time, and therefore uses inputs repeatedly



Filter



Output Fmap

y ₀₀	У ₀₁	У ₀₂	У ₀₃
y ₁₀	У ₁₁	У ₁₂	У ₁₂



Winograd Performance Varies

Optimal convolution algorithm depends on convolution layer dimensions



Meta-parameters (data layouts, texture memory) afford higher performance

Using texture memory for convolutions: 13% inference speedup

(GoogLeNet, batch size 1)



Winograd Summary

 Winograd is an optimized computation for convolutions

- It can significantly reduce multiplies
 For example, for 3x3 filter by 2.5X
- But, each filter size is a different computation.



Winograd as a Transform

$$B^{T} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$
$$G = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$
$$A^{T} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$
filter $g = \begin{bmatrix} g_{0} & g_{1} & g_{2} \end{bmatrix}^{T}$ input $d = \begin{bmatrix} d_{0} & d_{1} & d_{2} & d_{3} \end{bmatrix}^{T}$

Transform inputs

$$Y = A^{T} \begin{bmatrix} [GgG^{T}] \odot [B^{T}dB] \end{bmatrix} A$$
Dot-product
Transform output

 GgG^{T} can be precomputed



[Lavin et al., ArXiv 2015]

FFT Flow





FFT Overview

- Convert filter and input to frequency domain to make convolution a simple multiply then convert back to time domain.
- Convert direct convolution O(N_o²N_f²) computation to O(N_o²log₂N_o)

III ii

 So note that computational benefit of FFT decreases with decreasing size of filter

[Mathieu et al., ArXiv 2013, Vasilache et al., ArXiv 2014]

FFT Costs

- Input and Filter matrices are '0-completed',
 i.e., expanded to size E+R-1 x F+S-1
- Frequency domain matrices are same dimensions as input, but complex.
- FFT often reduces computation, but requires much more memory space and bandwidth



Optimization opportunities

- FFT of real matrix is symmetric allowing one to save ¹/₂ the computes
- Filters can be pre-computed and stored, but convolutional filter in frequency domain is much larger than in time domain
- Can reuse frequency domain version of input for creating different output channels to avoid FFT re-computations



cuDNN: Speed up with Transformations

60x Faster Training in 3 Years



AlexNet training throughput on:

CPU: 1x E5-2680v3 12 Core 2.5GHz. 128GB System Memory, Ubuntu 14.04

M40 bar: 8x M40 GPUs in a node, P100: 8x P100 NVLink-enabled

Source: Nvidia



GPU/CPU Benchmarking

- Industry performance website
- <u>https://github.com/jcjohnson/cnn-benchmarks</u>



- DeepBench
 - Profile layer by layer (Dense Matrix Multiplication, Convolutions, Recurrent Layer, All-Reduce)



GPU/CPU Benchmarking

- Minibatch = 16
- Image size 224x224
- cuDNN 5.0 or 5.1
- Torch

Speed (ms)

Platform	AlexNet	VGG-16	GoogLeNet (v1)	ResNet-50
Pascal Titan X (F+B)	14.56	128.62	39.14	103.58
Pascal Titan X (F)	5.04	41.59	11.94	35.03
GTX 1080 (F)	7.00	59.37	16.08	50.64
Maxwell Titan X	7.09	62.30	19.27	55.75
Dual Xeon E5-2630 v3	n/a	3101.76	n/a	2477.61

https://github.com/jcjohnson/cnn-benchmarks

GPU/CPU Benchmarking

- Minibatch = 1
- Image size 224x224

Speed (ms)

Platform	AlexNet	VGG-16
Maxwell Titan X	0.54	10.67
Exynos 7 Octa 7420	117	1926



DeepBench

- Profile layer by layer
 - Dense Matrix Multiplication, Convolutions, Recurrent Layer, All-Reduce (communication)

3.2.	Convol	lution	Resu	lts

Input Size	Filter Size	# of Filters	Padding (h, w)	Stride (h, w)	Application	Total Time (ms)	Fwd TeraFLOPS	Processor
W = 700, H =	R = 5,	32	0, 0	2, 2	Speech	2.98	6.63	TitanX
161, C = 1, N	S = 20				Recognition			Pascal
= 32								
W = 54, H =	R = 3,	64	1, 1	1, 1	Face	0.63	10.55	TitanX
54, C = 64, N	S = 3				Recognition			Pascal
= 8								
W = 224, H =	R = 3,	64	1, 1	1, 1	Computer	3.99	3.6	TitanX
224, C = 3, N	S = 3				Vision			Pascal
= 16								
W = 7, H = 7,	R = 3,	512	1, 1	1, 1	Computer	2.93	5.88	TitanX
C = 512, N =	S = 3				Vision			Pascal
16								
W = 28, H =	R = 5,	32	2, 2	1, 1	Computer	1.57	6.59	TitanX
28, C = 192, N	S = 5				Vision			Pascal
= 16								



https://svail.github.io/DeepBench/

DNN Accelerator Architectures

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html

Joel Emer, Vivienne Sze, Yu-Hsin Chen

Highly-Parallel Compute Paradigms

Temporal Architecture (SIMD/SIMT)



Spatial Architecture (Dataflow Processing)







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* multiply-and-accumulate

Worst Case: all memory R/W are **DRAM** accesses

• Example: AlexNet [NIPS 2012] has 724M MACs

→ 2896M DRAM accesses required









Opportunities: 1 data reuse



Types of Data Reuse in DNN

Convolutional Reuse

CONV layers only (sliding window)



