System Architecture for Mode-Matching

a MEMS Gyroscope

by

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ABSTRACT

MEMS gyroscopes are used to detect rotation rates and have enabled a variety of motion-based technologies in a range of industries. They are composed of micro-machined polysilicon structures that resonate and deflect when a rotation is experienced. The topic of this thesis surrounds a system architecture to optimize the performance of a gyroscope.

The MEMS gyroscope contains a resonator and an accelerometer, modeled as a two degree-of-freedom mass-spring system. When the resonant frequencies of each mode are matched, the mechanical output of the gyroscope is maximal. Feedback is used to match the two modes by automatically tuning the voltage on the poly-silicon structure until the accelerometer resonant frequency matches that of the resonator. A square wave dither signal is introduced as quadrature error and is used to track the phase across the gyroscope’s accelerometer. At mode-match, the phase lag is 90°, so the feedback mechanism maintains this 90° of phase lag between the input acceleration and mechanical output to keep the modes matched.

Two controllers were tried in the feedback mechanism, a linear controller and a bang-bang controller. The bang-bang controller was found to produce better results, and was able to bring a pre-fabricated sensor die to mode-match and achieve a resolution floor of 12°/hr.

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# Table of Contents

Abstract.............................................................................................................................................1

1 Introduction ..........................................................................................................................................5

1.1 Importance of MEMS Gyroscopes ...............................................................................................5

1.2 Applications ...................................................................................................................................6

1.2.1 Automotive .............................................................................................................................6

1.2.2 Consumer ...............................................................................................................................7

1.2.3 Industrial ...................................................................................................................................8

1.2.4 Military ......................................................................................................................................8

1.3 Summary .......................................................................................................................................9

2 Technical Background ....................................................................................................................10

2.1 Coriolis Acceleration ...................................................................................................................10

2.2 MEMS Vibratory Rate Gyroscope Operation .............................................................................14

2.2.1 Understanding the Physics .....................................................................................................14

2.2.2 Understanding a VRG Implementation ..................................................................................20

2.3 Signal Components .....................................................................................................................23

2.4 Electronegative Spring Dampening ............................................................................................27

2.5 Mode-Matching ..........................................................................................................................30

2.6 Related Work ...............................................................................................................................38

2.7 Summary .......................................................................................................................................39

3 Proposed Approach .......................................................................................................................41

3.1 System Overview .........................................................................................................................41

3.2 Sensor Die ....................................................................................................................................45

3.3 Clock Controller, Block-level Overview .....................................................................................47

3.4 Dither Controller, Block-level Overview ....................................................................................50

3.5 Rate Controller, Block-level Overview .......................................................................................51

3.6 Quadrature Controller, Block-level Overview ............................................................................56

3.7 Shield Controller, Block-level Overview .....................................................................................59

3.7.1 Linear Shield Controller .......................................................................................................61

3.7.2 Bang-bang Shield Controller ................................................................................................62
3.8 Summary ........................................................................................................... 63

4 System Analysis ..................................................................................................... 65

4.1 Design Considerations ....................................................................................... 65

4.1.1 Setting the Dither Frequency ........................................................................ 65

4.1.2 Setting the Corner Frequency of the Rate Controller ......................... 66

4.1.3 Using Integrators in the Quadrature Controller and Shield Controller 66

4.2 System Modeling ............................................................................................... 66

4.2.1 Gyroscope Stage with Demodulator, H(s) .............................................. 69

4.2.2 System Transfer Function ......................................................................... 75

4.3 System Stability Analysis ................................................................................. 77

4.4 Summary ........................................................................................................... 78

5 Circuit Realization and Results ........................................................................... 79

5.1 Clock Controller ............................................................................................... 79

5.2 Dither Controller ............................................................................................. 83

5.3 Rate Controller ............................................................................................... 89

5.4 Quadrature Controller .................................................................................... 98

5.5 Shield Controller ............................................................................................. 108

5.5.1 Linear Shield Controller ............................................................................ 111

5.5.2 Bang-bang Shield Controller ..................................................................... 115

5.6 Error Characteristics ....................................................................................... 117

5.7 Summary ........................................................................................................... 121

6 Conclusion ........................................................................................................... 123

7 References............................................................................................................ 125
1 Introduction

1.1 Importance of MEMS Gyroscopes

Micromachined inertial sensors have contributed greatly to today's market. Of particular importance are MEMS accelerometers and MEMS gyroscopes. MEMS gyroscopes are silicon-based sensors that are used to measure angular rate. They are ideal for motion-based systems, where information about an object's orientation or current motion is important. For example, in a robotic arm, a MEMS gyroscope can be used to detect how many degrees the arm has moved. For a car, a MEMS gyroscope can be used to determine if the car is skidding. Gyroscopes are found in a wide range of products and offer advantages in many areas, including automotive, consumer, industrial, and military applications. They can offer high performance in miniature packages; their performance makes them ideal for safety-critical and time-sensitive applications, and their small size makes them easy to physically incorporate into systems.

Because of the numerous applications for MEMS gyroscopes in motion-based systems, there is a lot of incentive for research in this area. In particular, research for MEMS gyroscopes is focused on improving performance and lowering cost. A highly sensitive, low-cost MEMS gyroscope can be easily mass-produced, so they are an attractive technology to pursue. Some particular applications are discussed in the following section.
1.2 Applications

1.2.1 Automotive

In the automotive industry, MEMS gyroscopes are found in several applications. Some important ones are related to safety. For example, automotive electronic stability control (ESC) systems use MEMS gyroscopes to detect the rotation rate of a car around its vertical axis. When a car skids or spins on ice, for example, a MEMS gyroscope can be used to detect that the car has entered a skid condition. The ESC system interprets rotation data from the gyroscope before signaling that the brakes need to be applied to counter the skid and put the car back in the right direction. Without this rotation data, the car could potentially skid out of control and endanger the driver. It is also crucial that the gyroscope is high fidelity. It does not make sense to design a safety system that relies on a device that provides incorrect data or reports data after the accident has occurred. In that sense, gyroscopes are able to not only supply important rotational data about a car but also do so in a timely manner to support safety-critical systems.

Another important safety mechanism that MEMS gyroscopes have enabled is rollover detection. When a vehicle enters a tight turn too fast, it runs the risk of rolling over. Sport utility vehicles (SUVs) are a notorious example of this problem; their tall body and narrow wheel base increase their risk of rolling over during a turn. When a car starts to tip over, the vehicle’s system can either brake the car before rollover happens or deploy airbags to save the driver. In both of these measures, a MEMS gyroscope is responsible for providing data on how the car rocks side to side.

For GPS and other navigation systems, occasionally the navigation device is not able to detect a satellite signal because the car enters a tunnel or the view of the sky is
obstructed. Instead of completely failing and becoming useless, the device uses a dead-
reckoning system to approximate the motion of the car and uses that data to estimate how
far a driver has deviated along the route. GPS devices can do this by incorporating
accelerometers and gyroscopes into their design, so that they can monitor the car’s motion
as needed. Even when a signal is lost, the device does not give up navigating, and the driver
is not completely on his own. For convenience purposes, GPS devices are light and
portable. MEMS gyroscopes cater to these systems by having compact form factors that are
easily integrated into electronic systems.

1.2.2 Consumer

MEMS gyroscopes have been introduced into the consumer space to support
handheld devices and enable interesting forms of human-computer interaction. A popular
example of MEMS gyroscopes in the consumer market is their application in image
stabilization. Digital cameras are very sensitive to subtle motions that users make with
their hands. When someone takes a picture, it is likely that his hands might shake or move
a little. If these motions aren’t compensated for, the pictures come out blurry and
unintelligible. By using MEMS gyroscopes for image stabilization, unintentional motions
can be detected. By factoring this information into the camera shot, a camera can correct
for blurriness. Combining MEMS gyroscope technology with digital image capture helps
produce better quality photos and corrects for errors that would otherwise be present.

There have also been applications for MEMS gyroscopes in the area of human-
computer interactions (HCI). This area is broad and spans using MEMS gyroscopes in game
controllers to using them in multimodal interfaces, such as pen-based computing. There is
a large amount of research in HCI, and MEMS gyroscopes have enabled new forms of
interaction by being able to record rotational data from users' hands, feet, and bodies. By integrating MEMS gyroscopes into game controllers, users gain a more dynamic experience with their games. In other systems, MEMS gyroscopes enable more intuitive interfaces that respond to human motion; they are enabling the shift away from a typical keyboard/mouse interface toward more dynamic human-computer interactions.

1.2.3 Industrial

Industrial applications for MEMS gyroscopes relate to equipment monitoring, robotic systems, and platform stabilization. MEMS gyroscopes allow equipment managers to detect when machinery is acting irregularly or hydraulic systems aren't performing properly. For example, a large rotor in a machine may have some irregular motion and a MEMS gyroscope would provide information about odd rotation rates. The ability to diagnose problems before they become large can help an industrial company take preventative measures. In industries where large robotic arms are used in manufacturing, MEMS gyroscopes play the roles of determining how to position the arms at the proper angles and of monitoring arm movements. In industrial applications that rely on stable, level platforms, MEMS gyroscopes are used to detect when a platform becomes tilted and can prevent issues with products that are manufactured on the machinery.

1.2.4 Military

Military applications for MEMS gyroscopes have roots in the aerospace and defense sectors. Since MEMS gyroscopes measure angular rate changes, they are ideal for unmanned air vehicles (UAVs), missile guidance systems, inertial navigation, and other applications. All of these applications require a high degree of sensitivity and ease of integration. For UAVs, high-fidelity MEMS gyroscopes help remote pilots control the yaw,
pitch, and roll of their vehicles. By being able to monitor the motion of a UAV, a pilot gains more information about the vehicle's trajectory than a simple vision-based system would provide. Since air travel requires small, light components, MEMS gyroscopes are ideal for avionic applications. A missile guidance system requires that the MEMS gyroscope is robust and reliable. Any failure in the gyroscope could affect the course of the missile and have adverse effects at its final destination. Missiles may occasionally use dead-reckoning systems, so it is important that their gyroscopes are accurate and do not drift over time.

1.3 Summary

Gyroscopes have a multitude of applications. Only a handful of them have been identified in this introduction, but it should be clear that there is motivation for the improvement of MEMS technology in this area. Improving the resolution of a given gyroscope is an active area of research, and this thesis explores one possible method of achieving it.

The remainder of the thesis is arranged as follows. Chapter 2 focuses on the physics that are involved in the operation of a MEMS gyroscope. It also presents a simple gyroscope implementation in the hopes of capturing some of the criteria that are of particular interest to MEMS gyroscope designers. Chapter 3 introduces a method to improve the performance of a gyroscope by making it mode-matched. It presents a system architecture to make this possible. Chapter 4 presents some of the mathematics behind the proposed system architecture so that the reader has an idea of what circuit parameters were important in the final design. Chapter 5 shows the results of implementing the proposed architecture with some discussion. Lastly, Chapter 6 discusses some closing thoughts about the project and future directions that could be taken.
2 Technical Background

2.1 Coriolis Acceleration

The topic of this thesis involves the use of a MEMS vibratory rate gyroscope (VRG), but before discussing how it works, it will help to provide background on the concept of Coriolis acceleration, which is the guiding principle to MEMS gyroscope operation. The MEMS gyroscope contains a poly-silicon structure that experiences acceleration when subjected to a rotation rate. Therefore, understanding the origins of the acceleration is important.

Consider the two reference frames in Figure 2.1, one inertial (Frame A) and one rotating (Frame B). An object located at $x_A$ in Frame A is located at $x_B$ in Frame B. The origin of Frame B is located at $X_{AB}$ in Frame A, and the axes of Frame B are referred to by the unit vectors $u_j$ where $j = 1, 2, 3$. Using the coordinate axis of Frame B, the location of the object is described by $x_B = (x_1, x_2, x_3)$.

![Figure 2.1: Moving Coordinate System. An object in a moving coordinate system can be described as follows. The object that exists at $x_A$ in Frame A exists at $x_B$ in Frame B. The origin of Frame B is described at $X_{AB}$ in Frame A. Each of the $u_j$ with $j = 1, 2, 3$ describe the axes of Frame B, and the object’s location in Frame B can be described by $x_B = (x_1, x_2, x_3)$ using those coordinate axes.][1]
The question to answer is: how can we describe the acceleration of the object? To do this, describe the position of the object in Frame B as:

\[ x_B = \sum_{j=1}^{3} x_j u_j \]

From the perspective of Frame A, the object is located at:

\[ x_A = X_{AB} + \sum_{j=1}^{3} x_j u_j \]

After taking the time derivative, we can find a formula for the velocity of the object in Frame A:

\[ \frac{dx_A}{dt} = \frac{dX_{AB}}{dt} + \sum_{j=1}^{3} \frac{dx_j}{dt} u_j + \sum_{j=1}^{3} x_j \frac{du_j}{dt} \]

One more time differentiation gives us the acceleration of the object in Frame A:

\[ \frac{d^2 x_A}{dt^2} = \frac{d^2 X_{AB}}{dt^2} + \sum_{j=1}^{3} \frac{d^2 x_j}{dt^2} u_j + \sum_{j=1}^{3} \frac{dx_j}{dt} \frac{du_j}{dt} + \sum_{j=1}^{3} x_j \frac{d^2 u_j}{dt^2} \]

After recognizing a few things, we can rewrite this equation in more intuitive terms. The first term on the right represents the acceleration of Frame B in Frame A. The second term on the right represents the acceleration of the object in Frame B. The third and fourth terms are identical and can be grouped together; they represent the velocity of the object in Frame B multiplied by the rate of change of the axes.

\[ \frac{d^2 x_A}{dt^2} = a_{AB} + a_B + 2 \sum_{j=1}^{3} v_j \frac{du_j}{dt} + \sum_{j=1}^{3} x_j \frac{d^2 u_j}{dt^2} \]
If we consider that Frame B is rotating around a vector \( \Omega \)—which is pointed along the axis of rotation—with rate \( |\Omega| \), ignore translational movement between the two reference frames, and only consider rotational motion, we can make the following substitutions:

\[
a_{AB} = 0
\]

\[
d\mathbf{u}_j = \Omega \times \mathbf{u}_j
\]

\[
\frac{d^2 \mathbf{u}_j}{dt^2} = \frac{d\Omega}{dt} \times \mathbf{u}_j + \Omega \times \frac{d\mathbf{u}_j}{dt} = \frac{d\Omega}{dt} \times \mathbf{u}_j + \Omega \times (\Omega \times \mathbf{u}_j)
\]

So,

\[
\frac{d^2 x_A}{dt^2} = a_B + 2 \sum_{j=1}^{3} v_j (\Omega \times \mathbf{u}_j) + \sum_{j=1}^{3} x_j \left( \frac{d\Omega}{dt} \times \mathbf{u}_j + \Omega \times (\Omega \times \mathbf{u}_j) \right)
\]

\[
= a_B + 2\Omega \times \sum_{j=1}^{3} v_j \mathbf{u}_j + \frac{d\Omega}{dt} \times \sum_{j=1}^{3} x_j \mathbf{u}_j + \Omega \times \left( \sum_{j=1}^{3} x_j \mathbf{u}_j \right)
\]

\[
a_A = a_B + 2(\Omega \times v_B) + \left( \frac{d\Omega}{dt} \times x_B \right) + \Omega \times (\Omega \times x_B)
\]

Solving in terms of \( a_B \), the acceleration of the object referenced to the rotating Frame B is:

\[
a_B = a_A - 2(\Omega \times v_B) - \left( \frac{d\Omega}{dt} \times x_B \right) - \Omega \times (\Omega \times x_B)
\]  

(2.1)

(2.1) shows the acceleration of the object as seen by an observer in Frame B. In order to match Newton's laws, the force from this acceleration must be considered as two components: \( F_B = F_A + F_{\text{fict}} \). There is a so-called “fictitious force” at play in these equations, which is described as:
\[ F_{\text{fict}} = -2m(\Omega \times v_B) - m \left( \frac{d\Omega}{dt} \times x_B \right) - m\Omega \times (\Omega \times x_B) \] (2.2)

Furthermore, (2.2) consists of three terms. The first term is the Coriolis force, the second term is the Euler force, and the third term is the Centrifugal force.

\[ F_{\text{fict}} = F_{\text{Coriolis}} + F_{\text{Euler}} + F_{\text{Centrifugal}} \]

Where,

\[ F_{\text{Coriolis}} = -2m(\Omega \times v_B) \] (2.3)

\[ F_{\text{Euler}} = -m \left( \frac{d\Omega}{dt} \times x_B \right) \] (2.4)

\[ F_{\text{Centrifugal}} = -m\Omega \times (\Omega \times x_B) \] (2.5)

There is an implicit assumption that the rate of rotation does not vary, so the Euler force goes to 0. Also, there are mechanisms to cancel the linear acceleration caused by the centrifugal force term. The Coriolis force, however, is the driving principle behind operation of a MEMS VRG. The Coriolis term states that a moving poly-silicon structure that undergoes a rotation will experience an acceleration that is proportional to the rotation rate in a direction that is orthogonal to both the rotation axis and the direction of motion. In more descriptive terms, if a block moves along the x-axis and rotates about the z-axis, it will accelerate along the y-axis as observed in Frame B. Once that acceleration is detected and converted into a meaningful medium, it is possible to discern the original rotation rate.
2.2 MEMS Vibratory Rate Gyroscope Operation

2.2.1 Understanding the Physics

Before discussing the inner workings of a real MEMS vibratory rate gyroscope, we can first take a step back to understand the physics involved in the system. After this discussion of the concepts, it will be easier to explain why the gyroscope is constructed as it is.

