

Cooling and Trapping Neutral Atoms

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Introduction and overview

The observation of Bose-Einstein condensation (BEC) in dilute atomic gases in 1995 was the realization of many long-standing goals: (1) to cool neutral atoms into the ground state of the system, thus exerting ultimate control over the motion and position of atoms limited only by Heisenberg's uncertainty relation; (2) to generate a coherent sample of atoms all occupying the same quantum state (this was subsequently used to realize an atom laser, a device which generates coherent matter waves); and (3) to create a quantum fluid with properties quite different from the quantum liquids He-3 and He-4. This provides a test-ground for many-body theories of the dilute Bose gas which were developed many decades ago, but never tested experimentally. BEC offers intriguing possibilities for further research, in directions as varied as superfluidity in a gas and atom interferometry, precision measurements and atom optics.

In the year 2000, we continued our work on critical velocity and dissipation in a Bose-Einstein condensate. Our earlier calorimetric work [1] was considerably extended by directly observing the flow field around the stirrer [2]. Furthermore, we carefully compared dissipation above and below the BEC transition temperature [3]. Another aspect of superfluidity was investigated by scattering impurities in a condensate [4].

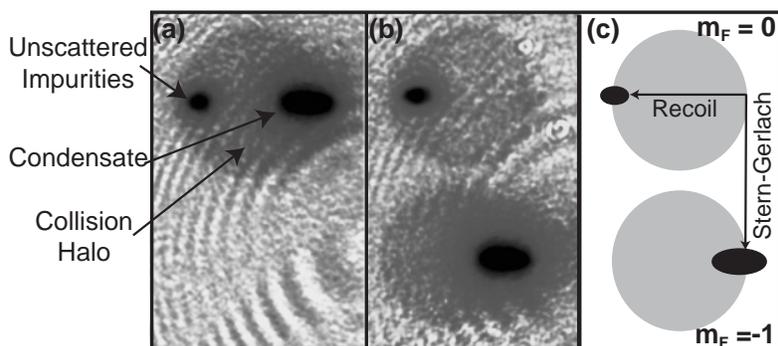
A condensate pumped by a laser beam is an amplifier for both light and atoms. After observing phase coherent amplification of atoms [5], we now studied the optical properties of this system and the interplay between atom and light amplification [6]. Two theoretical papers addressed conceptual questions raised by our work on light scattering in a BEC [7] and on atoms amplification [8].

Finally, in early 2000, we started to build a new two-chamber vacuum apparatus for transporting condensates, and a source of cold lithium atoms, which shall be mixed with sodium condensates. Both projects have advanced quickly, we have seen sodium condensates in the new vacuum chamber, and have obtained a nice magneto-optical trap of lithium atoms. Therefore, we are optimistic that first scientific results can be reported in the next progress report.

1. Superfluid suppression of impurity scattering in a Bose-Einstein condensate

The concept of superfluidity applies to both macroscopic and microscopic objects. In both cases, there is no dissipation or drag force as long as the objects move with a velocity less than the so-called critical velocity. A moving macroscopic object creates a complicated flow field. Above a certain velocity, vortices are created. In contrast, the physics of moving impurities, which are microscopic objects, is much simpler. At velocities larger than the Landau critical velocity, they will create elementary excitations, phonons or rotons in the case of liquid helium-4.

We could create impurity atoms in a trapped BEC by transferring some of the atoms into another hyperfine state using an optical Raman transition. The photon recoil and therefore the velocity of the impurity atoms was varied by the angle between the two laser beams. Collisions between the impurity atoms and the condensate were observed as a redistribution of momentum when the velocity distribution was analyzed with a ballistic expansion technique. The collisional cross section was dramatically reduced when the velocity of the impurities was reduced below the speed of sound of the condensate, in agreement with the Landau criterion for superfluidity [4].