Reuse: Activations Filter weights



Types of Data Reuse in DNN

Convolutional Reuse

CONV layers only (sliding window)



Fmap Reuse

CONV and FC layers



Reuse: Activations Filter weights

Reuse: Activations



Types of Data Reuse in DNN

Convolutional Reuse

CONV layers only (sliding window)



Fmap Reuse

CONV and FC layers



Filter Reuse

CONV and FC layers (batch size > 1)

Input Fmaps



Reuse: Activations Filter weights

Reuse: Activations

Reuse: Filter weights





Opportunities: 1 data reuse



Can reduce DRAM reads of filter/fmap by up to 500×**

** AlexNet CONV layers





Opportunities: 1 data reuse 2 local accumulation

- 100
- Can reduce DRAM reads of filter/fmap by up to 500×
- Partial sum accumulation does NOT have to access DRAM





Opportunities: 1 data reuse 2 local accumulation

- 102
- Can reduce DRAM reads of filter/fmap by up to 500×
- Partial sum accumulation does NOT have to access DRAM
- Example: DRAM access in AlexNet can be reduced from 2896M to 61M (best case)

Spatial Architecture for CNN



Low-Cost Local Data Access



* measured from a commercial 65nm process 14

Low-Cost Local Data Access

How to exploit **1** data reuse and **2** local accumulation with *limited* low-cost local storage?



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Low-Cost Local Data Access

How to exploit **1** data reuse and **2** local accumulation with *limited* low-cost local storage?

specialized **processing dataflow** required!



Шіт

Dataflow Taxonomy

- Weight Stationary (WS)
- Output Stationary (OS)
- No Local Reuse (NLR)



Weight Stationary (WS)



- Minimize weight read energy consumption
 - maximize convolutional and filter reuse of weights
- Broadcast activations and accumulate psums spatially across the PE array.


WS Example: nn-X (NeuFlow)

A 3×3 2D Convolution Engine





[Farabet et al., ICCV 2009]

WS Example: nn-X (NeuFlow)

Top-Level Architecture





Output Stationary (OS)



- Minimize partial sum R/W energy consumption
 - maximize local accumulation
- Broadcast/Multicast filter weights and reuse activations spatially across the PE array



OS Example: ShiDianNao

Input Fmap Dataflow in the PE Array







[Du et al., ISCA 2015]

OS Example: ShiDianNao





No Local Reuse (NLR)



- Use a large global buffer as shared storage
 - Reduce **DRAM** access energy consumption
- Multicast activations, single-cast weights, and accumulate psums spatially across the PE array



NLR Example: UCLA





NLR Example: DianNao





Taxonomy: More Examples

• Weight Stationary (WS)

[Chakradhar, *ISCA* 2010] [nn-X (NeuFlow), *CVPRW* 2014] [Park, *ISSCC* 2015] [ISAAC, *ISCA* 2016] [PRIME, *ISCA* 2016]

• Output Stationary (OS)

[Peemen, *ICCD* 2013] [ShiDianNao, *ISCA* 2015] [Gupta, *ICML* 2015] [Moons, *VLSI* 2016]

• No Local Reuse (NLR)

[**DianNao**, *ASPLOS* 2014] [**DaDianNao**, *MICRO* 2014] [**Zhang**, *FPGA* 2015]



Energy Efficiency Comparison

- Same total area
 256 PEs
- AlexNet CONV layers Batch size = 16



Energy Efficiency Comparison

- 256 PEs Same total area •
- AlexNet CONV layers • Batch size = 16 •



Energy-Efficient Dataflow: Row Stationary (RS)

- Maximize reuse and accumulation at RF
- Optimize for **overall** energy efficiency instead for *only* a certain data type



Row Stationary: Energy-efficient Dataflow



























- Maximize row convolutional reuse in RF
 Keep a filter row and fmap sliding window in RF
- Maximize row psum accumulation in RF

























Convolutional Reuse Maximized



Filter rows are reused across PEs horizontally



Convolutional Reuse Maximized



Fmap rows are reused across PEs diagonally



Maximize 2D Accumulation in PE Array



Partial sums accumulate across PEs vertically



Dimensions Beyond 2D Convolution



Filter Reuse in PE







Filter Reuse in PE





Filter Reuse in PE

1 Multiple Fmaps 2 Multiple Filters 3 Multiple Channels Filter 1 Fmap 1 Psum 1 Row 1 Row 1 Channel 1 * Row 1 C[≁]. н — R C[₹]. Filter 1 Fmap 2 Psum 2 ← R Row 1 Row 1 **Channel 1** Row 1 * share the same filter row Н

Processing in PE: concatenate fmap rows





Fmap Reuse in PE







Fmap Reuse in PE

Multiple Fmaps **2** Multiple Filters







Fmap Reuse in PE



Processing in PE: interleave filter rows





Channel Accumulation in PE





Channel Accumulation in PE





Channel Accumulation in PE



Processing in PE: interleave channels





DNN Processing – The Full Picture




Optimal Mapping in Row Stationary



[Chen et al., ISCA 2016]

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Dataflow Simulation Results



Evaluate Reuse in Different Dataflows

Weight Stationary

Minimize movement of filter weights

Output Stationary

Minimize movement of partial sums

No Local Reuse

- No PE local storage. Maximize global buffer size.