At the core of the MEMS gyroscope is a proof mass that has two degrees of freedom. It can move left and right along the x-axis, and it can move up and down along the y-axis. The mass is anchored via sets of springs. The gyroscope system is actually a classic example of a mass-spring system. Figure 2.2 demonstrates this system with a proof mass (m) and four springs defined by four spring constants (k_{x1}, k_{x2}, k_{y1}, k_{y2}).

![MEMS Gyroscope Mass-Spring System](image.png)

Figure 2.2: MEMS Gyroscope Mass-Spring System. Inside the MEMS gyroscope is a proof mass that has two degrees of freedom (x and y). The mass (m) is anchored via sets of springs, modeled here with spring constants k_{x1}, k_{x2}, k_{y1}, and k_{y2}. 
The fundamental principle that guides the operation of a MEMS gyroscope is the Coriolis force, as mentioned in 2.1. In the gyroscope, the proof mass is forced to oscillate along the x-axis. In response to a rotation around the z-axis, there is an acceleration in the y direction. This phenomenon arises because Coriolis acceleration is the cross product of the axis of rotation and the velocity. This ensures that the acceleration is orthogonal to both the rotational axis and the direction of motion.

Take, for example, that the mass is moving to the right, along the positive x-axis. If the mass is simultaneously rotated in the clockwise direction, the Coriolis acceleration is in the up direction, along the positive y-axis. If the mass is rotated in the opposite direction, the Coriolis acceleration is in the down direction, along the negative y-axis. We could reason out that the acceleration reverses sign in both cases if the mass is moving to the left instead.

Figure 2.3 shows that a moving mass undergoing rotation experiences an acceleration along the orthogonal axis. In Figure 2.3(a), the mass is moving to the right, rotated clockwise, and experiences an upward acceleration. In Figure 2.3(b), the mass is moving in the same direction, but since the direction of rotation has reversed, the Coriolis acceleration is in the opposite direction, down along the y-axis.
By sensing the magnitude of the accelerations, it is then possible to determine the input rotation rate that caused the Coriolis force. Section 2.2.2 will discuss how the mass is actually driven and how the accelerations are actually sensed.

Another important physical concept to understand is the nature of the mass-spring system. The mass-spring system is a second-order system that can be characterized with a transfer function that resembles (2.6), where $Y$ is the position of the mass, and $X$ is the input acceleration:

$$
\frac{Y(s)}{X(s)} = \frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}
$$

(2.6)
where,

\[
\omega_0 = \sqrt{\frac{k}{m}}
\]

(2.7)

\[k = \text{spring constant}, m = \text{mass}\]

and

\[Q = \text{quality factor}\]

What these equations imply is that the proof mass will deflect maximally if it is driven at an angular rate of \(\omega_0\). At this resonant frequency, we can achieve maximum mechanical output from the mass-spring system. This is desired because we will later see that the magnitude of the mechanical deflections will affect the magnitude of the electrical signal that we will convert it into. The transfer function in (2.6) can be graphed to produce a set of bode plots that will demonstrate the behavior of the system as the input drive frequency is swept. These plots are in Figure 2.4.

![Bode Diagram](image)

**Figure 2.4**: Mass-Spring System Transfer Function. (2.6) can be plotted in Bode diagram form to display magnitude and phase as a function of frequency. This plot was generated for \(\omega_0 = 1\) and \(Q=100\), and normalized so that Magnitude is 0dB at DC.
The plot in Figure 2.4 was generated for $\omega_0 = 1$, $Q = 100$, and has been normalized so that the magnitude of the output is 1 (0dB) at DC. Notice that at resonance, the magnitude of the transfer function is equal to $Q$ times the DC magnitude. This is the maximum value that the transfer function can take on. Another important thing to notice is that, at resonance, there is a $90^\circ$ phase delay through the transfer function. At resonance, if the input acceleration to the system is a sine wave, for example, the position of the mass will respond as an inverted cosine wave.

There are two second-order systems to consider in the gyroscope modeled in Figure 2.2. There is the system that describes motion along the x-axis and the system that describes motion along the y-axis. The mass is set up to oscillate along the x-axis at the resonant frequency for that axis. It is forcibly driven along that axis, so the x-axis is synonymously referred to as the drive axis or *drive mode*. The system that involves the mass resonating along the drive mode is referred to as the *resonator*. The y-axis is orthogonal to the drive mode and is called the *sense mode*, because Coriolis accelerations are sensed along the y-axis. The system that involves sensing accelerations along the sense mode is referred to as the *accelerometer*.

Each mode has a resonant frequency associated with it. In Figure 2.2, we introduced a proof mass with four unique springs. For simplicity, assume that the set of springs for each coordinate axis are equal ($k_{x1} = k_{x2} = k_R$ and $k_{y1} = k_{y2} = k_A$). We can simplify the gyroscope mass-spring system into two transfer functions—one to describe motion along the drive axis, and one to describe motion along the sense axis. In general, each of these transfer functions is defined by different resonant frequencies, $\omega_R$ and $\omega_A$ as follows:
\[
\omega_R = \text{resonant frequency of resonator} = \sqrt{\frac{k_R}{m}} \tag{2.8}
\]

\[
\omega_A = \text{resonant frequency of accelerometer} = \sqrt{\frac{k_A}{m}} \tag{2.9}
\]

Like Figure 2.4, the transfer function for each mode can be plotted as a function of frequency.

\[
\begin{align*}
\text{Resonator} & : \quad \frac{\omega_R}{s^2 + \frac{Q_R}{\omega_R} s + \omega_R^2} \\
\text{Accelerometer} & : \quad \frac{\omega_A}{s^2 + \frac{Q_A}{\omega_A} s + \omega_A^2}
\end{align*}
\]

Figure 2.5: Transfer Functions for Each Mode. Each mode has an associated transfer function that is governed by the resonant frequency for that mode. (a) The transfer function for the resonator. (b) The transfer function for the accelerometer.

It is not uncommon to design a gyroscope such that the resonant frequencies of both modes are close. When \( \omega_R = \omega_A \) then there is maximum energy transfer between the resonator axis and the accelerometer axis. That is, if the proof mass is resonating and it experiences a Coriolis acceleration, the deflection along the sense axis (accelerometer axis) is maximal when the two resonant frequencies are matched. This is ideal but difficult to
implement because it requires that the proof mass and springs are perfect and uniform. More discussion about this topic will occur later.

2.2.2 Understanding a VRG Implementation

Section 2.2.1 discussed some of the physical concepts involved in MEMS gyroscope operation. This section serves to discuss several key topics, including how the proof mass is driven and how accelerations are sensed.

At the core of the VRG is a poly-silicon structure, strategically lined with comb-like features. These comb-like features are interdigitated with other sets of comb-like features that are fixed to the poly-silicon substrate. One set of interdigitated “fingers” is responsible for generating the drive. Another set of interdigitated fingers is responsible for detecting accelerations.

Figure 2.6 demonstrates a cartoon depiction of a MEMS gyroscope structure. There are two main systems to look for—the resonator and the accelerometer. The inner box is the resonator, which is electrostatically driven. Comb-like features on the substrate switch polarity at the resonant frequency of the drive mode to cause the structure to be pulled back and forth along the drive axis. The entire structure is charged, so when the electrostatic comb drive elements change voltage, the resonator structure is electrostatically attracted and repelled, causing a mechanical motion.

The four flexures at each corner of the resonator can bend laterally and allow the resonator to move along the drive mode. When a Coriolis force is present, the resonator deflects vertically along the sense mode. This vertical motion is coupled into the accelerometer via the same four flexures. The accelerometer is the outer frame, and its deflections are sensed by sense elements that are fixed to the substrate. Not shown in the
cartoon are the anchors that tie the outer frame to the substrate. Those set of anchors set the spring constants for the accelerometer and allow the outer frame to almost float above the substrate below. There is some motion along the z-axis (into and out of the page), but it is factored out when performing final measurements.

Figure 2.6: Cartoon Depiction of MEMS Gyroscope Structure. The core of a gyroscope consists of a poly-silicon structure that can be described as two systems, a resonator and accelerometer. The resonator (the inner box) resonates horizontally; the four structures at the corners of the resonator can shear horizontally. The resonator’s motion is generated via electrostatic comb drive elements located at the center of the resonator frame. Vertical motion of the resonator (caused by Coriolis acceleration) is coupled into the accelerometer (the outer frame), whose deflections are sensed by fixed sense elements.

One question that can be asked is: how are deflections in the accelerometer translated into a meaningful electrical signal? The answer lies in closer examination of the
interdigitations. First, consider a parallel plate capacitor with area $A$, distance $d$ between the plates, and a dielectric constant $\varepsilon$ as in Figure 2.7.

![Figure 2.7: Parallel Plate Capacitor. A parallel plate capacitor is described by the area of overlap ($A$), the distance between the plates ($d$), and the dielectric constant ($\varepsilon$).](image)

It can be shown that the capacitance is:

$$C = \frac{\varepsilon A}{d}$$

As the distance between the two plates varies, the capacitance will also vary. This is exactly what is happening with the accelerometer in Figure 2.6. Capacitors are formed by the fingers on the accelerometer and the fingers fixed to the substrate. When the accelerometer frame deflects, the distance between the “plates” changes and can be translated into changes of capacitance. A transimpedance amplifier can take these changes and convert them into electrical signals.
Keep in mind that there are spring constants related to each axis of motion and that those spring constants give rise to resonant frequencies for each axis/mode. There are many mechanical factors that determine the spring constants of these springs. In a later section, there will be a discussion about how the spring constants can be slightly varied using electrical techniques. Consequently, changes in the spring constants will lead to changes in resonant frequencies. This is one of the key topics for this project.

2.3 Signal Components

The previous section discussed how Coriolis acceleration is translated into an electrical signal by detecting changes in capacitance. In this section, there will be a discussion on how to interpret the signals that are detected. In particular, the output from the accelerometer has two signal components, an in-phase component (I) and a quadrature component (Q).

The in-phase component is directly related to the Coriolis acceleration. However, manufacturing imperfections will give rise to a quadrature component. Let’s first consider the origin of the quadrature component. One assumption that was made was that the resonator is perfectly driven along the drive mode, and that there is no motion along the
sense mode until the resonator experiences a Coriolis acceleration. This can only occur if the spring constants along the sense mode are identical and perfectly cancel out—that is, there is no net force along the sense axis caused by mismatched springs. The motion of the resonator can be skewed if there is a net force on it along the sense mode, even if no Coriolis acceleration is present. For example, Figure 2.9 demonstrates ideal versus non-ideal behavior of the drive oscillation. In the ideal case, the resonator oscillates only along the drive mode; in the non-ideal case, the resonator has an orthogonal component to its motion. These unintentional deflections are maximal when the resonator is maximally away from the center of the frame and minimal when the resonator crosses the center of the frame. In other words, the deflections are in phase with the resonator’s position (relative to the center).

![Figure 2.9: Comparison of Ideal and Non-ideal Drive Behavior. (a) Ideal drive behavior with oscillation perfectly along drive axis. (b) Non-ideal drive behavior with orthogonal component to motion.](image)

The in-phase component, however, is related to the Coriolis acceleration. Coriolis acceleration is in phase with the velocity of the resonator since it is directly proportional to it as seen in (2.3). Using Figure 2.10 as a model, we can derive some equations to exemplify how to deal with the two different signal components.
The structure is moving back and forth along the drive mode at the resonant frequency of the drive mode ($\omega_R$). Using $x(t)$ as the deflection of the resonator along the drive mode and $X_R$ as the maximum deflection, we can model its motion as:

$$x(t) = X_R \sin(\omega_R t)$$

Taking it one step further, we can find the first derivative to get the velocity of the resonator:

$$v_x(t) = \frac{d}{dt} x(t) = \omega_R X_R \cos(\omega_R t)$$

Since our aim is to measure rotation about the $z$-axis, we know beforehand that the rotation axis is orthogonal to the drive mode. Since Coriolis acceleration is twice the cross product of angular rate and velocity, we can calculate it to be:

$$a_{Cor}(t) = -2(\Omega(t) \times v_x(t))$$

$$= 2\Omega(t) \omega_R X_R \cos(\omega_R t)$$

From here, we recognize that the rotation rate is modulated with a carrier frequency of $\omega_R$ and scaled by $2\omega_R X_R$ to produce Coriolis acceleration. For this project, we do not measure Coriolis acceleration directly. Instead, we actually measure the position of the

Figure 2.10: Vibrating Structure. The resonator oscillates along the drive mode (the x-axis here).
accelerometer, which is related to the input acceleration by the transfer function in Figure 2.5.

Next, consider that quadrature is introduced and that it is 90° out of phase with Coriolis acceleration. Acceleration along the sense mode is no longer just Coriolis acceleration; it is now the sum of Coriolis acceleration and quadrature:

\[ a_y = a_{\text{Cor}} + a_{\text{Quad}} \Leftrightarrow I \cos(\omega_R t) + Q \sin(\omega_R t) \]

Using synchronous demodulation, it is possible to isolate the original I and Q signals. I contains important rate information, and Q contains information about the quadrature error.