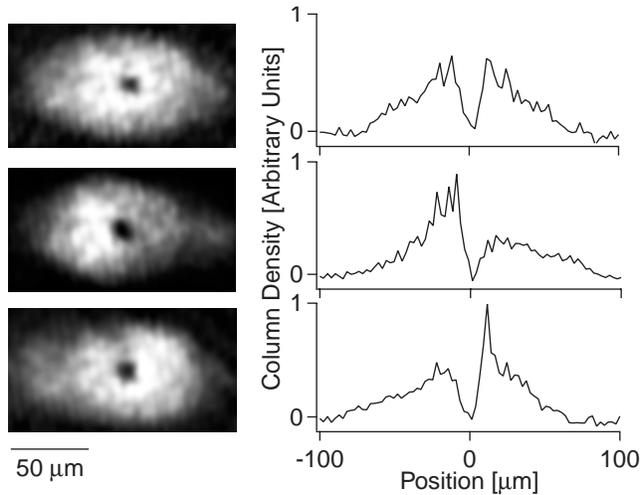


Observation of collisions between the condensate and impurities. The impurities in the $m=0$ hyperfine state traveled at 6 cm/s to the left. Collisions redistribute the momentum over a sphere in momentum space, resulting in the observed halo (a). In figure (b), the impurities and the condensate in the $m=-1$ state were separated with a magnetic field gradient during the ballistic expansion. The effect of collisions is to slow down some of the impurity atoms and speed up the condensate atoms. The absorption images are 4.5 mm times 7.2 mm in size.

2. Dissipationless flow and superfluidity in gaseous Bose-Einstein condensates

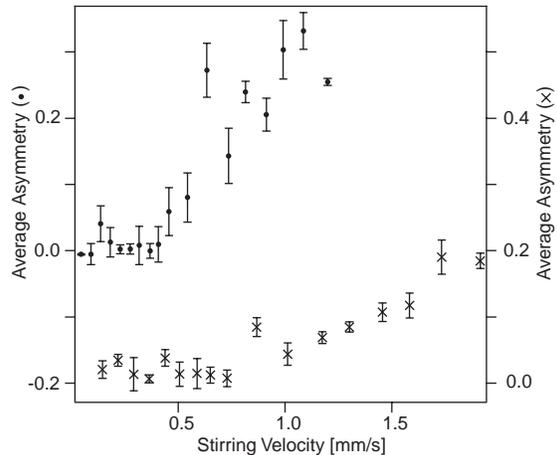
In previous work [1] we found evidence for a critical velocity in a condensate. The Bose-Einstein condensate was stirred with a laser beam at variable velocity, and the onset of dissipation was observed by monitoring the temperature of the sample. We have studied the same system by observing the condensate during the stirring using repeated *in situ* non-destructive imaging of the condensate. These images show the distortion of the density distribution around the moving object, thus directly probing the dynamics of the flow field [2].

The distortion or asymmetry of the flow is proportional to the drag force and a sensitive indicator for dissipation. The onset of dissipation was found at a critical velocity of about 10 % of the speed of sound which corrects the higher value found previously with a less sensitive method [1]. A comparison of the new technique observing the drag force to the calorimetric method showed good agreement.



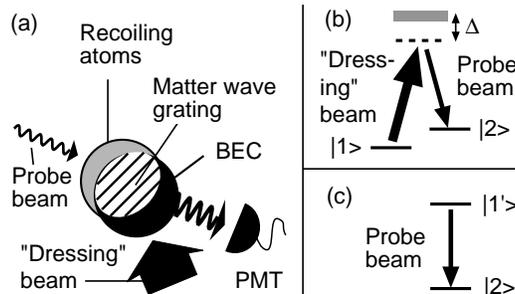
Pressure difference across a laser beam moving through a condensate. On the left side *in situ* phase contrast images of the condensate are shown, strobed at each stirring half period: beam at rest (top); beam moving to the left (middle) and to the right (bottom). The profiles on the right are horizontal cuts through the center of the images. The stirring velocity and the maximum sound velocity were 3.0 mm/s and 6.5 mm/s, respectively.

Density dependence of the critical velocity. The onset of the drag force is shown for two different condensate densities, corresponding to maximum sound velocities of 4.8 mm/s (solid circles, left axis) and 7.0 mm/s (crosses, right axis). The stirring amplitudes are 29 μm and 58 μm , respectively. The two vertical axes are offset for clarity. The bars represent statistical errors.



3. Amplification of light and atoms in a Bose-Einstein condensate

Bose-Einstein condensates illuminated by an off-resonant laser beam (“dressed condensates”) were used to realize phase-coherent amplification of matter waves [5, 9]. The amplification process involved the scattering of a condensate atom and a laser photon into an atom in a recoil mode and a scattered photon. This four-wave mixing process between two electromagnetic fields and two Schrödinger fields became a self-amplifying process above a threshold laser intensity, leading to matter wave gain. However, the symmetry between light and atoms indicates that a dressed condensate should not only amplify injected atoms, but also injected light.