Row Stationary

Evaluation Setup

- same total area
- 256 PEs
- AlexNet
- batch size = 16



Variants of Output Stationary





Dataflow Comparison: CONV Layers



RS optimizes for the best **overall** energy efficiency

Dataflow Comparison: CONV Layers



RS uses 1.4× – 2.5× lower energy than other dataflows

Dataflow Comparison: FC Layers



RS uses at least **1.3× lower** energy than other dataflows











Hardware Architecture for RS Dataflow



Eyeriss Deep CNN Accelerator



Data Delivery with On-Chip Network

_ink Clock Clock

DCNN Accelerator



How to accommodate different shapes with fixed PE array?



Logical to Physical Mappings





Physical PE Array

Logical to Physical Mappings



Multicast Network Design



Data Delivery with On-Chip Network

_ink Clock Clock

DCNN Accelerator



Compared to Broadcast, **Multicast** saves >80% of NoC energy



Chip Spec & Measurement Results

Technology	TSMC 65nm LP 1P9M
On-Chip Buffer	108 KB
# of PEs	168
Scratch Pad / PE	0.5 KB
Core Frequency	100 – 250 MHz
Peak Performance	33.6 – 84.0 GOPS
Word Bit-width	16-bit Fixed-Point
	Filter Width: 1 – 32
	Filter Height: 1 – 12
Natively Supported	Num. Filters: 1 – 1024
DNN Shapes	Num. Channels: 1 – 1024
	Horz. Stride: 1–12
	Vert. Stride: 1, 2, 4





Benchmark – AlexNet Performance

Image Batch Size of **4** (i.e. 4 frames of 227x227) Core Frequency = 200MHz / Link Frequency = 60 MHz

Layer	Power (mW)	Latency (ms)	# of MAC (MOPs)	Active # of PEs (%)	Buffer Data Access (MB)	DRAM Data Access (MB)
1	332	20.9	422	154 (92%)	18.5	5.0
2	288	41.9	896	135 (80%)	77.6	4.0
3	266	23.6	598	156 (93%)	50.2	3.0
4	235	18.4	449	156 (93%)	37.4	2.1
5	236	10.5	299	156 (93%)	24.9	1.3
Total	278	115.3	2663	148 (88%)	208.5	15.4

To support 2.66 GMACs [8 billion 16-bit inputs (**16GB**) and 2.7 billion outputs (**5.4GB**)], only requires **208.5MB** (buffer) and **15.4MB** (DRAM)

Benchmark – AlexNet Performance

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5	236	10.5	299	156 (93%)	24.9	1.3
Total	278	115.3	2663	148 (88%)	208.5	15.4

51682 operand* access/input image pixel

→ 506 access/pixel from buffer + 37 access/pixel from DRAM



*operand = weight, activation, psum

Comparison with GPU

	This Work	NVIDIA TK1 (Jetson Kit)
Technology	65nm	28nm
Clock Rate	200MHz	852MHz
# Multipliers	168	192
On-Chin Storage	Buffer: 108KB	Shared Mem: 64KB
on omp otorage	Spad: 75.3KB	Reg File: 256KB
Word Bit-Width	16b Fixed	32b Float
Throughput ¹	34.7 fps	68 fps
Measured Power	278 mW	Idle/Active ² : 3.7W/10.2W
DRAM Bandwidth	127 MB/s	1120 MB/s ³

- 1. AlexNet CONV Layers
- 2. Board Power
- 3. Modeled from [Tan, SC 2011]



From Architecture to System



https://vimeo.com/154012013



Summary of DNN Dataflows

Weight Stationary

- Minimize movement of filter weights
- Popular with processing-in-memory architectures

Output Stationary

- Minimize movement of partial sums
- Different variants optimized for CONV or FC layers

No Local Reuse

- No PE local storage \rightarrow maximize global buffer size

Row Stationary

- Adapt to the NN shape and hardware constraints
- Optimized for overall system energy efficiency



MICRO 2016 Papers in the Taxonomy

- **Stripes:** bit-serial computation in a **NLR**-like engine (based on DaDianNao)
- **NEUTRAMS**: a toolset for accelerators running the **WS** dataflow (synaptic weight memory array)
- Fused-layer: exploit inter-layer data reuse in a NLR engine (based on [Zhang, FPGA 2015])



Fused Layer

Dataflow across multiple layers





[Alwani et al., MICRO 2016]

Advanced Technology Opportunities

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html



Joel Emer, Vivienne Sze, Yu-Hsin Chen

Advanced Storage Technology

- Embedded DRAM (eDRAM)
 - Increase on-chip storage capacity
- 3D Stacked DRAM
 - e.g. Hybrid Memory Cube Memory (HMC), High Bandwidth Memory (HBM)
 - Increase memory bandwidth



eDRAM (DaDianNao)

- Advantages of eDRAM
 - 2.85x higher density than SRAM
 - 321x more energy-efficient than DRAM (DDR3)
- Store weights in eDRAM (36MB)
 - Target fully connected layers since dominated by weights



[Chen et al., DaDianNao, MICRO 2014]

Stacked DRAM (NeuroCube)

- NeuroCube on Hyper Memory Cube Logic Die
 - 6.25x higher BW than DDR3
 - HMC (16 ch x 10GB/s) > DDR3 BW (2 ch x 12.8GB/s)
 - Computation closer to memory (reduce energy)



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[Kim et al., NeuroCube, ISCA 2016]