In an ideal world, I and Q would be separate and distinct. Demodulation at the correct frequency and phase would perfectly isolate each. However, realistically, it is important to not only identify quadrature but also to correct for it. In fact, there are mechanisms to null out quadrature error. If we consider the cartoon from Figure 2.6 again, we can create a more refined version that adjusts for quadrature error. The key is to correct for when the resonator deflects along the sense axis when it is furthest from the center. By introducing four new structures at the corners of the resonator, as in Figure 2.11, we can electrostatically control quadrature. If we apply the appropriate voltages on the quadrature correction elements, then the resonator will tend to follow a path along the drive mode and not orthogonal to it.
This section will focus on a concept called electronegative spring dampening. As mentioned in earlier sections, there are two key spring constants involved in a MEMS gyroscope—one characterizing the drive mode and one characterizing the sense mode. When these spring constants are perfectly matched, there is an increase in performance from the gyroscope. However, these are mechanical spring constants, so it is difficult to adjust them after a gyroscope has been built. One technique to adjust the mechanical spring constants is to adjust “electrostatic” spring constants. It, then, becomes possible to reduce the effective spring constant for an axis. The ability to adjust the spring constant of the sense mode, for example, allows us to bring the resonant frequencies of the sense and drive modes closer together even if that is not how they started. This method of tuning is
called electronegative spring dampening. We will first look at the forces that act on the structure and then see how the forces result in an effective spring constant that differs from the mechanical spring constant.

First, consider the frame of the accelerometer. There are two dominant forces acting on the frame. There is a mechanical spring force acting on the frame (because the accelerometer frame is tethered to the substrate via flexures), and there are also electrostatic forces acting on the frame from the poly-silicon elements that form capacitors with each of its fingers. All of the fingers on the accelerometer frame are rigid and move in tandem, so we can zoom in on one of the fingers to identify what forces are present. The analogy applies equally to all of the fingers and the frame as a whole. Figure 2.12 presents the forces at play. It models one of the accelerometer fingers with a sense element as a set of parallel plates, where one plate is tethered by a spring and the other is fixed. The tethered plate represents the accelerometer finger, and the fixed plate represents the sense elements that are fixed to the substrate and interdigitated between the accelerometer fingers.

Figure 2.12: Forces on Accelerometer Finger. There are two dominant forces on each finger of the accelerometer frame—a mechanical force $F_{\text{mech}}$ and an electrostatic force $F_{\text{elec}}$. Varying $V$ has the net effect of reducing the effective spring constant.
If the tethered plate is deflected from its equilibrium position at \(y_0\), there is a net force on the structure. We can compute the forces on the tethered plate as follows:

\[
F_{\text{mec}} = -k_{\text{mec}}(y - y_0)
\]

\[
F_{\text{elec}} = -\frac{1}{2} \frac{A\varepsilon}{y^2} V^2
\]

In reality, the accelerometer does not deflect much from its equilibrium position, so we can perform a linear approximation on \(F_{\text{mec}}\) and \(F_{\text{elec}}\) [2]:

\[
F_{\text{mec}} \approx \left(\frac{dF_{\text{mec}}}{dy}\right)_{y=y_0} y = -k_{\text{mec}} y
\]

\[
F_{\text{elec}} \approx \left(\frac{dF_{\text{elec}}}{dy}\right)_{y=y_0} y = \frac{A\varepsilon}{y_0^3} V^2 y
\]

If we combine the mechanical force with the electrostatic force on the accelerometer finger, we can find the total force and arrange the terms such that the electrostatic force looks like it introduces a voltage controlled spring constant:

\[
F_{\text{total}} = F_{\text{mec}} + F_{\text{elec}}
\]

\[
= -k_{\text{mec}} y + \frac{A\varepsilon}{y_0^3} V^2 y
\]

\[
= -\left(k_{\text{mec}} - \frac{A\varepsilon}{y_0^3} V^2\right) y
\]

\[
= -k_{\text{eff}} y
\]

Where,

\[
k_{\text{eff}} = \left(k_{\text{mec}} - \frac{A\varepsilon}{y_0^3} V^2\right)
\]  \(2.10\)

The original mechanical spring constant, \(k_{\text{mec}}\), is essentially altered to create an effective spring constant \(k_{\text{eff}}\) that is reduced by the square of the voltage applied to the finger. By changing the voltage on the accelerometer, it becomes possible to decrease the
effective spring constant for the sense axis. The implication is that we can decrease the
effective spring constant and change the resonant frequency for that axis until it matches
the resonant frequency of the orthogonal drive axis, since:

\[
\omega_A = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{k_{\text{mech}} - \frac{Ae}{y_0^2} V^2}{m}} \tag{2.11}
\]

One interesting note is that we can only decrease the effective spring constant by varying
voltage; there is a negative effect when changing the electric potential between the plates.
There is not any way to vary the voltage to cause \( k_{\text{eff}} \) to become stiffer. We can only soften
the springs. For this example, we only modeled a single accelerometer finger with a single
sense element. In reality, the same forces apply to every set of fingers and interdigitated
sense elements as the entire accelerometer frame deflects.

### 2.5 Mode-Matching

This section will describe what it means for a gyroscope to be mode-matched and how
we can achieve this condition. When a gyroscope is mode-matched, the benefits are
improved performance and increased sensitivity. This occurs when the resonant frequency
of the resonator axis (drive axis) exactly matches the resonant frequency of the
accelerometer axis (sense axis). During this period, the spring constants for the respective
modes are identical and there is maximum mechanical output from the accelerometer.
What are the benefits of this? There is inherent electronic noise in the gyroscope system.
By increasing the mechanical output of the accelerometer, it is possible improve the signal
to (electronic) noise ratio. This improvement means that we can design more sensitive
MEMS gyroscopes that are more robust to electronic noise.
The output of the accelerometer is directly coupled with the motion of the resonator along the sense axis. When there is a Coriolis force present, the acceleration is modulated by the resonant frequency of the drive axis ($\omega_R$). As the motion is coupled into the accelerometer, the magnitude of the mechanical response becomes dependent on the relation between the drive frequency ($\omega_R$) and the resonant frequency of the accelerometer axis ($\omega_A$). Considering the transfer function from Figure 2.4, we can see how the relationship between $\omega_R$ and $\omega_A$ affects the mechanical output of the accelerometer. When $\omega_R < \omega_A$ (also referred to as the *spring-limited case*), then the magnitude of the mechanical response is modest, and the phase of the response is near $0^\circ$. When $\omega_R > \omega_A$ (also referred to as the *mass-limited case*), then the magnitude of the mechanical response falls with increasing $\omega_R$. At this point, the phase of the mechanical output to the input acceleration is near $-180^\circ$ shifted. At the mode-matched point, when $\omega_R = \omega_A$, the accelerometer output is maximal because it operates right under the Q-peak. At this point, the phase of the mechanical output is $-90^\circ$ shifted from the input acceleration. Figure 2.13 shows how the relation between $\omega_R$ and $\omega_A$ changes the magnitude of the output for the accelerometer.
How do these phase shifts relate to the output of the gyroscope? First, remember that there are two components to the output of the accelerometer, an in-phase component and a quadrature component. The in-phase component results from a Coriolis acceleration, which is in phase with the velocity of the resonator (by definition). In-phase can be 0°-shifted, -90°-shifted, or -180°-shifted from the actual Coriolis acceleration depending on whether the gyroscope is in the spring-limited, mode-matched, or mass-limited case, respectively. Hence, it is important to know under what conditions the gyroscope is operating, if we want to demodulate in-phase properly. The quadrature error is in
quadrature with the in-phase component, so whatever clock is used to demodulate in-phase needs to be shifted by 90° to demodulate the quadrature error.

In Figure 2.14(a), the gyroscope operates in the spring-limited case, so when there is 0° phase shift across the accelerometer, in-phase (I) comes out the top channel and quadrature error (Q) comes out the bottom channel. In Figure 2.14(c) (the mass-limited case), when there is -180° of phase shift, I continues to come out the top channel and Q continues to come out the bottom channel; in both cases, there is a sign reversal. However, when there is -90° of phase shift (the mode-matched case), I and Q switch channels; I comes out the bottom channel and Q comes out the top channel. This is seen in Figure 2.14(b). If we always wanted in-phase to show up in the top channel and quadrature to show up in the bottom channel, we could simply switch the clock inputs to the mixers when the phase shift is -90°.

\[
I \cos(\omega_R t) + Q \sin(\omega_R t)
\]

\[
\cos(\omega_R t)
\]

\[
\sin(\omega_R t)
\]

(a)
Figure 2.14: In-phase and Quadrature Dependence on Phase Shift. (a) When the phase shift is 0°, P comes out the top channel and Q comes out the bottom channel. (b) When the phase shift is -90°, -Q comes out the top channel and P comes out the bottom channel. (c) When the phase shift is -180°, P and Q come out the same channels as in (a) but with sign reversal. The sine-cosine axes in each figure are meant to illustrate the phase rotations from the input acceleration signal to the output position signal.

To visually see what happens when a gyroscope becomes mode-matched, consider the following plots. Figure 2.15 shows the power spectrum of the accelerometer output of a gyroscope in the spring-limited case. Two peaks are visible in the spectrum, one for the resonator and one for the accelerometer. The resonator has a resonant frequency of 17.9 kHz. The voltage on the structure, colloquially called the shield voltage, has been set such that the accelerometer peak is to the right of the resonator's peak at 18.6 kHz. This
situation occurs when the shield voltage is below the mode-matching voltage, causing $\omega_R < \omega_A$.

Figure 2.15: Power Spectrum, Spring-limited Case ($\omega_R < \omega_A$). Power spectrum of the accelerometer output of a gyroscope with resonator frequency of 17.9 kHz and a peak caused by the accelerometer resonant frequency at 18.6 kHz.

By increasing the shield voltage, the gyroscope can be set to operate mode-matched. In Figure 2.16, the resonator and accelerometer peaks are indistinguishable because they are overlap at 17.9 kHz. This is a characteristic of mode-matching, since $\omega_R = \omega_A$. When the gyroscope is mode-matched, its frequency spectrum is symmetric about the resonator's frequency.
Figure 2.16: Power Spectrum, Mode-matched Case ($\omega_R = \omega_A$). This plot shows the power spectrum of the accelerometer output of a gyroscope with resonator frequency of 17.9 kHz. It also shows how the overlap of the resonator peak with the accelerometer peak makes them indistinguishable.

Lastly, if the shield voltage is increased past the mode-match voltage, then the gyroscope will operate in the mass-limited case. Figure 2.17 shows a resonator peak at 17.9 kHz as before, but the accelerometer peak has moved to its left at 17.1 kHz. This situation corresponds to ($\omega_R > \omega_A$).

Figure 2.17: Power Spectrum, Mass-limited Case ($\omega_R > \omega_A$). This plot shows the power spectrum of the accelerometer output of a gyroscope with resonator frequency of 17.9 kHz. It also shows the accelerometer peak shifted to 17.1 kHz.
What methods are available to achieve the mode-matched condition? One possible way to mode-match a gyroscope is to design the MEMS structure such that the mechanical spring constants along each mode are perfectly identical. This requires precise design of the mechanical structure. Even with a perfect design, however, it is difficult to guarantee that the spring constants will be identical after manufacturing. In Figure 2.13, the sensitivity of the gyroscope to design and process variations is dependent on the width of the Q peak; for a very high, narrow Q peak, it is very difficult to maintain mode-match because the window of operating frequencies is narrow. The point at which a gyroscope is mode-matched is very sensitive to manufacturing variations, and any imperfections that are introduced can have a dramatic effect on the resonant frequencies of each mode. Additionally, even if the structure was designed perfectly and even if the gyroscope was manufactured perfectly, variations in temperature and other unpredictable operating conditions could also have a dramatic impact on the resonant frequencies of the modes.

In the previous section (Section 2.4), we discussed the possibility of altering spring constants with electronegative spring dampening. In fact, this technique is used to adjust the resonant frequency of the sense mode after manufacturing. In some cases, the gyroscope is calibrated once after it is fabricated. The shield voltage is increased until the resonant frequencies of the drive and sense modes approach each other. Then, the voltage value is set, stored, and used for the life of the gyroscope. Even though this technique relaxes the stress on the precision of the mechanical element and relaxes the stress on manufacturing tolerances, there is still the issue of dealing with changes in operating conditions. Large variations could cause the gyroscope to lose mode-match.
A better technique is to introduce a feedback system that dynamically maintains mode-match lock. Indeed, that is the topic of this project, and the overarching goal is to develop a system that will improve the sensitivity of a MEMS gyroscope by using a feedback system that dynamically adjusts the resonant frequency of the sense axis by applying electronegative spring dampening.

2.6 Related Work

There are numerous literature sources that review the operation of MEMS gyroscopes, which is a broad topic in itself. This section hopes to highlight some work that has been done in the realm of mode-matching with respect to MEMS gyroscopes.

In [3], Chang et al. propose a design for a control loop that automatically mode-matches a MEMS gyroscope. Their method builds upon the concept of a phase-locked-loop (PLL) and its application to MEMS. In general, a PLL takes a reference frequency and the output of a voltage controlled oscillator (VCO), discerns the phase difference between them, and outputs a compensatory signal so that the output of the VCO matches that of the reference. For Chang et al., they build upon the knowledge that at mode-match there should be 90° of phase difference between the driving signal and the accelerometer position signal. Their method is to use a multiplier to detect the phase difference between the two signals and use the error in phase to drive a controller which adjusts the sense mode of the accelerometer to match the resonant frequency of the drive mode.

The authors of [4] use a different technique to mode-match a gyroscope. Their method is based on the fact that the mechanical output of the gyroscope is maximal when it is mode-matched; this means that both the desired Coriolis signal and the undesired quadrature error are maximized. First the authors minimize the amount of quadrature
error that is present after fabrication with a set of quadrature correction elements on the sensor. In reality, however, it is very difficult to completely null out quadrature, so during operation, their system tries to identify when the sensor is mode-matched when quadrature error is maximal. They, then, employ a feedback to maintain the shield voltage that maximizes the quadrature error. Through I-Q demodulation, they are then able to separate in-phase Coriolis acceleration from quadrature error. Ultimately, their tuning fork gyroscope is able to achieve sub-degree-per-hour (0.2°/hr bias drift) performance. The sensor in their system is a tuning fork gyroscope with a sense quality factor of 36,000, which is significantly higher than the sense quality factor for the sensor in this project, which is roughly 50.

In [5], Ezekwe and Boser accomplish mode-matching with a technique that enables a vibratory gyroscope with a 0.004°/s/√Hz noise floor over a 50 Hz band. The guiding principle in their method is to inject a calibration signal that will allow them to detect when the drive and sense resonant frequencies align. Their calibration signal introduces tones at the output, and the amplitude difference between them is used to tune out frequency mismatches. Additionally, they focus on power dissipation reduction techniques that rely on boxcar sampling.

2.7 Summary

The focus of this chapter was to present some of the physics and technical details that are important to MEMS gyroscope design. The guiding concept is that a gyroscope senses Coriolis acceleration through an accelerometer mechanism. After sensing, it is important to distinguish what signal components are attributable to the in-phase rotation signal and what signal components are attributable to errors in fabrication. After understanding
those details, another key topic to grasp is the concept of mode-matching. It is the process of aligning the resonant frequencies of a two degree-of-freedom system, and this technique is the highlight of this project. The following section will discuss a possible way to achieve mode-match, whose effects are improved gyroscope performance and sensitivity.
3 Proposed Approach

3.1 System Overview

The overarching goal of this project is to design a feedback system that will enable a MEMS gyroscope to operate mode-matched. Under the mode-matched condition, the resonant frequency of the drive mode matches that of the sense mode, so when a Coriolis acceleration is experienced, the mechanical output of the accelerometer is maximal. By increasing the mechanical output of the gyroscope, we can make the gyroscope more robust to electronic noise, which results in creating a more sensitive gyroscope.