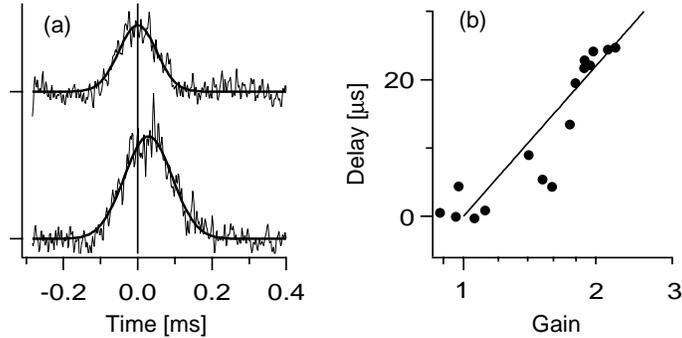


Amplification of light and atoms by off-resonant light scattering. (a) The fundamental process is the absorption of a photon from the “dressing” beam by an atom in the condensate (state $|1\rangle$), which is transferred to a recoil state (state $|2\rangle$) by emitting a photon into the probe field. The intensity in the probe light field was monitored by a photomultiplier. (b) The two-photon Raman-type transition between two motional states ($|1\rangle$, $|2\rangle$) gives rise to a narrow resonance. (c) The dressed condensate is the upper state ($|1'\rangle$) of a

two-level system, and decays to the lower state (recoil state of atoms, $|2\rangle$) by emitting a photon. Since the system is fully inverted, there is gain for the probe beam.

We have studied the optical properties of a dressed condensate above and below the threshold for the matter wave amplification [6]. The optical gain below the threshold has a narrow bandwidth due to the long coherence time of a condensate. The gain represents the imaginary part of the complex index of refraction. A sharp peak in the gain implies a steep dispersive shape for the real part of the index of refraction $n(\omega)$. This resulted in an extremely slow group velocity for the amplified light. The figure shows that light pulses were delayed by about $20 \mu\text{s}$ across the $20 \mu\text{m}$ wide condensate corresponding to a group velocity of 1 m/s . This is one order of magnitude slower than any value reported previously [10].

Above the threshold to matter wave amplification, we observed non-linear optical behavior. Thus we could map out the transition from single-atom gain to collective gain.

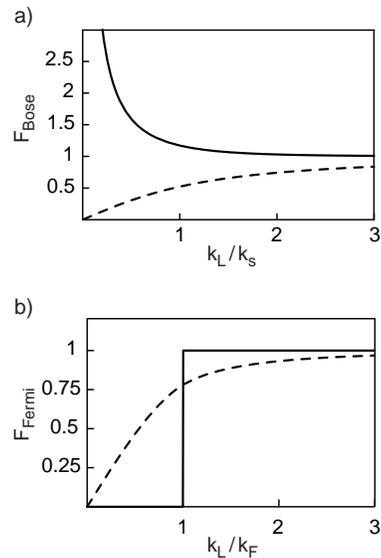


Pulse delay due to light amplification. (a) Amplification and $20 \mu\text{s}$ delay were observed when a Gaussian probe pulse of about $140 \mu\text{s}$ width and 0.11 mW/cm^2 peak intensity was sent through the dressed condensate (bottom trace). The top trace is a reference taken without the dressed condensate. Solid curves are Gaussian fits to guide the eyes. (b) The observed delay time was proportional to $\ln(g)$, where g is the observed gain.

4. Enhancement and suppression of spontaneous emission and light scattering by quantum degeneracy

Quantum degeneracy modifies light scattering and spontaneous emission. For fermions, Pauli blocking leads to a suppression of both processes. In contrast, in a weakly interacting Bose-Einstein condensate, we found spontaneous emission to be enhanced, while light scattering is suppressed [7]. This difference is attributed to many-body effects and quantum interference in a Bose-Einstein condensate.