Analog Computation

V1

V2

G1

 $I1 = V1 \times G1$

G2

 $12 = V2 \times G2$

| = |1 +

= V1×G1 + V2×G2

- Conductance = Weight
- Voltage = Input
- Current = Voltage × Conductance
- Sum currents for addition

$$Output = \sum Weight \times Input$$

Filter Weights = G1, G2, ... (conductance)

Weight Stationary Dataflow



Figure Source: ISAAC, ISCA 2016

Memristor Computation

Use memristors as programmable weights (resistance)

- Advantages
 - High Density (< 10nm x 10nm size*)</p>
 - ~30x smaller than SRAM**
 - 1.5x smaller than DRAM**
 - Non-Volatile
 - Operates at low voltage
 - Computation within memory (in situ)
 - Reduce data movement



Memristor



[Chi et al., ISCA 2016]

Resistive Memory Devices



Figure Source: Han Wang, USC

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Challenges with Memristors

- Limited Precision
- A/D and D/A Conversion
- Array Size and Routing
 - Wire dominates energy for array size of 1k × 1k
 - IR drop along wire can degrade read accuracy
- Write/programming energy
 - Multiple pulses can be costly
- Variations & Yield
 - Device-to-device, cycle-to-cycle
 - Non-linear conductance across range

ISAAC

- eDRAM using memristors
- 16-bit dot-product operation
 - 8 x 2-bits per memristors
 - 1-bit per cycle computation



I = I1 + I2 = V1.G1 + V2.G21-bit 2-bit - 2-bit - 2-bit - 2-bit - 2-bit - 2-bit - 2-bit 16 1-bit 2-bit 2-bit 2-bit 2-bit 2-bit 2-bit 2-bit 2-bit iterations 2-bit a 2-bit 1-bit 2-bit 🔩 2-bit 🔩 2-bit 🔩 2-bit 🔩 2-bit 🔩 2-bit 🔩 2-bit 1-bit S&H S&H S&H S&H S&H S&H S&H S&H **Input Neurons** ADC

MiT 📀

[Shafiee et al., ISCA 2016]

Shift & ADD
ISAAC



Eight 128x128 arrays per IMA

12 IMAs per Tile



PRIME

- Bit precision for each 256x256 ReRAM array
 - 3-bit input, 4-bit weight (2x for 6-bit input and 8-bit weight)
 - Dynamic fixed point (6-bit output)
- Reconfigurable to be main memory or accelerator
 - 4-bit MLC computation; 1-bit SLC for storage



[Chi et al., ISCA 2016]



Fabricated Memristor Crossbar

- Transistor-free metal-oxide 12x12 crossbar
 - A single-layer perceptron (linear classification)
 - 3x3 binary image

 10 inputs x 3 outputs x 2 differential weights = 60 memristors











[Prezioso et al., Nature 2015]

Network and Hardware Co-Design

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html



Joel Emer, Vivienne Sze, Yu-Hsin Chen

Network Optimization

- <u>Reduce precision</u> of operations and operands
 - Fixed and Floating point
 - Bit-width
- <u>Reduce number</u> of operations and storage of weights
 - Compression
 - Pruning
 - Network Architectures



Number Representation





Image Source: B. Dally

Cost of Operations



[Horowitz, "Computing's Energy Problem (and what we can do about it)", ISSCC 2014]

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©,

N-bit Precision





Methods to Reduce Bits

Example: 16-bit \rightarrow 8-bits **Quantization/Rounding** $2^{11} + 2^9 + 2^6 + 2^1 = 2626$ (overflow) **Dynamic Fixed Point Rescale and Reduce bits** 0100101010010000010 **Fine-tuning: Retrain Weights** $2^{10} + 2^7 + 2^5 + 2^2 + 2^0 = 1189$ 500 min/max min/max PDF PDF AlexNet 2.5 400 AlexNet (Layer 1) (Layer 6) ⊥ ³⁰⁰ 400 200 [_] ______ ______ Image Source: Moons et al, WACV 2016 100 0.5 _0∟ _0.5 -0.06 0.5 -0.04 -0.02 0 0.02 0.04 0.06 Parameter value [-] Parameter value [-] Dynamic range = 1 Dynamic range = 0.125

Batch normalization important to 'center' dynamic range

Impact on Accuracy



[Gysel et al., Ristretto, ICLR 2016]

Google's Tensor Processing Unit (TPU)

"With its TPU Google has seemingly focused on delivering the data really quickly by <u>cutting</u> <u>down on precision</u>. Specifically, it doesn't rely <u>on floating point</u> <u>precision like a GPU</u>

. . . .

Instead the chip uses integer math...TPU used **<u>8-bit integer</u>**."

- Next Platform (May 19, 2016)





Nvidia PASCAL

"New half-precision, 16-bit floating point instructions deliver over 21 TeraFLOPS for unprecedented training performance. With 47 TOPS (tera-operations per second) of performance, new 8-bit integer instructions in Pascal allow AI algorithms to deliver real-time responsiveness for deep learning inference."