The actual system consists of several blocks, each of which will be described with greater detail in subsequent sections. Figure 3.1 presents the system architecture that is used to mode-match the gyroscope. A packaged sensor die that contains minimal electronics is used. It contains the MEMS structure that will serve to mechanically detect Coriolis accelerations. The mechanical motions are sensed and translated into electrical signals for other controllers in this system. A clock controller is responsible for both generating clocks that will be used in the system as well as providing a drive signal for the resonator. The rate controller is responsible for demodulating the in-phase component from the sensor’s accelerometer outputs. The actual quadrature correction is handled by a quadrature controller, which is responsible for integrating quadrature errors and reacting by driving the quadrature correction elements to the proper voltage. Lastly, the heart of the
Figure 3.1: System-level Diagram. This system was designed to mode-match a MEMS gyroscope by introducing a dither signal onto the quadrature correction elements. When mode-matched, the dither will return through the appropriate channels. When not mode-matched, the dither will return through the rate controller. The error will be detected and corrected for by the shield controller.
project focuses on the development of the shield controller, which is responsible for driving the voltage on the structure to a value that will enable mode-match.

In order to do mode-matching, the system has a behavior similar to a phase locked loop (PLL). When the gyroscope is mode-matched, Section 2.5 discussed that there is a -90° phase shift in the transfer function of the accelerometer. The mechanical output of the accelerometer has a -90° phase shift from the input Coriolis acceleration. The PLL-like behavior of the feedback system maintains this -90° phase shift. When the resonant frequency of the drive mode is less than the resonant frequency of the sense mode ($\omega_R < \omega_A$), the mechanical output is in-phase with the Coriolis acceleration (and the drive signal), and we know that the gyroscope is not operating mode-matched. Similarly, when the resonant frequency of the drive mode is greater than the resonant frequency of the sense mode ($\omega_R > \omega_A$), the mechanical output is anti-phase with the Coriolis acceleration (and drive signal), and we also know that the gyroscope is outside of mode-match. By detecting these conditions, then it becomes possible to correct for them.

A drive signal is required to force the resonator to oscillate along the drive mode. It happens that when the resonator oscillates at resonance, the drive signal is in phase with its velocity. When the gyroscope is not mode-matched and $\omega_R < \omega_A$, then the in-phase component (I) can be demodulated using the drive signal since it is in phase with Coriolis acceleration which is in phase with the velocity of the resonator. The quadrature signal (Q) can be demodulated using a clock that is 90 degrees shifted from the drive signal. When the gyroscope is not mode-matched and is operating where $\omega_R > \omega_A$ the same situation occurs, but the signs of P and Q are reversed.
In the mode-matched case, we must be more careful about what clocks are used to demodulate I and Q. Because of the -90° shift, I is now in quadrature with the drive signal and Q is now in phase with the drive signal. Demodulating with the same clocks as in the previous two cases will invert which channels I and Q come out. To demodulate I, we cannot use the drive clock; we must a 90°-phase-delayed version of it. To demodulate Q, the drive clock should now be used. Essentially, in the mode-matched case, to avoid I and Q coming out the wrong controllers, the clocks that are used in the un-mode-matched case must be switched. Figure 2.14 might clear up why this is the case.

When the clocks are fixed for the mode-matched condition and the gyroscope is mode-matched, I comes out the rate controller and Q comes out the quadrature controller. In order to detect the state of the structure, a square wave dither signal is introduced onto the quadrature correction elements mentioned in Section 2.3 and is monitored to see which controller it passes through. It is generated by the dither controller and introduced onto the quadrature correction elements on the sensor. The dither acts as a Q signal. When the gyroscope is mode-matched, Q can be demodulated by the quadrature controller. If, however, the dither bleeds into the demodulated output of the rate controller, there is an error, meaning that the dither is corrupting the rate output. This will occur when the phase through the accelerometer is not perfectly -90°. Furthermore, if the dither comes through the rate controller, it is rectified by demodulating it a second time at the dither frequency and low-pass filtered to produce an error signal that is used to correct the shield voltage.

By using the error to correct the shield voltage, it is then possible to tune the resonant frequency of the sense mode to match that of the drive mode. The process continues until the error is null and we are mode-matched. In the ideal case, since the dither is introduced
as quadrature, the in-phase component should not be corrupted by it. This approach has the added benefit that the pure Coriolis acceleration output is unaffected. It continues to output on the rate channel when the gyroscope is in mode-match, while the dither passes through the quadrature channel.

### 3.2 Sensor Die

The purpose of the sensor die is to provide the system with a micromachined gyroscope structure that is sensitive to Coriolis acceleration. The project focus is in designing an electrical system that will optimize the operation of the sensor, not in designing a better mechanical structure, so a pre-fabricated sensor die, whose design is beyond the scope of this project, is used. The sensor die serves as the interface for picking up Coriolis accelerations and providing those resulting signals to other electronics down the chain. The rest of the system is designed to reduce inherent errors in the sensor and force the sensor to operate mode-matched. Figure 3.2 shows some of the key inputs and outputs of the sensor die block.

![Sensor Die Block](image)

Figure 3.2: Sensor Die Block. The sensor die contains the MEMS structure necessary to detect rotation rates and a minimal amount of electronics to support its operation.
The sensor die contains the resonator-accelerometer MEMS structure with minimal electronics to broadcast the motion of the resonator and accelerometer as electrical signals. That is, there is a set of outputs from the resonator that relate its position as it oscillates along the drive mode, and there is a set of outputs from the accelerometer that relate its position as it experiences accelerations along the sense mode. Each of these modes is subject to the same second-order characteristics discussed in Section 2.2.1. The outputs of the resonator are sinusoidal with a frequency of $\omega_R$, and the outputs of the accelerometer are also sinusoidal with a frequency of $\omega_R$ but with amplitude proportional to the rotation rate.

The sensor die also contains several voltage inputs for it to operate properly. For example, a drive signal must be provided to the sensor in order to cause the resonator to oscillate. At resonance, the drive signal and the velocity of the resonator are in phase with each other. The appropriate drive frequency is determined by the clock controller, which will be discussed in Section 3.3.

There is also a means to correct for quadrature error. There are a set of differential inputs that allow a quadrature correction signal to be applied. Normally, quadrature would be corrected by applying a DC voltage to these inputs. This mode-matching system introduces a dither signal onto the quadrature correction inputs instead, such that the resulting signal is a composite of the DC value and the dither. The quadrature correction input voltages are provided by the quadrature controller, which will be discussed in Section 3.6. In addition to the quadrature correction inputs, there are a set of differential in-phase correction inputs that can be used to correct for offset errors at the output. These inputs
are currently driven with a manually-tuned DC value, but future work could investigate a feedback system to drive these voltages appropriately.

Lastly, a reference input on the sensor die allows the shield voltage to be set. This voltage determines whether or not the sensor operates mode-matched or not. If it is too high or too low, then the gyroscope will operate outside of mode-match. Among other things, the shield voltage adjusts how much electronegative spring dampening we apply. There is an inverse relationship between the shield voltage and the direction the accelerometer resonant frequency ($\omega_A$) moves. Suppose the sensor die is designed such that $\omega_A > \omega_R$, and the shield voltage is set to some value. When we increase the shield voltage, $\omega_A$ decreases; when we decrease the shield voltage, $\omega_A$ increases. This can be seen by varying the voltage parameter in (2.11). At some point, $\omega_A$ will cross $\omega_R$. By using a feedback system to control this voltage, we can make the sensor operate at mode-match regardless of operating conditions.

It should be mentioned that although the inputs are on the order of 1V, internally these voltages go through high gain amplifiers that charge internal structures to the order of 20V. Outputs are on the order of 100mV with roughly 1.5V offsets. Though not crucial at this point, it might help to have a better understanding of the sensor die.

3.3 Clock Controller, Block-level Overview

The clock controller serves several purposes in this system. It is responsible for generating the drive signal to force the resonator into resonance. It is also responsible for deriving clocks for the rate controller and quadrature controller. Finally, it is responsible for providing the dither controller a clock from which the dither signal is derived.
The clock controller generates the drive signal through a feedback loop which causes the resonator to self-resonate. The resonator output of the sensor die is the input to the clock controller. The output of the clock controller is a drive signal that all clocks in the system are phase-referenced to. To generate the drive signal, the clock controller consists of a comparator to convert the sinusoidal resonator signal into a square wave, a phase-locked loop (PLL) to lock the resonator’s natural frequency to the drive frequency, and a circuit of D flip-flops for generating in-phase and quadrature clocks. Figure 3.3 demonstrates the components in the clock controller.

![Figure 3.3: Clock Controller Block. The clock controller consists of a comparator, a phase-locked loop, and a circuit of flip-flops to generate the in-phase and quadrature clocks.](image)

On startup, the resonator is kick-started with a voltage on the drive input. As its motion crosses from one extreme to the other, the output of the resonator results in a sinusoidal output. The comparator passively tracks the natural frequency of the resonator and outputs a square waveform with the same frequency of the sinusoid from the resonator output. At steady state, the sinusoid oscillates at \( \omega_R \), and the comparator outputs at the same frequency. The PLL following the comparator ensures that the clock controller stays locked to the resonant frequency of the drive mode. It compares the output of the resonator to the drive clock and adjusts its output until the drive signal is at the resonant
frequency of the drive mode. It also generates the first “kick” that gets the resonator moving. The output of the PLL is another square wave that is passed to the I/Q clock generator which will generate in-phase and quadrature clocks and their inverses. At the output of the clock controller are four square waves. There is the drive signal ($\phi=0^\circ$), its inverse ($\phi=180^\circ$), the quadrature clock ($\phi=90^\circ$), and the quadrature clock’s inverse ($\phi=270^\circ$).

The clock in the quadrature controller uses the drive signal ($\phi=0^\circ$) to demodulate out the quadrature component of the acceleration. Similarly, the clock in the rate controller uses the drive signal shifted by 90-degrees ($\phi=90^\circ$) to demodulate out the Coriolis acceleration. In Figure 3.4, idealized waveforms of the resonator velocity, the $\phi=0^\circ$ clock, and the $\phi=90^\circ$ clock are all plotted for comparison. The figure depicts a resonator with resonant frequency of 18 kHz that leads to clock signals with amplitude 1.

The drive clock is also passed to the dither controller to be frequency divided so that the dither can be generated and applied to the sensor die.
3.4 Dither Controller, Block-level Overview

The dither controller is responsible for taking an input clock and frequency-dividing it down to generate the dither signal. This dither signal is then passed to the quadrature controller so that it can be introduced onto the quadrature correction elements. Additionally, the dither controller provides the dither signal to the shield controller, so that it can perform its second demodulation of the rate controller’s demodulated output to detect whether or not the dither is appearing as in-phase error. Figure 3.5 shows a block-level diagram of the components in the dither controller. It consists of a frequency divider that divides the input drive signal into a dither signal. The current division factor is 256. This division factor provides a dither signal with a long enough period to allow the accelerometer to respond. Additionally, the dither signal is provided with its inverse so
that the quadrature controller can apply the dither differentially across the quadrature correction elements.

3.5 Rate Controller, Block-level Overview

The rate controller is designed to take the accelerometer’s output and demodulate the in-phase (I) component. When Coriolis acceleration is present, the rate controller will see a sinusoid whose amplitude is proportional to the rotation rate. After mixing and low-pass filtering, a DC voltage that is proportional to the rotation rate is achieved.

Figure 3.6 shows a block diagram of the rate controller. It consists of a mixer and a low-pass filter. At mode-match, I will be in quadrature with the drive signal, so the rate controller takes the accelerometer signal and uses the $\phi=90^\circ$ clock to demodulate in-phase. This will produce a rectified sinusoid whose average DC value is output after low-pass filtering the rectified waveform. When the gyroscope is outside of mode-match, the dither (introduced as Q) appears as in-phase error after the mixer stage. This will only occur if Q becomes in phase with the $\phi=90^\circ$ clock, which is what we are trying to avoid. When this is the case, the signal after the mixer is also passed to the shield controller and is used as an error signal to correct for being out of mode-match.
It helps to idealize the waveforms we expect to see at the input and output of the rate controller. First, consider the case when no dither is introduced, but the gyroscope is mode-matched. Figure 3.7 shows some example waveforms that we would expect. The upper two plots show a signal from the accelerometer that contains only an in-phase component. Overlaid on that signal are the $\phi=90^\circ$ clock (on the left) and the $\phi=0^\circ$ clock (on the right). The results of demodulating the accelerometer signal with the respective clocks are the lower two plots. Demodulating $I$ with the $\phi=90^\circ$ clock yields curves that will low-pass filter to produce a finite DC value. However, demodulating $I$ with the $\phi=0^\circ$ clock will yield a set of curves that low-pass filter to a zero DC value, since the clock is in quadrature with the signal.

Figure 3.6: Rate Controller Block. The rate controller demodulates the accelerometer output to produce a signal that is proportional to the input rotation rate. Any dither that bleeds into this channel is fed to the shield controller to correct the error.
Figure 3.7: Rate Controller Outputs During Mode-Match. Above, an idealized accelerometer waveform that contains only an in-phase (I) component is overlaid with the $\phi=90^\circ$ clock (on the left) and the $\phi=0^\circ$ clock (on the right). Below, the results of demodulating with the respective clocks are shown. On the left, the curves produce a finite DC value when low-pass filtered. On the right, the curves average out to 0 when low-pass filtered.

We can develop the model a bit more by introducing the dither and observing what we should expect to see. For this example, there will be no I component; only the Q component, which is a square wave dither, will be introduced as the accelerometer signal. Figure 3.8 shows what happens in this scenario. At the top of the figure, the purely-Q-
component accelerometer signal is overlaid with the input dither. Demodulating the signal with the $\phi=90^\circ$ clock yields sets of curves that will low-pass filter to 0 after demodulating a second time at the dither frequency. Demodulating the signal with the $\phi=0^\circ$ clock yields a set of curves with an envelope that resembles the input dither. These curves can be demodulated a second time at the dither frequency and low-pass filtered to produce a DC value. The latter case occurs when the dither feeds through the rate controller because the gyroscope has fallen out of mode-match. The actual second demodulation is not handled by the rate controller; the shield controller is responsible for the second demodulation and feeding the final DC value back to the sensor.
Figure 3.8: Rate Controller Output with Dither Enabled. (a-b) Accelerometer response to a dither signal with dither signal overlaid. (c) Using the in-phase ($\phi=90^\circ$) clock to demodulate the accelerometer output yields curves that will average to 0. (d) Using the quadrature ($\phi=0^\circ$) clock to demodulate the accelerometer output yields curves that will average to a non-zero value when a second demodulation at the dither frequency and phase is performed.
3.6 Quadrature Controller, Block-level Overview

The quadrature controller corrects for inherent anisoeelastic errors in the sensor that give rise to a signal that is in quadrature to the in-phase component. Additionally, the quadrature controller introduces the dither on the quadrature correction elements so that the phase across the accelerometer can be tracked. Figure 3.9 shows a block level diagram of the quadrature controller. It consists of a mixer, an integrator, and a summing circuit for compositing the dither with the quadrature error.

At mode-match, quadrature error becomes in phase with the drive signal. Thus, the mixer uses a clock signal with $\phi = 0^\circ$. When quadrature error is present, the output of the mixer is a rectified sinusoid. The integrator then integrates the error and sets the DC values on the quadrature correction elements. Following the integration stage are a set of adders that composite the dither with those DC values, giving an AC component to the quadrature correction values.
Figure 3.9: Quadrature Controller Block. The quadrature controller demodulates the accelerometer output to identify quadrature and feeds back a quadrature error signal to the quadrature correction elements to cancel it. Additionally, the quadrature controller is responsible for introducing the dither signal onto the quadrature correction elements.