Modification of spontaneous emission (solid line) and light scattering (dashed line) due to quantum degeneracy. In (a) we have plotted the enhancement factor for spontaneous emission and the suppression factor for light scattering for a weakly interacting Bose-Einstein condensate as a function of the light wave vector k_L in units of k_s , the wave vector of an atom moving at the speed of sound. In (b) the suppression factors for spontaneous emission and light scattering in a Fermi gas at zero temperature are plotted as a function of k_L in units of the Fermi wave vector k_F .



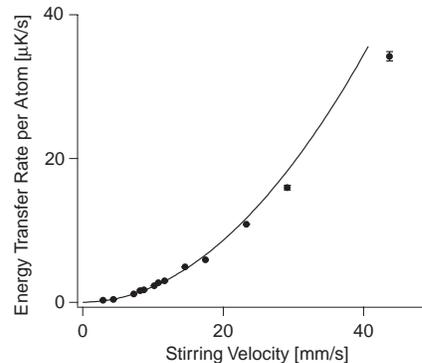
Generally, transition rates between an initial state with population N_1 and a final state with population N_2 are proportional to $N_1 (1+N_2)$ for bosons and to $N_1 (1-N_2)$ for fermions. This simple derivation of transition rates using occupation numbers becomes subtle or even invalid for correlated many-body states such as an interacting Bose-Einstein condensate ground state. We analyzed under which circumstances the simple approach can be used to reproduce the correct results for the interaction between light and a BEC. We showed theoretically that spontaneous emission in a weakly interacting BEC is enhanced, consistent with the description using occupation numbers, and calculated the enhancement factor. We compared this result to light scattering in a BEC, which is suppressed due to many-body interference effects not included in the simple derivation, as we have shown experimentally and theoretically in previous work [11]. In contrast, in fermionic systems quantum degeneracy leads to a suppression of *both* spontaneous emission and light scattering.

5. Dissipation in a stirred dilute Bose gas

We studied dissipation in a dilute Bose gas induced by the motion of a macroscopic object [3]. A blue-detuned laser beam focused on the center of a trapped gas of sodium atoms was scanned both above and below the BEC transition temperature. Measurements below the transition temperature confirmed the existence of a critical velocity [1, 2]. Above the transition temperature, we found a quadratic dependence of the dissipated energy on velocity, in agreement with a simple model.

These measurements allowed for a comparison between the heating rates for the superfluid and normal gas. Outside the superfluid regime, well above the critical velocity, the intrinsic heating of the condensate (normalized for geometry) was seen to be similar to the heating of the normal component. In both cases, each collision with the stirrer at velocity v transfers an energy of about Mv^2 to the gas.

Heating of the normal gas. The energy transfer rate per particle versus velocity of the moving laser beam shows a quadratic dependence.



6. Does matter wave amplification work for fermions?

Several recently observed phenomena Bose-Einstein condensates, superradiance of atoms, four-wave mixing and matter wave amplification were described as processes which are bosonically stimulated, i.e., their rates are proportional to $(N+1)$, where N is the number of identical bosons in the final state. However, we had pointed out that atomic superradiance does not depend on Bose-Einstein statistics and would occur for thermal atoms or even for fermions, although with a much shorter coherence time [12]. These suggestions have stirred a controversy among researchers.

In Ref. [8], we reconciled the different physical descriptions with the central result that the stimulated processes mentioned above do not rely on quantum statistics, but rather on symmetry and coherence. Bosonic quantum-degeneracy is sufficient, but not necessary for these processes. It represents only one special way to prepare a system in a cooperative state which shows coherent and collective behavior.

References

1. C. Raman, M. Köhl, R. Onofrio, D.S. Durfee, C.E. Kuklewicz, Z. Hadzibabic, and W. Ketterle, Phys. Rev. Lett. **83**, 2502 (1999).