- Nvidia.com (April 2016)



Precision Varies from Layer to Layer



[Judd et al., ArXiv 2016]

[Moons et al., WACV 2016]

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Bitwidth Scaling (Speed)

Bit-Serial Processing: Reduce Bit-width → Skip Cycles Speed up of 2.24x vs. 16-bit fixed





[Judd et al., Stripes, CAL 2016]

Bitwidth Scaling (Power)



[Moons et al., VLSI 2016]



Binary Nets



Classification Accuracy(%)									
Binary-Weight				Binary-Input-Binary-Weight				Full-Precision	
BWN		BC[11]		XNOR-Net		BNN[11]		AlexNet[1]	
Top-1	Top-5	Top-1	Top-5	Top-1	Top-5	Top-1	Top-5	Top-1	Top-5
56.8	79.4	35.4	61.0	44.2	69.2	27.9	50.42	56.6	80.2

BinaryConnect (BC) = [Courbariaux et al., ArXiv 2015] Binary Neural Networks (BNN) = [Courbariaux et al., ArXiv 2016]

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[Rastegari et al., BWN & XNOR-Net, ECCV 2016]

Reduce Number of Ops and Weights

- Network Compression
 - Low Rank Approximation
 - Weight Sharing and Vector Quantization
- Pruning
 - Weights
 - Activations
- Network Architectures



Low Rank Approximation

- Low Rank approximation
 - Tensor decomposition
 based on singular value
 decomposition (SVD)
 - Filter Clustering with modified K-means
 - Fine Tuning



- Speed up by 1.6 2.7x on CPU/GPU for CONV1, CONV2 layers
- Reduce size by 5 13x for FC layer
- < 1% drop in accuracy

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[Denton et al., NIPS 2014]

Low Rank Approximation on Phone

- Rank selection per Layer
- Tucker Decomposition (extension of SVD)
- Fine tuning

Model Top-5		Weights	FLOPs	S 6		Titan X
AlexNet	80.03	61M	725M	117ms	245mJ	0.54ms
AlexNet*	78.33	11 M	272M	43ms	72mJ	0.30ms
(imp.)	(-1.70)	$(\times 5.46)$	$(\times 2.67)$	$(\times 2.72)$	$(\times 3.41)$	(×1.81)
VGG-S	84.60	103M	2640M	357ms	825mJ	1.86ms
VGG-S*	84.05	14 M	549M	97ms	193mJ	0.92ms
(imp.)	(-0.55)	(×7.40)	(×4.80)	$(\times 3.68)$	$(\times 4.26)$	$(\times 2.01)$
GoogLeNet	88.90	6.9M	1566M	273ms	473mJ	1.83ms
GoogLeNet*	88.66	4.7M	760M	192ms	296mJ	1.48ms
(imp.)	(-0.24)	$(\times 1.28)$	$(\times 2.06)$	$(\times 1.42)$	$(\times 1.60)$	$(\times 1.23)$
VGG-16	89.90	138M	15484M	1926ms	4757mJ	10.67ms
VGG-16*	89.40	127M	3139M	576ms	1346mJ	4.58ms
(imp.)	(-0.50)	$(\times 1.09)$	$(\times 4.93)$	$(\times 3.34)$	$(\times 3.53)$	$(\times 2.33)$

[Kim et al., ICLR 2016]

Weight Sharing + Vector Quantization

Trained Quantization: Weight Sharing via K-means clustering (reduce number of unique weights)



Exploit Data Statistics



Sparsity in Fmaps

Many zeros in output fmaps after ReLU



I/O Compression in Eyeriss

Link Clock Core Clock

DCNN Accelerator



Compression Reduces DRAM BW



Simple RLC within 5% - 10% of theoretical entropy limit



[Chen et al., ISSCC 2016]

Data Gating / Zero Skipping in Eyeriss



[Chen et al., ISSCC 2016]

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Cnvlutin

- Process Convolution Layers
- Built on top of DaDianNao (4.49% area overhead)
- Speed up of 1.37x (1.52x with activation pruning)



[Albericio et al., ISCA 2016]

Pruning Activations

Remove small activation values



[Albericio et al., ISCA 2016]

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[Reagen et al., ISCA 2016]

Pruning – Make Weights Sparse

- Optimal Brain Damage
- 1. Choose a reasonable network architecture
- 2. Train network until reasonable solution obtained
- 3. Compute the second derivative for each weight
- 4. Compute saliencies (i.e. impact on training error) for each weight
- 5. Sort weights by saliency and delete low-saliency weights
- 6. Iterate to step 2



[Lecun et al., NIPS 1989]

Pruning – Make Weights Sparse

Prune based on magnitude of weights





[Han et al., NIPS 2015]

Pruning of VGG-16

Pruning has most impact on Fully Connected Layers



Speed up of Weight Pruning on CPU/GPU

On Fully Connected Layers

Average Speed up of 3.2x on GPU, 3x on CPU, 5x on mGPU



Intel Core i7 5930K: MKL CBLAS GEMV, MKL SPBLAS CSRMV NVIDIA GeForce GTX Titan X: cuBLAS GEMV, cuSPARSE CSRMV NVIDIA Tegra K1: cuBLAS GEMV, cuSPARSE CSRMV

Batch size = 1

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[Han et al., NIPS 2015]

Energy-Aware Pruning

- # of Weights alone is not a good metric for energy
 - Example (AlexNet):
 - # of Weights (FC Layer) > # of Weights (CONV layer)
 - Energy (FC Layer) < Energy (CONV layer)
- Us energy evaluation method to estimate DNN energy
 - Account for data movement
- Prune based on energy rather than weights
 - Reduce <u>overall energy (ALL layers)</u> by 3.7x for AlexNet
 - 1.8x more efficient than previous magnitude-based approach
 - 1.6x energy reduction for GoogleNet



Compression of Weights & Activations

- Compress weights and fmaps between DRAM and accelerator
- Variable Length / Huffman Coding