Figure 3.10 shows the result at each stage in the quadrature controller. At the start, a waveform from the accelerometer containing only Q is depicted. The input clock signal to the mixer is also overlaid. Then, the demodulation produces a set of rectified sinusoidal curves which are passed into the integrator. The integrator produces a DC value which sets a voltage bias to correct the inherent quadrature error (that produced Q in the first place). Lastly, the adders will composite onto the DC bias a square wave whose frequency is the dither frequency.
Figure 3.10: Quadrature Controller Signal Chain. From top to bottom: The accelerometer signal with only a Q component and demodulating clock overlaid, the rectified sinusoid at the output of the demodulator with average DC value overlaid, the average DC value with dither signal composited, a zoomed out figure of the square wave dither offset by the DC value.
Once the dither is applied, the signal chain outputs exactly match the right-side chain in Figure 3.8.

### 3.7 Shield Controller, Block-level Overview

At this point, a dither has been introduced onto the quadrature correction elements, and the rate controller has demodulated the output of the accelerometer with the $\phi=90^\circ$ clock. The purpose of the shield controller is to monitor this intermediate step of the rate controller, generate an error signal that is a function of the phase across the accelerometer, and use the error signal to correct the shield voltage.

When the dither appears as in-phase error the shield controller corrects this so that the dither only appears as quadrature error. Figure 3.11 exemplifies two possible extremes that could result from the rate controller's first demodulation. The upper row shows inputs that would come from the rate controller (Figure 3.8); the plot on the left represents how quadrature error would appear entering the shield controller. If this is what the shield controller sees, then the gyroscope is mode-matched. If, however, the shield controller receives the plot on the right as its input, it is seeing the dither introducing in-phase error. In this latter case, a second demodulation at the dither frequency and phase will lead to an error signal that can be used to correct any mode mismatches. If the gyroscope is mode-matched, low-pass filtering the result of a second demodulation will average out to 0. If the gyroscope is not mode-matched, the effect of demodulation and low-pass filtering is to rectify the dither and produce a signal to adjust the shield voltage appropriately.
Figure 3.11: Shield Controller Idealized Signal Chain. (Left) Idealized waveforms through the shield controller when the dither only introduces quadrature controller (gyroscope is mode-matched). (Right) Idealized waveforms through the shield controller when the dither introduces in-phase error (gyroscope is not mode-matched).
Two incarnations of the shield controller were designed and compared. The first performs the second demodulation and directly low-passes the result to produce an error signal from the rectified waveform. It is called the *linear shield* controller. The second is a bang-bang controller. It is identical to the first except it uses a comparator stage between the second demodulation and the LPF. It is referred to as the *bang-bang shield controller*.

### 3.7.1 Linear Shield Controller

The first design of the shield controller consists of a mixer to demodulate the dither from the rate controller’s demodulated signal and a low-pass filter (integrator) to get the DC component of the newly rectified signal. Figure 3.12 shows the design for the linear shield controller. The DC value from the integrator is then offset by a nominal shield voltage. This nominal voltage is necessary for the gyroscope to start up. If no shield voltage is supplied on startup, then the gyroscope does not operate properly.

![Shield Controller Block, Linear Controller](image)

**Figure 3.12:** Shield Controller Block, Linear Controller. One of the shield controller designs involves trying to demodulate the dither from the rate controller’s rectified internally rectified signal.

One challenge in the design of this controller is to ensure that the filter values (gain, cutoff frequency) are selected in such a way that ensures the feedback loop is stable. If the gain is set too low, the system can be over-damped, and there will seem to be no effect on the
shield voltage. If the gain is set too high, it might not be possible for the shield controller to settle on the correct value.

### 3.7.2 Bang-bang Shield Controller

Conceptually, the bang-bang demodulating shield controller serves the same purpose of the linear shield controller implementation. The purpose of using a bang-bang controller is to minimize the time it takes for the shield controller to settle on the correct shield voltage. Figure 3.13 shows the bang-bang design for the shield controller. It consists of a mixer, a comparator, a low-pass filter, and a means to offset the output by a nominal reference voltage.

The comparator has two possible outputs: high or low. It only cares about the sign of the difference of its inputs, not the magnitude of the difference. Therefore, small input differences have the same weight as large input differences. By introducing a comparator between the mixer and the low-pass filter, phase errors are quickly amplified even if the errors are small, since the controller now has a magnitude-independent characteristic. The low-pass filter stage essentially integrates the output of the comparator, giving the controller a type of sigma-delta behavior. Over a long enough time period, the low-pass filter will average out the outputs of the comparator, and this final value will be what corrects the shield voltage.
3.8 Summary

All of the blocks in this system work together to bring the sensor to operate mode-matched. The blocks work intricately and very dynamically. If we were interested only in making the sensor operate without any concern about whether or not it was mode-matched, all we would really need are the clock controller and rate controller. The clock controller provides a signal to drive the resonator, and the rate controller provides the end user with a voltage that is proportional to the rotation rate. The target of all the other blocks is to improve the performance of the sensor die.

The quadrature controller improves the performance of the sensor by introducing feedback to correct for quadrature error. Additionally, in order to expand the mode-matching capabilities of the system, the quadrature controller is responsible not only for correcting quadrature but also for introducing a dither signal which is used to monitor the phase delay through the accelerometer. Since the dither is introduced as quadrature error, we know that the gyroscope is not mode-matched if we detect it as an in-phase signal. The remaining blocks serve to detect this situation and provide the proper feedback to move the gyroscope toward mode-match.
Chapter 4 serves to overview some of the system level parameter that affect the operation of the feedback loops. In particular, it serves to identify some key parameters that play a role in the stability of the shield controller feedback loop.
4 System Analysis

4.1 Design Considerations

The previous chapter provided a description of the system architecture. This and the following sections will focus on an analysis of the system in order to identify parameters that are important in the final design. The system calls for several controller blocks, each of which consists of filters of some sort. One challenge in the design is to decide what filter values are appropriate for each block and how to choose them so that the system works reliably. For example, the rate controller, quadrature controller, and shield controller all include a low-pass filter/integrator. Knowing how to set the gain and corner frequencies for each filter is important. Another subject worth discussing is how to choose the divide ratio that determines the dither frequency. Lastly, some consideration needs to be given toward ensuring that the shield voltage feedback loop is stable.

4.1.1 Setting the Dither Frequency

Much of the design hinges on qualities about the sensor die. For example, this project uses a gyroscope sensor die with a resonator frequency of roughly 18 kHz (actual measurements showed it to be 17.9 kHz). Additionally, Q-factors characterizing the resonator and accelerometer were on the order of 10-100. Since the dither introduces quadrature error into the gyroscope, there is a transient response every time the dither transitions. In order to ensure that the shield controller feedback system reacts to the steady-state values of the accelerometer output, the dither frequency is chosen to be at least a factor of 1/Q of the drive frequency. For this project the division factor is 256, and the resulting dither is 70Hz.
$$\omega_D = \frac{\omega_R}{N}, N = 256$$

$$2\pi f_D = \frac{2\pi f_R}{256} \Rightarrow f_D = \frac{17.9kHz}{256} = 70Hz$$

4.1.2 Setting the Corner Frequency of the Rate Controller

The rate controller contains a low-pass filter to bandwidth-limit the output. Since the dither has a frequency of 70Hz, it made sense to set the corner frequency of the low-pass filter to be below that value. It was chosen to be 10Hz.

4.1.3 Using Integrators in the Quadrature Controller and Shield Controller

In both the quadrature controller and shield controller, integrators are used in place of low-pass filters. Integrators were chosen because they provide infinite gain at DC. This is important for the feedback loops to quickly correct any errors present. It should be noted that the linear shield controller uses an integrator but the bang-bang shield controller does not because of the very high gain that is inherent with a bang-bang controller. The gain of each of the integrators was set such that there was stability in the feedback loops.

4.2 System Modeling

The aim of this section is to present a model that will be used to derive the system transfer function for a feedback loop involving the sensor die and the shield controller. For this project, the shield controller loop is of particular importance, so an analysis of the transfer function that characterizes the controller’s behavior is useful for identifying parameters that play a role in the stability of the feedback. The entire system is highly non-linear, but linearization about the operating point and some reasonable approximations
will at least aid in the design process and give some idea on how to set different parameters.

The model for this analysis is depicted in Figure 4.1. At the input is the resonant frequency of the accelerometer \((\omega_A(s))\) which is determined by the shield voltage. It is passed into the gyroscope stage which uses the dither, modulated by the resonator frequency, to generate a response that is demodulated to calculate the phase through the accelerometer \((\varphi(s))\). At mode-match the phase through the accelerometer will be \(-90^\circ\), and the phase error signal will indicate how unmatched the modes are when the phase is not \(-90^\circ\). The dither, though it actually is a square wave, is assumed to have a significantly lower frequency than the resonant frequencies of either the resonator or accelerometer, so it is modeled as a constant term for the purposes of approximation. The shield controller appears as a proportional-integrator (PI) block in this model, and its purpose it to take the phase error, integrate it, and correct the resonant frequency of the accelerometer. In order to assist with the analysis, the forward path is grouped together to form the transfer function \(H(s)\), which will be derived in the following section.
In the sections that follow, each of the stages will be investigated and the closed-loop transfer function will be derived. Some of the variables and their meanings are listed in Table 4.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Associated Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(s)</td>
<td>Transfer function of the accelerometer</td>
</tr>
<tr>
<td>Q_a</td>
<td>Quality factor of the accelerometer</td>
</tr>
<tr>
<td>Q_d</td>
<td>Amplitude of the dither signal introduced as quadrature</td>
</tr>
<tr>
<td>K_s</td>
<td>Gain of the shield controller</td>
</tr>
<tr>
<td>X_0</td>
<td>Amplitude of the resonator motion</td>
</tr>
<tr>
<td>w_R</td>
<td>Resonator angular frequency</td>
</tr>
<tr>
<td>w_A</td>
<td>Accelerometer angular frequency</td>
</tr>
<tr>
<td>ϕ</td>
<td>Phase measurement signal</td>
</tr>
</tbody>
</table>

Table 4.1: System Transfer Function Variables

Figure 4.1: Shield Voltage Loop System Model. The feedback loop involving the gyroscope and the shield controller contains two stages. The forward stage is characterized by the gyroscope’s accelerometer transfer function and its response to the dither (which is modeled as a constant); its output is a measure of the phase through the accelerometer. The controller stage corresponds to the integration of the phase error that is detected, which is fed back and used to correct the resonant frequency of the accelerometer.
4.2.1 Gyroscope Stage with Demodulator, H(s)

The strategy for identifying H(s) in the model is to consider the response of the gyroscope to a sinusoidal input when $\omega_A$ is fixed. Here, the input is the dither modulated by the resonator. We will first identify this response, then demodulate it to arise at a measure of the phase through the accelerometer. Ultimately, our interest is to discover a transfer function H(s) that relates the phase measurement, $\phi(s)$, to the resonant frequency of the accelerometer, $\omega_A(s)$. Even though the accelerometer frequency affects the transfer function of the accelerometer, breaking the implicit assumption that $A(s)$ is time-invariant, we can still approximate the behavior of the system by assuming that changes in $\omega_A$ occur slowly, relative to the oscillations we are considering.

First, consider the input to the accelerometer, $x(t)$:

$$x(t) = Q_D X_0 \cos(\omega_R t) \quad (4.1)$$

The response $y(t)$ of the accelerometer is determined by the differential equation:

$$y'' + \frac{\omega_A}{Q_A} y' + \omega_A^2 y = Q_D X_0 \cos(\omega_R t) \quad (4.2)$$

whose solution is composed of two parts—a particular solution and a homogeneous solution.

The particular solution $y_p(t)$ is solved by first recognizing that the input $x(t)$ is the real part of a complex exponential, so we can first find the particular solution to:

$$\dddot{y}_p + \frac{\omega_A}{Q_A} \ddot{y}_p + \omega_A^2 y_p = Q_D X_0 e^{i\omega_R t}$$

and then derive:

$$y_p(t) = \Re\{\dddot{y}_p(t)\}$$
The following derivation finds $\dot{y}_p$:

$$
\dot{y}_p = \frac{Q_DX_0e^{i\omega Rt}}{(i\omega_R)^2 + \frac{\omega_A}{Q_A}(i\omega_R) + \omega_A}
$$

$$
= \frac{Q_DX_0\left((\omega_A^2 - \omega_R^2) - i\frac{\omega_A\omega_R}{Q_A}\right)e^{i\omega Rt}}{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A\omega_R}{Q_A}\right)^2}
$$

$$
= \frac{Q_DX_0}{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A\omega_R}{Q_A}\right)^2} \left((\omega_A^2 - \omega_R^2) - i\frac{\omega_A\omega_R}{Q_A}\right)(\cos(\omega_R t) + i \sin(\omega_R t))
$$

Then, only considering the real part of $\dot{y}_p$:

$$
y_p(t) = \Re\{\dot{y}_p(t)\} = \frac{Q_DX_0}{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A\omega_R}{Q_A}\right)^2} \left((\omega_A^2 - \omega_R^2) \cos(\omega_R t) + \frac{\omega_A\omega_R}{Q_A} \sin(\omega_R t)\right)
$$

$$
= \frac{Q_DX_0}{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A\omega_R}{Q_A}\right)^2} \sqrt{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A\omega_R}{Q_A}\right)^2} \cos(\omega_R t + \phi_1)
$$

Simplifying, we arrive at the particular solution to (4.2):

$$
y_p(t) = \frac{Q_DX_0}{\sqrt{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A\omega_R}{Q_A}\right)^2}} \cos(\omega_R t + \phi_1) \quad (4.3)
$$

where $\phi_1$ is the phase through the accelerometer:

$$
\phi_1 = \tan^{-1}\left(\frac{\frac{\omega_A\omega_R}{Q_A}}{\omega_A^2 - \omega_R^2}\right) \quad (4.4)
$$
Next, we solve for the associated homogeneous solution of (4.2) which is:

\[ y'' + \frac{\omega_A}{Q_A} y' + \omega_A^2 y = 0 \]  

(4.5)

In order to find the solution to the associated homogeneous solution, we find the roots to the characteristic equation:

\[ \lambda^2 + \frac{\omega_A}{Q_A} \lambda + \omega_A^2 = 0 \]

Solving for \( \lambda \) using the quadratic equation:

\[
\lambda = -\frac{\omega_A}{2Q_A} \pm \frac{1}{2} \sqrt{\left(\frac{\omega_A}{Q_A}\right)^2 - 4\omega_A^2} = -\frac{\omega_A}{2Q_A} \pm \frac{\omega_A}{2} \frac{\sqrt{1 - \frac{4Q_A^2}{Q_A^2}}}{Q_A}
\]

It is known beforehand that the quality factor of the accelerometer \( Q_A \) is much larger than \( \frac{1}{2} \), so the two roots to the characteristic equation end up as:

\[
\lambda_1 = -\frac{\omega_A}{2Q_A} + \frac{i\omega_A}{2} \frac{\sqrt{4 - \frac{1}{Q_A^2}}}{Q_A}
\]

\[
\lambda_2 = -\frac{\omega_A}{2Q_A} - \frac{i\omega_A}{2} \frac{\sqrt{4 - \frac{1}{Q_A^2}}}{Q_A}
\]

Knowing these roots, the solution to the associated homogeneous solution \( y_H(t) \) is:

\[ y_H(t) = A e^{\lambda_1 t} + B e^{\lambda_2 t} \]  

(4.6)

where \( A \) and \( B \) are unknown constants that we will solve for shortly.