2. R. Onofrio, C. Raman, J.M. Vogels, J. Abo-Shaeer, A.P. Chikkatur, and W. Ketterle, Phys. Rev. Lett. **85**, 2228 (2000).
3. C. Raman, R. Onofrio, J.M. Vogels, J.R. Abo-Shaeer, and W. Ketterle, J. Low Temp. Phys. **122**, 99 (2001).
4. A.P. Chikkatur, A. Görlitz, D.M. Stamper-Kurn, S. Inouye, S. Gupta, and W. Ketterle, Phys. Rev. Lett. **85**, 483 (2000).
5. S. Inouye, T. Pfau, S. Gupta, A.P. Chikkatur, A. Görlitz, D.E. Pritchard, and W. Ketterle, Nature **402**, 641 (1999).
6. S. Inouye, R.F.Löw, S. Gupta, T. Pfau, A. Görlitz, T.L. Gustavson, D.E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **85**, 4225 (2000).
7. A. Görlitz, A.P. Chikkatur, and W. Ketterle, Phys. Rev. A **36**, 041601(R) (2001).
8. W. Ketterle and S. Inouye, Phys. Rev. Lett., in print; e-print cond-mat/0008232.
9. M. Kozuma, Y. Suzuki, Y. Torii, T. Sugiura, T. Kuga, E.W. Hagley, and L. Deng, Science **286**, 2309 (1999).
10. L.V. Hau, S.E. Harris, Z. Dutton, and C.H. Behroozi, Nature **397**, 594 (1999).
11. D.M. Stamper-Kurn, A.P. Chikkatur, A. Görlitz, S. Inouye, S. Gupta, D.E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **83**, 2876 (1999).
12. S. Inouye, A.P. Chikkatur, D.M. Stamper-Kurn, J. Stenger, D.E. Pritchard, and W. Ketterle, Science **285**, 571 (1999).

Publications

Papers (in refereed journals) and major book chapters

W. Ketterle and S. Inouye:

Collective enhancement and suppression in Bose-Einstein condensates.

Special issue of Comptes rendus de l'académie des sciences, Série IV - Physique Astrophysique, in print; e-print cond- mat/0101424 29.

C. Raman, R. Onofrio, J.M. Vogels, J.R. Abo-Shaeer, and W. Ketterle:

Dissipationless flow and superfluidity in gaseous Bose-Einstein condensates.

J. Low Temp. Phys. **122**, 99-116 (2001).

W. Ketterle and S. Inouye:

Does matter wave amplification work for fermions?

Phys. Rev. Lett., in print; e-print cond-mat/0008232

A. Görlitz, A.P. Chikkatur, and W. Ketterle:

Enhancement and suppression of spontaneous emission and light scattering by quantum degeneracy.

Phys. Rev. A **36**, 041601(R), 2001.

S. Inouye, R.F.Löw, S. Gupta, T. Pfau, A. Görlitz, T. L. Gustavson, D. E. Pritchard and W. Ketterle:

Amplification of Light and Atoms in a Bose-Einstein Condensate.

Phys. Rev. Lett. **85**, 4225-4228 (2000).

R. Onofrio, C. Raman, J. M. Vogels, J. Abo-Shaeer, A.P. Chikkatur, and W. Ketterle:

Observation of Superfluid Flow in a Bose-Einstein Condensed Gas.

Phys. Rev. Lett. **85**, 2228-2231 (2000).

A.P. Chikkatur, A. Görlitz, D.M. Stamper-Kurn, S. Inouye, S. Gupta, and W. Ketterle:

Suppression and enhancement of impurity scattering in a Bose-Einstein condensate.

Phys. Rev. Lett. **85**, 483-486 (2000).

D.M. Stamper-Kurn and W. Ketterle:
Spinor Condensates and Light Scattering from Bose-Einstein Condensates.
in: *Coherent Atomic Matter Waves*, Proceedings of the Les Houches Summer School, Course LXXII in 1999, edited by R. Kaiser, C. Westbrook, and F. David (Springer, New York, 2001), e-print cond-mat/0005001.

R. Onofrio, D.S. Durfee, C. Raman, M. Köhl, C.E. Kuklewicz, and W. Ketterle:
Surface excitations of a Bose-Einstein condensate.
Phys. Rev. Lett. **84**, 810-813 (2000).

W. Ketterle:
Bose-Einstein condensation in dilute atomic gases: atomic physics meets condensed matter physics.
Proceedings of the 22nd International Conference on Low Temperature Physics (LT 22).
Physica B **280**, 11-19 (2000).

Articles in Proceedings

W. Ketterle, A.P. Chikkatur, and C. Raman:
Collective enhancement and suppression in Bose-Einstein condensates.
in: *Atomic Physics 17*, edited by E. Arimondo, P. DeNatale, and M. Inguscio (American Institute of Physics, Melville, New York, 2001), pp. 337-355; e-print cond-mat/0010375.