Example:

Value: $16'b0 \rightarrow$ Compressed Code: $\{1'b0\}$

Value: 16'bx \rightarrow Compressed Code: {1'b1, 16'bx}

Tested on AlexNet → 2× overall BW Reduction

Layer	Filter / Image bits (0%)	Filter / Image BW Reduc.	IO / HuffIO (MB/frame)	Voltage (V)	MMACs/ Frame	Power (mW)	Real (TOPS/W)
General CNN	16 (0%) / 16 (0%)	1.0x		1.1	_	288	0.3
AlexNet 11	7 (21%) / 4 (29%)	1.17x / 1.3x	1 / 0.77	0.85	105	85	0.96
AlexNet 12	7 (19%) / 7 (89%)	1.15x / 5.8x	3.2 / 1.1	0.9	224	55	1.4
AlexNet 13	8 (11%) / 9 (82%)	1.05x / 4.1x	6.5 / 2.8	0.92	150	77	0.7
AlexNet 14	9 (04%) / 8 (72%)	1.00x / 2.9x	5.4 / 3.2	0.92	112	95	0.56
AlexNet 15	9 (04%) / 8 (72%)	1.00x / 2.9x	3.7 / 2.1	0.92	75	95	0.56
Total / avg.	_	—	19.8 / 10	_	_	76	0.94
LeNet-5 11	3 (35%) / 1 (87%)	1.40x / 5.2x	0.003 / 0.001	0.7	0.3	25	1.07
LeNet-5 12	4 (26%) / 6 (55%)	1.25x / 1.9x	0.050 / 0.042	0.8	1.6	35	1.75
Total / avg.	_	_	0.053 / 0.043	-	-	33	1.6



[Moons et al., VLSI 2016; Han et al., ICLR 2016]

Sparse Matrix-Vector DSP

Use CSC rather than CSR for SpMxV



Reduce memory bandwidth by 2x (when not M >> N)

[Dorrance et al., FPGA 2014]

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EIE: A Sparse Linear Algebra Engine

- Process Fully Connected Layers (after Deep Compression)
- Store weights column-wise in Run Length format
 - Non-zero weights, Run-length of zeros
 - Start location of each column since variable length
- Read relative column when input is non-zero

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Network Architecture

Reduce size and computation with 1x1 Filter



Used in Network In Network(NiN) and GoogLeNet

[Lin et al., ArXiV 2013 / ICLR 2014] [Szegedy et al., ArXiV 2014 / CVPR 2015]


Network Architecture

Reduce size and computation with 1x1 Filter



Used in Network In Network(NiN) and GoogLeNet

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Network Architecture

Reduce size and computation with 1x1 Filter



Used in Network In Network(NiN) and GoogLeNet

[Lin et al., ArXiV 2013 / ICLR 2014] [Szegedy et al., ArXiV 2014 / CVPR 2015]



SqueezeNet



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[F.N. landola et al., ArXiv, 2016]

Energy Consumption of Existing DNNs

- Maximally reducing # of weights does not necessarily result in optimized energy consumption
- Deeper CNNs with fewer weights (e.g. GoogleNet, SqueezeNet), do not necessarily consume less energy than shallower CNNs with more weights (e.g. AlexNet)
- Reducing # of weights can provide equal or more reduction than reducing the bitwidth of weights (e.g. BWN)



* Energy-aware Pruning (This Work)

Benchmarking Metrics for DNN Hardware

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html



Joel Emer, Vivienne Sze, Yu-Hsin Chen

Metrics Overview

- How can we compare designs?
- Target Metrics
 - Accuracy
 - Power
 - Throughput
 - Cost
- Additional Factors
 - External memory bandwidth
 - Required on-chip storage
 - Utilization of cores

Download Benchmarking Data

- Input (<u>http://image-net.org/</u>)
 - Sample subset from ImageNet Validation Dataset
- Widely accepted state-of-the-art DNNs (Model Zoo: <u>http://caffe.berkeleyvision.org/</u>)
 - AlexNet
 - VGG-16
 - GoogleNet-v1
 - ResNet-50



Metrics for DNN Algorithm

- Accuracy
- Network Architecture
 - # Layers, filter size, # of filters, # of channels
- # of Weights (storage capacity)

Number of non-zero (NZ) weights

- # of MACs (operations)
 - Number of non-zero (NZ) MACS



Metrics of DNN Algorithms

Metrics	AlexNet	VGG-16	GoogLeNet (v1)	ResNet-50
Accuracy (top-5 error)*	19.8	8.80	10.7	7.02
Input	227x227	224x224	224x224	224x224
# of CONV Layers	5	16	21	49
Filter Sizes	3, 5,11	3	1, 3 , 5, 7	1, 3, 7
# of Channels	3 - 256	3 - 512	3 - 1024	3 - 2048
# of Filters	96 - 384	64 - 512	64 - 384	64 - 2048
Stride	1, 4	1	1, 2	1, 2
# of Weights	2.3M	14.7M	6.0M	23.5M
# of MACs	666M	15.3G	1.43G	3.86G
# of FC layers	3	3	1	1
# of Weights	58.6M	124M	1M	2M
# of MACs	58.6M	124M	1M	2M
Total Weights	61M	138M	7M	25.5M
Total MACs	724M	15.5G	1.43G	3.9G