The total solution to (4.2) is the sum of the particular solution (4.3) and the homogeneous solution (4.6):

\[ y(t) = y_p(t) + y_H(t) \]

\[ y(t) = \frac{Q_D X_0}{\sqrt{(\omega_A^2 - \omega_R^2)^2 + \left(\frac{\omega_A \omega_R}{Q_A}\right)^2}} \cos(\omega_R t + \phi) + e^{\lambda_1 t} + e^{\lambda_2 t} \]  

(4.7)
In order to solve for the unknown constants $A$ and $B$, we need to set some initial conditions. Assume that the output for all time $t<0$ is 0, and that at $t=0$, the slope of the response is 1. That is:

\begin{align*}
y(0) &= 0 \quad (4.8) \\
y'(0) &= 1 \quad (4.9)
\end{align*}

At mode-match, the phase through the accelerometer is $-90^\circ$, so we can estimate $\phi_1$ as $-90^\circ$, which causes the first term in (4.7) to go to 0, leaving:

\[ y(0) = 0 = A + B \Rightarrow B = -A \]

After finding $y'(t)$ to be:

\[
y'(t) = -\frac{QDX_0\omega_R}{\sqrt{(\omega_A^2 - \omega_R^2)^2 + (\omega_A\omega_R/Q_A)^2}} \sin(\omega_Rt + \phi_1) + A(\lambda_1 e^{\lambda_1t} - \lambda_2 e^{\lambda_2t})
\]

and plugging in $t=0$:

\[
y'(0) = 1 = \frac{QDX_0\omega_R}{\sqrt{(\omega_A^2 - \omega_R^2)^2 + (\omega_A\omega_R/Q_A)^2}} + A(\lambda_1 - \lambda_2)
\]

Then, the constant $A$ is solved as (after also plugging in $\lambda_1$ and $\lambda_2$):

\[
A = \frac{1}{2i\omega_A/2} \sqrt{4 - \frac{1}{Q_A^2}} \left(1 - \frac{QDX_0\omega_R}{\sqrt{(\omega_A^2 - \omega_R^2)^2 + (\omega_A\omega_R/Q_A)^2}}\right)
\]

Lastly, plugging in the constants we have derived back into (4.7) yields the following:

\[
y(t) = \frac{QDX_0}{\sqrt{(\omega_A^2 - \omega_R^2)^2 + (\omega_A\omega_R/Q_A)^2}} \cos(\omega_R t + \phi_1) + A(e^{\lambda_1 t} - e^{\lambda_2 t})
\]
\[ y(t) \approx \hat{y}(t) = \frac{Q_D X_0}{Q_A} \cos(\omega_R t + \phi_1) + \frac{1 - \frac{Q_D X_0 \omega_R}{\omega_A \omega_R}}{Q_A} e^{-\frac{\omega_A^2}{2 Q_A^2}t} \sin(\omega_A t) \]

\[ \approx \frac{Q_D X_0}{\omega_A^2} \cos(\omega_R t + \phi_1) - \frac{Q_D X_0}{\omega_A^2} e^{-\frac{\omega_A^2}{2 Q_A^2}t} \sin(\omega_A t) \]
\[ \hat{y}(t) = \frac{Q_D X_0}{Q_A} \left( \cos(\omega_R t + \phi_1) - e^{- \frac{\omega_A}{2Q_A} t} \sin(\omega_A t) \right) \]  

(4.10) is an approximation to the response of the accelerometer to the modulated dither input. Applying a demodulation stage to the result in (4.10) with a clock that is 90° (\(\phi=90°\)) shifted from the drive signal will yield a measurement of the phase through the accelerometer \(\varphi(t)\). Since the model assumed that the drive signal is a cosine term, the 90° shift in the clock means demodulation occurs with a negative sine term:

\[
\varphi(t) = \hat{y}(t) (-\sin(\omega_R t)) \\
= -\frac{Q_D X_0}{2Q_A} \left\{ \sin(2\omega_R t + \phi_1) - \sin(\phi_1) \right\} - e^{- \frac{\omega}{2Q_A} t} \left[ \cos((\omega_R - \omega) t) - \cos((\omega + \omega_R) t) \right]
\]

Again, because we are operating around mode-match, \(\omega_A \approx \omega_R\), so the argument in the first cosine term \((\omega_A - \omega_R)\) is taken to be 0. The integrator in the controller block will attenuate the high frequency terms, so another approximation we will make is to eliminate the high frequency terms from the above equation since they will tend to 0 post-integration.

\[
\varphi(t) \approx -\frac{Q_D X_0}{2Q_A} \left\{ -\sin(\phi_1) \right\} - e^{- \frac{\omega}{2Q_A} t} \left[\right]
\]

(4.11)  

* Using the trigonometric identities: \(\cos \theta \sin \phi = \frac{\sin(\theta + \phi) - \sin(\theta - \phi)}{2}\) and \(\sin \theta \sin \varphi = \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{2}\)
At this point, we have identified the step response of the accelerometer to the dither in (4.11). By differentiating (4.11) with respect to time, we can obtain the impulse response to the dither:

\[
\frac{d\phi(t)}{dt} = -\frac{Q_D X_0 \left( \frac{\omega_A}{\omega_A^2} \right)}{\frac{2Q_A}{\omega_A^2}} e^{-\frac{\omega_A}{2Q_A} t} \tag{4.11}
\]

\[
\frac{d\phi(t)}{dt} = -\frac{Q_D X_0}{4\omega_A} e^{-\frac{\omega_A}{2Q_A} t} \tag{4.12}
\]

Our next step is to take the Laplace transform of (4.12):

\[
L\left\{ \frac{d\phi(t)}{dt} \right\} = -\frac{Q_D X_0}{4\omega_A} \frac{1}{s + \frac{\omega_A}{2Q_A}} \tag{4.13}
\]

Our final step to identify \( H(s) \), the transfer function from the input accelerometer resonant frequency to the output phase measurement, is to linearize (4.13) with respect to \( \omega_A \) around mode-match:

\[
H(s) = \frac{\varphi(s)}{\omega_A(s)} \approx \frac{dL\left\{ \frac{d\phi(t)}{dt} \right\}}{d\omega_A} \bigg|_{\omega_A = \omega_R} = \frac{Q_D X_0}{4\omega_R^2} \left( \frac{1}{s + \frac{\omega_R}{2Q_A}} \right) \tag{4.14}
\]

(4.14) is an important finding because it shows that the accelerometer introduces a pole into the transfer function of the forward path.

### 4.2.2 System Transfer Function

Section 4.2.1 was able to describe the open-loop transfer function, \( H(s) \), of our model (Figure 4.1). In this section, we consider the closed-loop transfer function of the system when the PI controller from the shield controller is introduced.

Figure 4.1 transforms into something simpler and is modeled in Figure 4.2.
Figure 4.2 abstracts the details of $H(s)$ that were shown in Figure 4.1 and models the PI controller as some $G(s)$. We can use Black’s equation to identify the system transfer function for the simplified model and plug in for $H(s)$ and $G(s)$.

\[
\frac{\varphi(s)}{\omega_A(s)} = \frac{H(s)}{1 + H(s)G(s)}
\]

\[
= \frac{Q_D X_0}{4\omega_R^2} \left( \frac{1}{s + \frac{\omega_R}{2Q_A}} \right)
\]

\[
= \frac{1 + Q_D X_0}{4\omega_R^2} \left( \frac{1}{s + \frac{\omega_R}{2Q_A}} \right) \frac{K_S}{s}
\]

\[
= \frac{Q_D X_0}{4\omega_R^2} \frac{S}{s + \frac{\omega_R}{2Q_A}} + \frac{Q_D X_0 K_S}{4\omega_R^2}
\]

\[
\frac{\varphi(s)}{\omega_A(s)} = \frac{Q_D X_0}{4\omega_R^2} \frac{S}{s^2 + \frac{\omega_R}{2Q_A}S + \frac{Q_D X_0 K_S}{4\omega_R^2}}
\]

(4.15) is the system transfer function for the model. In the next section, an analysis will be performed on the stability of the system transfer function.
4.3 System Stability Analysis

With any feedback system, stability is an important criterion. The previous section produced a system transfer function, and this section will show plots of the transfer function. For the closed-loop transfer function in (4.15), the parameters in Table 4.2 were used to generate the bode diagram in Figure 4.3. These parameters match those in the actual system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_A</td>
<td>50</td>
</tr>
<tr>
<td>Q_D</td>
<td>3.3V</td>
</tr>
<tr>
<td>K_S</td>
<td>-10/[(10kΩ)(1µF)] = -1kHz</td>
</tr>
<tr>
<td>X_0</td>
<td>1µm</td>
</tr>
<tr>
<td>(\omega_R)</td>
<td>18kHz</td>
</tr>
</tbody>
</table>

Table 4.2: Parameters used in generating the bode plot for the closed-loop transfer function

![Bode Diagram for Closed-Loop Transfer Function](image)

Figure 4.3: Bode diagram for the closed-loop system transfer function. These plots were generated using the parameters in Table 4.2 for (4.15). Because of the zero in the transfer function, the phase of the transfer function never drops below -90°, keeping the system stable.

Correspondingly, we can generate a root locus plot to show the location of the poles as the gain of the controller is varied. This plot is shown in Figure 4.4.
Figure 4.4: Root-locus plot for the system model. This root-locus plot shows the location of the poles as the gain in the controller is varied. One pole is near 0, and this corresponds to the pole contributed by the integrator. The other pole is at 1130 rad/s, and this comes from the accelerometer.

4.4 Summary

The purpose of this chapter was to identify some of the mathematics involved in the feedback loop associated with the gyroscope/shield-controller stages. The closed-loop transfer function was plotted to demonstrate that the system is stable. This little bit of knowledge is promising and shows that the architecture for the system has merit. In Chapter 5, we’ll see a realization of the system architecture from Chapter 3, and we’ll see that it provides results with good stability.
5 Circuit Realization and Results

5.1 Clock Controller

As discussed in Section 3.3, the clock controller is part of a feedback loop that serves to generate the drive signal to cause the resonator to self-resonate. It takes the resonator signal and matches a square wave drive signal to it. More specifically, at resonance, the drive signal is in phase with the velocity of the resonator, so we would expect it to be 90° shifted from the actual resonator signal, which measures position.

The clock controller is shown in Figure 5.1 and is implemented with a comparator (ADCMP609), a PLL (74HCT4046), and D flip-flops (74HCT175). The inputs (ResP_buff and ResN_buff) come from the sensor die and are band-pass filtered using a low-pass filter \( f_{\text{cutoff}} = 2 \text{ kHz} \) cascaded with a high-pass filter \( f_{\text{cutoff}} = 130 \text{ kHz} \). Each filter stage is composed of a simple RC circuit. AC hysteresis is used to prevent the comparator from double-triggering, which explains why there are 20pF capacitors in positive feedback around the comparator. The resistor and capacitor values around the PLL stage were chosen to set the operating frequency range of the 74HCT4046. Lastly, the D flip-flop circuit serves to generate both an in-phase and quadrature clock as well as their inverses. Of the four clocks at the output, the drive signal is designated as \( \phi=0^\circ \), and all the other clocks are referenced in terms of their phase relative to the drive \( (\phi=0^\circ, \phi=180^\circ, \phi=90^\circ, \text{ and } \phi=270^\circ) \).
Figure 5.1: Clock Controller Schematic. The clock controller consists of an RC network to band-pass filter the input, an ADCMP609 comparator, a 74HCT4046 phase-locked-loop, and D flip-flops in the 74HCT175 package. Four clocks are output by the clock controller. The $\phi=0^\circ$ (PHI=0) clock acts as the drive signal.
Figure 5.2 contains plots of the input to the clock controller. The inputs come from the outputs of the resonator of the sensor die. They come as a differential set of measurements \((\text{ResP}_{\text{buff}} \text{ and } \text{ResN}_{\text{buff}})\), which contain artifacts due to common mode noise. The artifacts are spikes that occur at the minima and maxima of each signal, and are a result of clock edges that are internal to the sensor die.

Each waveform is about 200mV P-P and is anti-phase with the other. For convenience, a waveform that is the difference of the two signals is plotted to show a more smoothed-out form of the resonator signal with common mode noise canceled out. Because of the anti-phase quality of the resonator signals, the difference of the two creates a waveform that is twice the amplitude of any of the individual signals.

![Figure 5.2: Resonator Signals. This plot depicts waveforms that are output from the sensor die and fed to the clock controller. The resonator signal is differential (\text{ResP}_{\text{buff}} \text{ and } \text{ResN}_{\text{buff}}) and subject to common mode noise. The difference of the differential signals is also plotted.](image)

A more significant plot is Figure 5.3, which shows the clock controller generating the square waves that are used as clocks for the rest of the circuit. The resonator signals show a resonant frequency of roughly 17.9 kHz and so do the resulting square waves. Two
square waves are depicted. The upper waveform ($\phi=0^\circ$) is fed back as the drive signal and also used to demodulate quadrature in the quadrature controller. Notice that it is in quadrature with the resonator signal, which tracks the position of the resonator. It follows, then, that the drive signal is in phase with the velocity of the resonator as expected. The lower waveform ($\phi=90^\circ$) is used in the rate controller to demodulate in-phase.

For completeness, the clock controller also generates the inverses of the $\phi=0^\circ$ and $\phi=90^\circ$ clocks. They are depicted together in Figure 5.4.

Figure 5.3: Clock Generation. The clock controller uses the resonator signal to generate two sets of clocks which are in quadrature with each other. The upper square wave is used as the drive signal as well as the quadrature controller demodulating clock. The lower square wave is used as the rate controller demodulating clock.
The dither controller is not particularly complex. It consists of digital logic to frequency divide the drive clock. The result is a square wave dither signal that is applied to the quadrature correction elements and is also used to perform a second demodulation following the first demodulation done in the rate controller.

The actual circuit consists of a cascade of D flip-flop ICs that provide division factors of 8, 4, 2, and 4 in that order. The dither controller was implemented this way to allow experimentation with different divide ratios until a satisfactory one could be found. Ultimately, a divide-by-256 ratio was settled upon because it provided a slow enough dither that the gyroscope was able to reach steady-state outputs between every clock transition. This helped both in debugging as well as in identifying phenomena that occur in the accelerometer signal as a result of the dither. For example, slowing down the dither frequency made it possible to notice the exponential rise and fall of the accelerometer's

Figure 5.4: Clock Controller Clocks. The clock controller generates all four clocks—φ=0°, φ=180°, φ=90°, and φ=270°.

5.2 Dither Controller

The dither controller is not particularly complex. It consists of digital logic to frequency divide the drive clock. The result is a square wave dither signal that is applied to the quadrature correction elements and is also used to perform a second demodulation following the first demodulation done in the rate controller.

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output in response to the forced quadrature error. We will see these effects in a later section. A schematic of the dither controller is available in Figure 5.5.

In the schematic, the drive signal is input to the dither controller and frequency divided down by a factor of 256 to generate two sets of dither clocks, where each set is in quadrature with the other. Each pair contains two anti-phase square waves so that the dither can be applied differentially across the quadrature correction elements. The outputs, Dither+ and Dither- are fed to the quadrature controller. Also, the four dither clocks are fed to the shield controller to perform the second demodulation. All the frequency division is handled by quad package D flip-flop ICs (74HCT175) as seen in the schematic.
The dither controller consists of four 74HCT175 D flip-flop ICs. Each IC is a quad-package and contains four flip-flops. The input to the dither controller is the drive signal, and the first stage frequency divides the drive signal by a factor of 8. The following two stages are cascaded to perform frequency divisions of 4 and 2, in that order. The final stage performs a frequency division by a factor of 4 but is also designed to generate in-phase and quadrature dither clocks and their inverses. Ultimately, the drive signal is frequency divided by a factor of 256 and provided differentially as Dither+ and Dither-.
The 1/256 divide ratio means that with a resonator frequency of 17.9 kHz, the dither frequency is 70Hz. In fact, this is what we see at the output of the dither controller in Figure 5.6.