W. Ketterle:
Experimental Studies of Bose-Einstein Condensates in Sodium.
in: *Bose Einstein Condensates and Atom Lasers*, edited by S. Martellucci, A.N. Chester, A. Aspect, and M. Inguscio (Kluwer Academic, New York, 2000), pp. 1-29.

W. Ketterle and C. Raman:
Collisions at nanokelvin temperatures in Bose-Einstein condensates.
In: *The Physics of Electronic and Atomic Collisions*, Proceedings of the XXI International Conference in Sendai, Japan, 1999, editors Y. Itikawa, K. Okuno, H. Tanaka, A. Yagishita, M. Matsuzawa (American Institute of Physics, Melville, New York, 2000) pp. 23-43.

Popular articles

W. Ketterle
Bose-Einstein-Kondensation – Quantenmechanik am absoluten Nullpunkt.
Jahrbuch der Akademie der Wissenschaften zu Göttingen 1999 (Vandenhoeck&Ruprecht, Göttingen, 2000).

Conference contributions with published abstracts

A. Görlitz, A.P. Chikkatur, S. Inouye, S. Gupta, T. Pfau, D.M. Stamper-Kurn, D.E. Pritchard, and W. Ketterle:
Probing and manipulating Bose-Einstein condensates with laser light.
16th Interdisciplinary Laser Science Conference (ILS-XVI), Providence, RI, October 22-26, 2000, Conference Program, p. 96.

S. Inouye, R. Löw, T. Pfau, S. Gupta, A.P. Chikkatur, A. Görlitz, D.E. Pritchard, and W. Ketterle:
Amplification of light and atoms in a Bose-Einstein condensate.
16th Interdisciplinary Laser Science Conference (ILS-XVI), Providence, RI, October 22-26, 2000, Conference Program, p. 96.

W. Ketterle
Collective Enhancement and Suppression in Bose-Einstein condensates.
2000 Atomic, Molecular and Optical Physics Research Meeting, U.S. Department of Energy, Office of Basic Energy Sciences, Warrenton, Virginia, September 26-29, 2000, Book of Abstracts.

T.L. Gustavson, J. Abo-Shaeer, A.P. Chikkatur, A. Görlitz, S. Gupta, Z. Hadzibabic, S. Inouye, R. Löw, R. Onofrio, C. Raman, J.M. Vogels, D.E. Pritchard, and W. Ketterle:

Recent experiments with Bose-Einstein condensates: Amplification of light and matter waves, superfluidity, and RF transitions in an optical trap.

6th Workshop on Atom Optics and Interferometry, Cargese, Corse (France), July 26-29, 2000, Invited Talks Book of Abstracts.

S. Gupta, A.P. Chikkatur, A. Görlitz, T.L. Gustavson, S. Inouye, A.E. Leanhardt, R. Löw, T. Pfau, T. Rosenband, D.E. Pritchard, and W. Ketterle:

Superradiant Rayleigh Scattering, Matter Wave Amplification and Slow Light Propagation in a Sodium BEC.

6th Workshop on Atom Optics and Interferometry, Cargese, Corse (France), July 26-29, 2000, Book of Poster Abstracts.

Z. Hadzibabic, J.R. Abo-Shaeer, D.S. Durfee, J. Gore, M. Köhl, C.E. Kuklewicz, R. Onofrio, C. Raman, J.M. Vogels, and W. Ketterle:

Surface Excitations and Critical Velocity in a Sodium BEC.

6th Workshop on Atom Optics and Interferometry, Cargese, Corse (France), July 26-29, 2000, Book of Poster Abstracts.

J.M. Vogels and W. Ketterle:

Experimental Studies of Superfluidity in Gaseous Bose-Einstein condensates.

Workshop on Rotating Bose-Einstein condensates, Trento, June 11-13, 2000, Book of abstracts p. 13,

W. Ketterle:

New light on Bose-Einstein condensates.

ICAP 2000, Florence, June 4-9, 2000, Book of Abstracts, p. 47.

S. Inouye, T. Pfau, R. Loew, S. Gupta, A.P. Chikkatur, A. Görlitz, T. Gustavson, A.E. Leanhardt, D.E. Pritchard, and W. Ketterle:

Optical and Atom-Optical Properties of a Dressed Bose-Einstein condensate.