*Single crop results: <u>https://github.com/jcjohnson/cnn-benchmarks</u>

Metrics of DNN Algorithms

Metrics	AlexNet	VGG-16	GoogLeNet (v1)	ResNet-50
Accuracy (top-5 error)*	19.8	8.80	10.7	7.02
# of CONV Layers	5	16	21	49
# of Weights	2.3M	14.7M	6.0M	23.5M
# of MACs	666M	15.3G	1.43G	3.86G
# of NZ MACs**	394M	7.3G	806M	1.5G
# of FC layers	3	3	1	1
# of Weights	58.6M	124M	1M	2M
# of MACs	58.6M	124M	1M	2M
# of NZ MACs**	14.4M	17.7M	639k	1.8M
Total Weights	61M	138M	7M	25.5M
Total MACs	724M	15.5G	1.43G	3.9G
# of NZ MACs**	409M	7.3G	806M	1.5G

*Single crop results: <u>https://github.com/jcjohnson/cnn-benchmarks</u>



**# of NZ MACs computed based on 50,000 validation images

Metrics of DNN Algorithms

Metrics	AlexNet	AlexNet (sparse)	
Accuracy (top-5 error)	19.8	19.8	
# of Conv Layers	5	5	
# of Weights	2.3M	2.3M	
# of MACs	666M	666M	
# of NZ weights	2.3M	863k	
# of NZ MACs	394M	207M	
# of FC layers	3	3	
# of Weights	58.6M	58.6M	
# of MACs	58.6M	58.6M	
# of NZ weights	58.6M	5.9M	
# of NZ MACs	14.4M	2.1M	
Total Weights	61M	61M	
Total MACs	724M	724M	
# of NZ weights	61M	6.8M	
# of NZ MACs	409M	209M	

Metrics for DNN Hardware

- Measure energy and DRAM access relative to number of non-zero MACs and bit-width of MACs
 - Account for impact of sparsity in weights and activations
 - Normalize DRAM access based on operand size
- Energy Efficiency of Design
 - pJ/(non-zero weight & activation)
- External Memory Bandwidth
 - DRAM operand access/(non-zero weight & activation)
- Area Efficiency
 - Total chip mm²/multi (also include process technology)
 - Accounts for on-chip memory



ASIC Benchmark (e.g. Eyeriss)

ASIC Specs	
Process Technology	65nm LP TSMC (1.0V)
Total core area (mm ²) /total # of multiplier	0.073
Total on-Chip memory (kB) / total # of multiplier	1.14
Measured or Simulated	Measured
If Simulated, Syn or PnR? Which corner?	n/a



ASIC Benchmark (e.g. Eyeriss)

Layer by layer breakdown for AlexNet CONV layers

Metric	Units	L1	L2	L3	L4	L5	Overall*
Batch Size	#				4		
Bit/Operand	#				16		
Energy/ non-zero MACs (weight & act)	pJ/MAC	16.5	18.2	29.5	41.6	32.3	21.7
DRAM access/ non-zero MACs	Operands/ MAC	0.006	0.003	0.007	0.010	0.008	0.005
Runtime	ms	20.9	41.9	23.6	18.4	10.5	115.3
Power	mW	332	288	266	235	236	278



* Weighted average of CONV layers

Website to Summarize Results

- <u>http://eyeriss.mit.edu/benchmarking.html</u>
- Send results or feedback to: <u>eyeriss@mit.edu</u>

		Metric	Units	Input
ASIC Specs	Input	Name of CNN	Text	AlexNet
Process	Process 65nm LP TSMC	# of Images Tested	#	100
Technology	(1.0V)	Bits per operand	#	16
Chip area (mm ²) /	0.095	Batch Size	#	4
multiplier		# of Non Zero MACs	#	409M
On-Chip memory (kB) / multiplier	1.14	Runtime	ms	115.3
Measured or Measured	Power	mW	278	
Simulated		Energy/non-zero	pJ/MAC	21.7
If Simulated, Svn	n/a	MACs		
or PnR? Which corner?		DRAM access/non- zero MACs	operands /MAC	0.005



Implementation-Specific Metrics

Different devices may have implementation-specific metrics

Example: FPGAs

Metric		Units	AlexNet
Device		Text	Xilinx Virtex-7 XC7V690T
Utilization	DSP	#	2,240
	BRAM	#	1,024
	LUT	#	186,251
	FF	#	205,704
Performance Density		GOPs/slice	8.12E-04



Hardware for Training

MICRO Tutorial (2016)

Website: http://eyeriss.mit.edu/tutorial.html

Joel Emer, Vivienne Sze, Yu-Hsin Chen



Cost function for Model Training



What do we vary to minimize the error?



Training Optimization Problem

- Model parameters θ (include bias, weights, ...)
- Model output $y(\theta) = f(x, \theta)$
- Desired output
- Error $e(\theta) = y(\theta) z$
- Cost function^{*}
- Minimization $dE(\theta)/d\theta = 0$ (but no closed form)

Ζ

* Over all inputs in the training set

 $E(\theta) = \Sigma e(\theta)^2$



Steepest descent

Classical first order iterative optimization scheme: Gradient is steepest descent – follow it!



 $\theta^{n+1} = \theta^n - \alpha \cdot dE(\theta^n)/d\theta$

where α is the step size along the gradient...