![Figure 5.6: Dither Controller Outputs. The dither controller outputs a differential set of square wave dither signals (Dither+ and Dither-). They are each at 1/256 of the drive frequency.](image)

We can zoom in to see the drive clock more closely and also to see the transitions in the dither. This is depicted in Figure 5.7. Again, we see the resonant frequency of the drive near 17.9 kHz, and the dither shows signs of finite transition bands. For the most part, these transition bands do not affect the mode-match system considerably.
The dither introduces some interesting anomalies in the frequency domain as well. Because the dither is modulated by the frequency of the resonator, it introduces tones into the power spectrum, which are clearly visible in Figure 5.8. The plots were all generated by manually tuning the shield voltage, so the feedback loop from the shield controller is disabled. Figure 5.8(a) shows the frequency spectrum in the spring-limited case ($\omega_R < \omega_A$) with dither tones and without. Figure 5.8(b) and Figure 5.8(c) do the same but for the mode-matched case ($\omega_R = \omega_A$) and mass-limited case ($\omega_R < \omega_A$), respectively. In all cases, the dither and its odd harmonics\(^2\) are spaced 140Hz (2x70Hz) apart as expected. Only in the mode-matched case is there symmetry in the power spectrum.

\(^2\)A square wave is the infinite sum of odd integer harmonics:

$$x_{\text{square}}(t) = \sum_{k=1}^{\infty} \frac{4}{\pi} \sin((2k-1)2\pi ft) = \frac{4}{\pi} \left( \sin(1 \times (2nft)) + \frac{1}{3} \sin(3 \times (2nft)) + \frac{1}{5} \sin(5 \times (2nft)) + \ldots \right)$$

Figure 5.7: Zoomed-in Dither Controller Outputs. This figure zooms in on the waveforms in Figure 5.6 to show the drive signal ($\phi=0^\circ$ clock) more clearly.
Figure 5.8: Power Spectrum with Dither-Introduced Tones. After the dither is introduced, it is noticeable that it introduces tones into the power spectrum spaced 140Hz apart. The plots show the output power spectrum with and without the dither tones overlaid for the (a) spring-limited case, (b) mode-matched case, and (c) mass-limited case.

5.3 Rate Controller

The rate controller is responsible for producing a signal that is proportional to the rotation rate from the output of the accelerometer. It does this by demodulating the accelerometer’s output with the in-phase clock and low-pass filtering the result to produce a DC value. Waveforms that result from the rate controller are presented in this section.

In order to perform the demodulation, the rate controller uses a mixer that is implemented as a cross-switch (using the ADG609) that either feeds the input through or inverts it at the output. The frequency of the switching must occur in phase with the $\phi=90^\circ$ clock for the mixer to demodulate the rate information and not the quadrature error. (For the gyroscope to operate outside of mode-match, the $\phi=0^\circ$ clock would be used instead.)

Figure 5.9 depicts how the mixer is structured. It works because the outputs of the accelerometer are waveforms that are anti-phase. Since the switching occurs at the zero
crossings of the input signals, the resulting waveforms resemble rectified sinusoids, which is exactly what is to be expected from demodulation. Figure 5.12 shows some of the results of the mixer.

![Mixer Circuit](image)

Figure 5.9: Mixer Circuit. The mixer circuit either feeds the input through or inverts it at the output.

A schematic diagram of the rate controller is available in Figure 5.10. Following demodulation by the ADG609 is a low-pass filter stage. The low-pass filter is implemented with an OPA1632 differential amplifier, and provides a bandwidth limited output of approximately 10Hz.

\[
f_{\text{cut off}} = \frac{1}{2\pi R_F C_F} = \frac{1}{2\pi(16k\Omega)(1\mu F)} = 9.95 \text{Hz} \approx 10 \text{Hz}
\]

The demodulated output is also passed to the shield controller to perform a second demodulation at the dither frequency. When the gyroscope is mode-matched, this waveform will be non-zero, but will integrate to zero before and after the second demodulation is performed.
Figure 5.10: Rate Controller Schematic. \textit{AccP\_buff\_decoup} and \textit{AccN\_buff\_decoup} are inputs from the accelerometer. The $\phi=0^\circ$ clock ($\text{PHI}=0$) is used to demodulate the in-phase signal, which is fed both to a low-pass filter (with a corner frequency of 10Hz) and to the shield controller (for a second demodulation). At the output, \textit{RateP} and \textit{RateN} provide a signal that is proportional to the input rotation rate.
To demonstrate the operation of the rate controller, a rotation rate is introduced on the sensor die. This is synonymous with introducing in-phase error on the sensor die, which is feasible by adjusting the DC values on the in-phase correction inputs. After doing this, we can monitor the outputs of the rate controller. For the following plots, the mode-match system is disabled and the gyroscope is forced to operate in the spring-limited case \((\omega_R < \omega_A)\) by setting the shield voltage below the mode-match voltage, which was experimentally found beforehand.

In Figure 5.11, the signal chain through the rate controller is displayed. Only one of the buffered accelerometer outputs has been plotted along with the demodulating clock \((\phi=0^\circ)\) and the result of the demodulation. The presence of in-phase error yields a sinusoidal output with peak-to-peak amplitude of roughly 100mV. Since the gyroscope is not mode-matched in the figure, the drive clock \((\phi=0^\circ\text{ clock})\) needs to be used for demodulation. The result is a waveform of rectified sinusoids.
We can also compare what happens to the output of demodulation when the wrong input clock is used. In Figure 5.12, we see that demodulation with the $\phi=0^\circ$ clock (on the left) and with the $\phi=90^\circ$ clock (on the right) yield radically different waveforms. The former will low-pass filter to produce a DC value, and the latter will low-pass filter to a zero value. In both cases, the waveforms are the difference of the differential outputs at each stage instead of the single-ended waveforms.

Figure 5.11: Rate Controller Signal Chain. After applying in-phase error, disabling the mode-matching mechanism, and setting the gyroscope to operate in the spring-limited case, the gyroscope's accelerometer output (top) shows a sinusoid with 100mV P-P amplitude. After demodulating with the $\phi=0^\circ$ clock (middle), the result is a set of rectified sinusoids (bottom), which will low-pass filter to a DC value proportional to the rotation rate.
Figure 5.12: Comparison of Demodulation with In-phase and Quadrature Clock. (a) Demodulating the accelerometer output with the in-phase clock. (b) Demodulating the accelerometer output with the quadrature clock.

For completeness, we can show that the outputs of the rate controller are two DC values whose difference is proportional to the rotation rate. This is depicted in Figure 5.13. \textit{RateP} outputs 1.482V and \textit{RateN} outputs 1.539V. The difference, -57mV, is proportional to the in-phase error introduced. The in-phase error introduced was arbitrary and was only meant to demonstrate demodulation through the rate controller. Thus, there won’t be a discussion here about what angular rate corresponds to this output value. Under normal circumstances, the in-phase error is manually tuned out. Future work would include the development of an in-phase correction feedback loop. This would serve to cancel out offset on the output for zero-rate input.
This next set of plots demonstrates the rate controller operating with the mode-match system enabled. Figure 5.14 shows two waveforms. The upper waveform is the difference of the accelerometer outputs; the lower waveform is the difference of each channel after performing in-phase demodulation with the $\phi=90^\circ$ clock. Figure 5.14 demonstrates that the dither signal elicits a response from the accelerometer as expected. At every transition in the dither signal, the output of the accelerometer exponentially falls and grows.
Figure 5.14: Accelerometer Response to Dither and Demodulation with $\phi=90^\circ$ Clock. When the dither signal is applied to the sensor die, the accelerometer responds with exponential rise and decay at the transitions in the dither signal (top). After demodulating with the $\phi=90^\circ$ clock, a waveform that averages to 0 is produced (bottom).

Figure 5.15 and Figure 5.16 serve to show zoomed in versions of Figure 5.14. They were taken at the high-to-low and low-to-high dither transitions respectively. Both of these figures demonstrate that the gyroscope is mode-matched. The S-like quality of the demodulated curves indicates that the dither has experienced 90° of phase shift through the accelerometer. The dither was introduced as quadrature error. If it did not experience 90° of phase shift, it would be in phase (or anti-phase in the mass-limited case) with the $\phi=90^\circ$ clock.
Figure 5.15: Accelerometer Response to the Dither and Demodulated Result Viewed from the Rate Controller, High-to-Low Transition.

Figure 5.16: Accelerometer Response to the Dither and Demodulated Result Viewed from the Rate Controller, Low-to-High Transition.
5.4 Quadrature Controller

In Section 3.6, there was a discussion on the purpose of the quadrature controller. In addition to integrating quadrature error, the quadrature controller also introduces the dither signal onto the quadrature correction elements on the sensor die.

In order to do this, the quadrature controller consists of a mixer plus a circuit that not only integrates quadrature error but also composites a dither signal on the output of the integration. The mixer is implemented as in the rate controller. The frequency of the switching must occur in phase with the $\phi=0^\circ$ clock for the mixer to demodulate the quadrature error when the gyroscope is mode-matched. Under un-mode-matched conditions, quadrature error is in quadrature with the drive signal, so the $\phi=90^\circ$ clock would be used if we did not want the mode-match system to operate.

It may help to explain how the integrating/compositing circuit works. The circuit in the quadrature controller takes on the following architecture:

![Integrating/Compositing Circuit](image)

We can derive the transfer function of the circuit as follows:

$$\frac{V_{IN+} - V_+}{R_G} + sC_G(V_{DITH+} - V_+) = sC_F(V_+ - V_{OUT+}) \quad (5.1)$$
\[
\frac{V_{IN-} - V_-}{R_g} + s C_G (V_{DITH-} - V_-) = s C_F (V_- - V_{OUT-})
\]  

Also, assuming the differential amplifier is high gain and has large input impedance we can apply:

\[
V_+ = V_-
\]  

After plugging in (5.3) into (5.1) and (5.2) and then taking the difference of the resulting equations, we come to a transfer function for the circuit:

\[
(V_{OUT+} - V_{OUT-})(s) = \frac{1}{sR_G C_F} (V_{IN+} - V_{IN-}) - \frac{C_G}{C_F} (V_{DITH+} - V_{DITH-})
\]  

From (5.4), we see that the output is a composite of the inputs at \(V_{IN+}\) and \(V_{IN-}\) and \(V_{DITH+}\) and \(V_{DITH-}\). Specifically, the difference between \(V_{IN+}\) and \(V_{IN-}\) is integrated and the difference between \(V_{DITH+}\) and \(V_{DITH-}\) is subject to a gain, which is dependent on the ratio of the capacitors.

\[
\frac{V_{OUT+} - V_{OUT-}}{V_{IN+} - V_{IN-}}(s) = -\frac{1}{sR_G C_F}
\]

\[
\frac{V_{OUT+} - V_{OUT-}}{V_{DITH+} - V_{DITH-}}(s) = \frac{C_G}{C_F}
\]

In Figure 5.18, a schematic of the quadrature controller is shown. The integrating/compositing circuit uses an OPA1632 differential amplifier, and its outputs are multiplexed through a switch to the Qp/Qn quadrature correction inputs on the sensor die. Looking at the capacitor values, it is apparent that the quadrature controller introduces the dither only after attenuating by a factor of 10.

\[
-\frac{C_G}{C_F} = -\frac{0.1 \mu F}{1 \mu F} = -\frac{1}{10}
\]

99
Figure 5.18: Quadrature Controller Schematic. AccP_buff_decoup and AccN_buff_decoup are the inputs to the quadrature controller. They come from the accelerometer and are mixed by the ADG609, using the $\phi=0^\circ$ (PHI=0) clock. The rectified differential outputs are passed to an integrator that is set up to also composite a square wave dither on the integrated result. Finally, Qp and Qn are the outputs of the quadrature controller. They are ultimately fed back to the sensor die and applied to the quadrature correction elements to null quadrature error.
At the input of the quadrature controller are \textit{AccP\_buff\_decoup} and \textit{AccN\_buff\_decoup}. These are the inputs from the accelerometer. Additionally, the $\phi=0^\circ$ clock ($\text{PHI}=0$) is provided to the mixer. The \textit{Dither+} and \textit{Dither-} inputs come from the dither controller and feed into the OPA1632 through the 0.1$\mu$F capacitors. The outputs of the quadrature controller are $Q_p$ and $Q_n$. These are fed back to the sensor die to correct for DC quadrature error and to introduce the dither onto the correction elements.

In order to demonstrate the functionality of the quadrature controller, two cases can be considered: when the mode-match system is disabled and when it is enabled. In Figure 5.19, the mode-match system is disabled, and the shield voltage is set such that the gyroscope operates in the spring-limited case ($\omega_R<\omega_A$). This means that the demodulating clock must be switched from the mode-matched case (i.e. the $\phi=90^\circ$ clock must be used for demodulation). Additionally, the quadrature controller’s output is not fed back to the sensor die. Because of it, quadrature error is not corrected, and we see a sinusoidal output with an approximately 100mV P-P amplitude from the accelerometer even though the gyroscope is stationary and the rotation rate is 0. The accelerometer output consists mainly of quadrature error because in-phase error is manually tuned out.
We can compare the effect of using the incorrect clock to demodulate quadrature, just as was done in Section 5.3. In Figure 5.20, we see the effects of performing demodulation with the $\phi=90^\circ$ clock (on the left) and the $\phi=0^\circ$ clock (on the right). The upper curves in both plots are the difference in the accelerometer outputs. The lower curves in each plot are the result of demodulating. Keep in mind that the plots were taken for the spring-limited case, not the mode-matched case. The clocks and results would be switched in the mode-matched case.
Next, consider what happens when the quadrature controller is enabled, but the gyroscope is still held in the spring-limited case. We would expect the output of the accelerometer to be close to zero and the output of the quadrature controller to reach some stable value. In Figure 5.21, we see that the accelerometer output has gone from the 100mV P-P sinusoid to almost 0. The demodulation results in 0 as well. At the output of the quadrature controller are two DC values, one output at $Q_p$ and one output at $Q_n$. Each of these is fed back to the sensor die, as mentioned before. These DC values settle to 1.497V and 1.612V, respectively, to null out quadrature error. The outputs are displayed in Figure 5.22.
Figure 5.21: Quadrature Controller Waveforms with Quadrature Controller Turned On. When the quadrature controller is enabled, the output from the accelerometer (top) goes to 0. Demodulating with the $\phi=90^\circ$ clock (middle) produces a waveform with close to zero amplitude (bottom).

Figure 5.22: Quadrature Controller Outputs. Qp and Qn are fed back to the sensor die to correct for inherent quadrature error. Qp is 1.49V, and Qn is 1.61V.
The next series of plots will show operation of the quadrature controller when the mode-match feedback system is enabled. For these cases, quadrature is demodulated with the $\phi=0^\circ$ clock and the dither is applied to the quadrature correction elements. Because the dither essentially introduces quadrature error, the accelerometer responds with a sinusoidal output that is proportional to the dither amplitude. Figure 5.23 shows that this happens. The upper curve shows the difference in the accelerometer outputs; the lower curve shows the result of demodulating with the $\phi=0^\circ$ clock. As the dither is applied to the quadrature correction elements, the accelerometer responds with exponential rise and decay between dither transitions. The demodulated waveform has an envelope that is the original dither signal.