Bull. Am. Phys. Soc. **45**, 109 (2000).

A.P. Chikkatur, A. Görlitz, D. Stamper-Kurn, S. Inouye, S. Gupta, D.E. Pritchard, and W. Ketterle:

Impurity scattering in a Bose-Einstein condensate.

Bull. Am. Phys. Soc. **45**, 48 (2000).

S. Inouye, T. Pfau, S. Gupta, A. Chikkatur, A. Görlitz, D.E. Pritchard, and W. Ketterle:

Observation of phase-coherent amplification of atomic matter waves.

Bull. Am. Phys. Soc. **45**, 48 (2000).

C. Raman, M. Köhl, R. Onofrio, D.S. Durfee, C.E. Kuklewicz, Z. Hadzibabic, J. Abo-Shaeer, J. Vogels, and W. Ketterle:

Stirring, heating and superfluidity in a Bose-Einstein condensate.

Bull. Am. Phys. Soc. **45**, 49 (2000).

R. Onofrio, D.S. Durfee, C. Raman, M. Köhl, C.E. Kuklewicz, J. Abo-Shaeer, J. Vogels, and W. Ketterle:

Bose condensates and optical dipole forces.

Bull. Am. Phys. Soc. **45**, 79 (2000).

W. Ketterle:

New Light on Bose-Einstein Condensates.

Bull. Am. Phys. Soc. **45**, 121 (2000).

W. Ketterle:

When coherent light interacts with coherent atoms – optical properties of Bose-Einstein condensates.

Technical Digest, Quantum Electronics and Laser Science Conference (QELS 2000), p. 215.

C. Raman, R. Onofrio, M. Köhl, D.S. Durfee, C.E. Kuklewicz, Z. Hadzibabic, and W. Ketterle:

Dipole forces, excitations and dissipation in Bose-Einstein condensates.

Technical Digest, Quantum Electronics and Laser Science Conference (QELS 2000), p. 227.

S. Inouye, T. Pfau, S. Gupta, A. P. Chikkatur, A. Görlitz, D.E. Pritchard, and W. Ketterle:

Observation of phase coherent amplification of matter waves.

Technical Digest, Quantum Electronics and Laser Science Conference (QELS 2000), p. 228.

A.P. Chikkatur, A. Görlitz, D.M. Stamper-Kurn, S. Inouye, S. Gupta, and W. Ketterle:
Suppression and enhancement of impurity scattering in a Bose-Einstein condensate.
Technical Digest, Quantum Electronics and Laser Science Conference (QELS 2000), post deadline paper
QPD 2-1/3.

W. Ketterle:
Bose-Einstein-Kondensation – Quantenmechanik am absoluten Nullpunkt.
Verhandl. DPG (2000).

A. Görlitz, A. P. Chikkatur, S. Gupta, S. Inouye, T. Pfau, D.M. Stamper-Kurn, D.E. Pritchard, and W.
Ketterle:
Spektroskopie und Manipulation von Bose-Einstein-Kondensaten durch Lichtstreuung.
Verhandl. DPG (2000).

W. Ketterle:
Exciting Bose-Einstein condensates with laser light.
18th General Conference of the Condensed Matter Division of the European Physical Society (CMD18),
Montreux, Switzerland, 3/13-17/2000, Book of Abstracts, p. 327.

W. Ketterle:
When Laser-Like Atoms Meet Laser Light.
2000 AAAS Annual Meeting and Science Innovation Exposition.
2/17-22/2000, Washington DC, Book of Abstracts, A 47.

A. Chikkatur, D. Stamper-Kurn, S. Inouye, J. Stenger, H.-J. Miesner, and W. Ketterle:
Spinor Bose-Einstein Condensates: Ground States, Metastability, and Quantum Tunneling.
RTC and ITAMP Workshop on Multi-Component and Spinor Bose-Einstein Condensates of Trapped
Dilute Vapors, Rochester, 1/6-8/2000, Book of Abstracts, p. 9.

Theses

Christopher E. Kuklewicz, "Surface Excitations and Critical Velocity of a Bose-Einstein Condensate"
Master thesis, Department of Physics, MIT, 2000.

Robert Löw, "Dressing and trapping Bose-Einstein condensates with light", Diploma Thesis, University of
Bonn, Germany, 2000.