Calculating Steepest Descent

Also called error back-propagation



• $dE(\theta)/d\theta = 2 \cdot \Sigma[(y(\theta)-z) \cdot dy(\theta)/d\theta]$

error **e** back-propagation

Chain rule -> Back propagation

 The chain rule of calculus allows one to calculate the derivative of a layered network, i.e., a composition of functions, iteratively working backwards through the layers using the (feature map) values of the layer, i.e., function, and the derivative from the next layer.

 Back propagation is the process of doing this calculation numerically for a given input.



Per Layer Calculations

Xi

 $\mathbf{y} = f(\mathbf{x})$

For layer k: Inputs: x^k Weights: w^k Outputs: y^k

So $y_i^k = f^k [\Sigma(w_{ij}^k x_j^k)]$

Where

 $x_{j}^{k} = y_{j}^{k-1}$

or $\mathbf{y}^{k} = f^{k}(\mathbf{y}^{k-1}, \mathbf{\theta})$





Layer Operation Composition

- Steepest descent $\theta^{k+1} = \theta^k \alpha \cdot dE/d\theta$
- Derivative (1) $dE/d\theta = 2 \cdot \Sigma[(y(\theta)-z) \cdot dy(\theta)/d\theta]$
- Model output $y^{n} = f(x)$ $y^{n} = f^{n}(y^{n-1}) = f^{n}(f^{n-1}(y^{n-2}))$
- Layer k $y^k = f^k(y^{k-1}) = f^k(f^{k-1}(y^{k-2}))$



Chain rule

Chain rule for functions

y = f(g(x)) y' = f'(g(x)) * g'(x)

$$y = f^{n}(y^{n-1}) = f^{n}(f^{n-1}(y^{n-2}))$$

y' =
$$f^{n'}(f^{n-1}(y^{n-2})) * f^{n-1'}(y^{n-2})$$

= $f^{n'}(y^{n-1}) * f^{n-1'}(y^{n-2})$



Back propagation

•
$$y_{00} = (a^*x_0 + b^*x_1)$$

 $y_{01} =$
 $y_{10} = y_{00}^*c + y_{01}^*d$

• $dy_{10}/da = dy_{10}/dy_{00} * dy_{00}/da = c * x_0$



Back Propagation for Addition

- y₀ = a + b
- $y_1 = f(y_0)$
- $dy_0/da = 1$
- $dy_0/db = 1$
- $dy_1/dy_0 = f'(y_0)$



dy1/dy0 * 1

- $dy_1/da = dy_1/dy_0 * dy_0/da = dy_1/dy_0 * 1 = dy_1/dy_0$
- $dy_1/db = dy_1/dy_0 * dy_0/db = dy_1/dy_0 * 1 = dy_1/dy_0$

Back Propagation for Multiplication

- y0 = a * b
- y1 = f(y0)

- dy0/da = b
- dy0/db = a
- dy1/dy0 = f'(y0)
- dy1/da = dy1/dy0 * dy0/da = dy1/dy0 * b
- dy1/db = dy1/dy0 * dy0/db = dy1/dy0 * a





Back propagation for Network

y00 = (a*x0 + b*x1) y01 =
y10 = y00*c + y01*d

• dy10/da = dy10/dy00 * dy00/da = c * x0



Back Propagation Recipe

Start point

• Select a initial set of weights (θ) and an input (x)

Forward pass

- For all layers
 - Compute layer outputs use as input for next layer (and save for later)

Backward pass

- For all layers (with output of previous layer and gradient of next layer)
 - Compute gradient, i.e., (partial) derivative, for layer
 - Back-propagate gradient to previous layer
 - Compute (partial) derivatives for (local) weights of layer

Calculate next set of weights

• $\theta^{k+1} = \theta^k - \alpha \cdot dE/d\theta$



Back Propagation



Precision on Training





- Beware truncating changes to zero
- Rounding can bias result -> use stochastic rounding



Back Propagation Batches



Issue:

N = 1 is often too noisy, weights may oscillate around the minimum

Solution:

- Use batches of N inputs...
- Max theoretical speed up: N



Parallel creation of gradient

- Steepest descent $\theta^{k+1} = \theta^k \alpha \cdot dE/d\theta$
- Derivative $dE/d\theta = 2 \cdot \Sigma[(y(\theta)-z) \cdot dy(\theta)/d\theta]$

Split sum of pieces of $dE/d\theta$ across different nodes!



Batch Parameter Update



[Dean et al., NIPS 2012]

Training Uses a Lot of Memory

GPU memory usage proportional to network depth

GPU memory







[Rhu et al., vDNN, MICRO 2016]
How Much Memory Is It?





GPU

memory

Reuse Distance of Feature Maps



MiT 📀

[Rhu et al., vDNN, MICRO 2016]

Problems with saturation



Issue

- A null gradient results in no learning, which happens if:
 - the sigmoid saturates, or
 - the ReLU saturates

Solution

- Initialize weighs so the average value is zero, i.e., work in the interesting zone of the activation functions
- Normalize data (zero mean)



Non-differential operations

Issue

- Discrete activation function / weights
 - extreme case is binary net
- Derivative not well defined

Solution

- Use approximate derivative, or
- Discretize a-posteriori



Model Overfitting

Problem:

- Neural net learns too specifically from input set, rather than generalizing from input, called overfitting
- Overfitting can be a result of too many parameters in model

Solution:

- Dropout turn off neurons at random; other neurons will take care of their job.
 - + Reliability
 - - Redundancy (-> pruning)



Architecture Challenges for Training

- Floating point accuracy
- Where to store the gradients
- Synchronization for parallel processing



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