Figure 5.23: Accelerometer Response to Dither and Demodulation with $\phi=0^\circ$ Clock. When the dither signal is applied to the sensor die, the accelerometer responds with exponential rise and decay at the transitions in the dither signal (top). After demodulating with the $\phi=0^\circ$ clock, a waveform enveloped by the dither signal is produced (bottom).

Figure 5.24 and Figure 5.25 serve to show zoomed in versions of Figure 5.23. They were taken at the high-to-low and low-to-high dither transitions respectively.
Figure 5.24:Accelerometer Response to the Dither and Demodulated Result Viewed from the Quadrature Controller, High-to-Low Transition.

Figure 5.25:Accelerometer Response to the Dither and Demodulated Result Viewed from the Quadrature Controller, Low-to-High Transition.
Looking at the zoomed in plots, it is apparent that after the dither was introduced, it manifested itself as quadrature error, which is why demodulating with the quadrature clock yields rectified sinusoids with the dither waveform as the envelope. Integrating the demodulated result produces a DC term that is fed back to the sensor die. The dither signal is superimposed to produce the waveforms seen in Figure 5.26. When the mode-match system was disabled and the gyroscope was forced into the spring-limited case, we saw that applying 1.497V and 1.612V to $Q_p$ and $Q_n$ had the effect of nulling quadrature error. Even with the mode-match system in place, the quadrature controller does not deviate much from these DC outputs. The only difference is that the dither is now composited with them. Now, the output of the quadrature controller is composed of a DC term (1.480V and 1.624V) and an AC term (a 70Hz square wave). The amplitude of the dither is roughly 330mV which coincides with attenuating a 3.3V clock signal by a factor of 10, as discussed earlier.

![Figure 5.26: Quadrature Controller Outputs, Mode-Match System Enabled](image)

When the mode-match system is enabled, the dither (a 70Hz square wave) is composited with the DC output of the quadrature controller.
The quadrature controller works as expected. It is able to integrate quadrature error and correct for it by feeding a compensatory signal back to the sensor die. Introducing the dither signal does not affect the normal operation of the quadrature controller significantly. This is shown by how the DC values of the quadrature controller output are not affected much by the dither. Ultimately, our concern is whether or not the dither can be recovered by the shield controller. If the dither manifests itself as in-phase error, the gyroscope is not mode-matched. Otherwise, the dither correctly brings about quadrature error, and the gyroscope should be mode-matched. Operation of the shield controller is the next topic of discussion.

5.5 Shield Controller

In Section 3.7, we discussed the purpose of the shield controller. It monitors the demodulated result from the rate controller and adjusts the shield voltage if it sees the dither create in-phase error when it should not. By performing a second demodulation at the dither frequency, the shield controller produces a rectified dither signal, low-pass filters it, and corrects the shield voltage accordingly. Two implementations of the shield controller were tested, a linear controller and a bang-bang controller. The bang-bang implementation differs slightly from the linear controller by introducing a comparator between the demodulation and low-pass filtering stages.

Figure 5.27 offers a schematic diagram of the circuit used to implement both shield controllers. The shield controller receives its input at the nodes marked Shield_inP and Shield_inN from the demodulated output of the rate controller. The mixing stage is shared by both shield controllers. Mixing occurs at the dither frequency. Since the dither is inverted when introduced by the quadrature controller (because of the negative gain
factor), demodulation is performed with the dither clock that is anti-phase with the clock used in the quadrature controller. After demodulation, the circuit branches into the linear shield controller (on the top) and the bang-bang shield controller (on the bottom). Each is described in the following sections. Ultimately, the output of one of the controllers is selected via a switch that feeds the output back to the sensor die to set the shield voltage.
Figure 5.27: Shield Controller Schematic. The inputs Shield_inP and Shield_inN are demodulated by the mixing circuit, using the dither clock that is anti-phase to the one used in the quadrature channel. After the mixing stage, each shield controller implementation branches out. The linear shield controller is located above, and the bang-bang shield controller is located below. A switch at the output allows a user to switch between each implementation or provide a steady reference voltage.
5.5.1 Linear Shield Controller

The linear shield controller is composed of a differential integrator with an instrumentation amplifier to perform a differential-to-single-ended measurement. By integrating the result of the demodulation stage, the shield controller produces a signal that is a function of the phase error in the accelerometer. This error is superimposed with a reference voltage and is driven back to the sensor.

Figure 5.28 shows the signal from the rate controller demodulated at the dither frequency. On the top is the low-to-high transition; on the bottom is the high-to-low transition. Because the shield controller is enabled, the dither only appears as quadrature error. Hence, the waveforms from the rate controller resemble S-curves. If, instead, the waveform resembled rectified sinusoids, the second demodulation would result in a signal that would have a non-zero average value.
Figure 5.28: Demodulated Output in Linear Shield Controller. The S-like quality of the curves in both (a) and (b) indicate that the shield controller is ensuring that the introduced dither only comes out as quadrature error.

Another interesting topic to investigate is how long it takes for the gyroscope to become mode-matched after start up. Figure 5.29 is a trace of the shield voltage that is fed back to the sensor die. Initially, there is a start-up transient in the first 20ms. The reference voltage ($V_{REF}$) provided to the shield controller is nominally at 1.12V, and it is up to the shield controller to bring this voltage up to the proper value. Before the shield voltage settles, there is a slow rise as the shield controller hones in on the proper voltage. In this case, the mode-match shield voltage is 1.179V, and it takes less than 300ms for the controller to reach that value.
In the frequency domain, it is possible to see how well the linear shield controller causes the accelerometer and resonator peaks to line up. Figure 5.30 shows that the controller is able to bring the resonant peaks very close together, though not perfectly. There is a slight asymmetry in the plot, indicating that the peaks aren’t perfectly lined up. The tone immediately to the right of the 17.9kHz mark is slightly taller than the tone to the left.

Figure 5.29: Settling Time for Linear Shield Controller. At t=0, the system turns on. After a 15ms transient, the reference voltage ($V_{REF}$) is set to 1.12V, and the shield controller takes over to bring the shield voltage input to 1.179V. Within approximately 300ms, the shield voltage has settled.
Figure 5.30: Power Spectrum with Linear Shield Controller Enabled. The linear shield controller is able to bring the resonant peaks of the accelerometer and resonator very close, as indicated by the symmetry of the power spectrum. The line-up is not perfect, however, as indicated by the fact that the tone to the right of 17.9kHz is slightly larger than the tone to the left.

Why does the linear shield controller have problems? There are a couple possibilities that could explain this. One plausible explanation is that components in the system introduce some phase error that causes the feedback to work correctly but settle at the incorrect value. For example, suppose some part of the circuit introduced an extra 2° of phase, so instead of the quadrature signal being 90° out of phase with the in-phase signal, it is now 92° out of phase. Since the feedback operates on the presumption that quadrature and in-phase will be perfectly 90° apart, added phase delays could contribute to an error in the settled shield voltage. Similarly, offset errors might also contribute to the problem.
5.5.2 Bang-bang Shield Controller

The bang-bang shield controller has the same purpose as the linear shield controller. It is composed of a comparator stage after the demodulator and a low-pass filter to average out the output of the comparator over time. The purpose of the bang-bang controller is to bring the shield voltage to the correct value quicker than the linear controller.

Since the plots for the demodulation stage are identical to those in Figure 5.28, they will not be repeated here. Instead, consider the settling time of the shield voltage when the bang-bang controller is used. Figure 5.31 shows a trace of this. Between 0 and 20ms, there is a start-up transient, and $V_{\text{REF}} = 1.12V$ is supplied as a reference at $t=20\text{ms}$. At $t=60\text{ms}$, the shield voltage has already shown signs of settling, so the gyroscope is able to achieve mode-match within 60ms after start-up, significantly faster than the linear shield controller.
Again, the quality of the bang-bang controller can be verified by analyzing the power spectrum of the accelerometer output. Figure 5.32 shows this plot with the accelerometer and resonator peaks lined up very well. There is close to perfect symmetry about the resonator frequency, 17.9kHz.
Figure 5.32: Power Spectrum with Bang-bang Shield Controller Enabled. The bang-bang shield controller is able to bring the resonant peaks of the accelerometer and resonator very close, as indicated by the symmetry of the power spectrum. The match-up is even better than with the linear shield controller.

The bang-bang controller shows great promise. It has a faster settling time than the linear shield controller, and it brings the resonant peaks of the accelerometer and resonator even closer than the linear implementation. It has actually become the implementation of choice for this project because of the fast settling time and ability to mode-match the gyroscope.

5.6 Error Characteristics

As with many real-world systems, there are errors associated with the output caused by noise and other factors. The MEMS gyroscope is also susceptible to sources of error, and this next section will briefly introduce some sources and tools that are used to characterize
them. Lastly, results on the resolution of the gyroscope implementation for this project will be shown.

One source of error on a gyroscope’s output is a constant bias. A constant bias is the average output of the gyroscope when the gyroscope is not experiencing a rotation rate. This constant bias error, when integrated, causes an angular error that grows linearly with time[6]. For example, if the rate bias error is \( \varepsilon \), then the angular error will be \( \theta(t) = \varepsilon \cdot t \). If the bias is constant and known, it can be compensated for by simply subtracting it from the output.

Another source of error is thermo-mechanical white noise [6]. Thermo-mechanical noise perturbs the gyroscope’s output and gives rise to a white noise sequence. This creates angular random walk (ARW) error, which is typically measured in units of \(^\circ/\sqrt{\text{h}}\). For example, a gyroscope with \( \text{ARW} = 0.2^\circ/\sqrt{\text{h}} \) will have a standard deviation of the orientation error of \( 0.2^\circ \) after 1 hour and a standard deviation of the orientation error of \( \sqrt{2} \cdot 0.2^\circ = 0.28^\circ \).

A third source of error is flicker noise in the electronics and other components [6]. Flicker noise causes bias fluctuations, and a bias stability measurement is an indication on how the gyroscope’s bias will change over a specific period of time.

One tool that is used to measure a gyroscope’s resolution is to calculate the Allan variance. Computing the Allan variance produces a plot that is a time-domain analysis of the uncertainty in a gyroscope’s output. Figure 5.33 shows a sample of this chart. It is called a Green chart and measures Allan deviation (square root of Allan variance) on the \( y \)-axis with averaging time on the \( x \)-axis. Longer and longer averaging times bottom out at the minimum of the graph where further averaging cannot eliminate any more error.
sources. At this point, where the slope m=0, flicker noise dominates, and this is the resolution floor of the gyroscope.

![Figure 5.33: Sample Green Chart for Gyroscope Noise Analysis. The Green Chart is a measure of the Allan Deviation averaged over time. Longer average times reduce the uncertainty of the gyroscope only to a certain point where flicker noise dominates. This is indicated by the minimum of the graph where the slope is 0. This point sets the resolution floor of the gyroscope. [6]](image)

The mode-matched gyroscope in this project also has a Green chart characterizing its performance. This is shown in Figure 5.34, and represents data collected using the bang-bang shield controller. There is no quantization noise region shown, which would have a slope of -1. The angle random walk (white noise) is 0.019°/√s. The resolution floor is slightly below that of Earth’s rotation rate (15°/h = 0.004°/s). The minimum is at 12°/h = 0.003°/s.
Figure 5.34: Green Chart for Mode-matched Gyroscope and System. The mode-matched gyroscope has a resolution just below the Earth’s rotation rate. The Earth’s rotation rate is $15^\circ/h = 0.004^\circ/s$. The resolution of this mode-matched gyroscope is at $12^\circ/h = 0.003^\circ/s$.

It also helps to put into perspective how the mode-matching system compares against an un-mode-matched gyroscope and a gyroscope that is manually mode-matched (by manually tuning the shield voltage) but doesn’t have the feedback mechanism in place. Figure 5.35 shows this comparison with the x-axis adjusted to show the other two curves more clearly. The un-mode-matched gyroscope is the pre_mm curve, and has a resolution floor of 0.004°/s, which is slightly worse than the mode-matched case. The manually-tuned mode-matched gyroscope (open_loop) has significantly worse performance than either of the other cases. It reaches a resolution floor of 0.01°/s. This anomaly occurs because even though the gyroscope is mode-matched it does not maintain mode-match lock since it is under an open-loop condition and may come out of mode-match from time to time.
Figure 5.35: Green Chart Showing Comparison of un-mode-matched gyroscope (pre_mm), a manually-tuned mode-matched gyroscope with no feedback mechanism (open_loop), and the mode-matched gyroscope with feedback system in place (mm). The mode-matched gyroscope with feedback has the lowest resolution floor because mode-match causes the gyroscope to be more sensitive. Even though the manually-tuned gyroscope should be mode-matched, since it is running in an open loop configuration, it is unstable and does not maintain mode-match, which worsens its performance compared to the other two cases.

5.7 Summary

The purpose of this chapter was to present the findings of implementing the mode-matching system with all of its control loops. Every controller in the system behaves well and provides its expected output. The clock controller is able to reliably provide a drive signal to the sensor die to keep the resonator oscillating. The dither controller divides the drive signal down by a factor of 256 and is introduced as quadrature to the gyroscope. The gyroscope responds to the applied error with exponential rise and decay times, and the shield controller is able to detect when the dither does not undergo the expected phase shift of -90°.

Two implementations of the shield controller were tested. The bang-bang implementation had some advantages over the linear implementation. It had a faster settling time, and visual inspection showed that it brought the resonator and accelerometer...
frequencies closer together than the linear controller. Because of these findings, the bang-bang controller is the choice implementation, and it even showed that it could improve gyroscope performance to resolve rotation rates down to 12°/hr, which is below the Earth's rotation rate.
6 Conclusion

This project presented a new architecture to mode-match a gyroscope—the benefits of which are improved gyroscope performance and better rotation rate resolution. By introducing a square wave dither signal as quadrature error into a gyroscope, it became possible to detect the phase shift across the gyroscope’s accelerometer and then use this error to correct the gyroscope’s shield voltage. When the quadrature was shifted by anything other than -90°, it was apparent that mode-match had not been reached. The result was an error signal that was fed back to the sensor die until the error reached zero.

The system architecture resulted in a gyroscope with a resolution floor of 12°/hr, which is already below the rotation rate of the Earth (15°/hr). It is impressive to be able to reach this kind of resolution, because it opens up more possible applications for gyroscopic technology.

Several steps could be taken to further research in this project. For example, the mode-matched system aims to improve the signal to electronic noise ratio. However, it does not lead to any improvement in the signal to mechanical noise ratio. Brownian noise in the mechanical structure gives rise to errors on the output. If anything, mode-matching amplifies both the mechanical signal and noise, so eliminating mechanical noise sources would be ideal to improve the performance of the gyroscope even more.

An additional step for improvement involves developing a feedback loop to cancel in-phase offset. In-phase offset occurs when the gyroscope outputs a non-zero value for zero input rotation. The quadrature controller was able to correct for quadrature error, but no in-phase correction mechanism was implemented for this project. Future work could be invested in that area.
In conclusion, this project presents one possible method for achieving mode-match in a MEMS gyroscope. It uses a feedback mechanism that has proven to be stable, reliable, and promising for future generations of gyroscopes. The ability to design and manufacture better and better gyroscopes has enabled the emergence of new technologies in the world market, and it continues to be an interesting area of research.
7 References


  http://en.wikipedia.org/wiki/Coriolis_effect