

Multi Packet Reception and Network Coding[†]

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Abstract—We consider throughput and delay gains resulting from network coding used to complement multi-packet reception in a fully connected network with broadcast traffic. The network is comprised of N nodes, J of which have data packets to be distributed to all other nodes. Owing to half-duplex constraints, a transmitting node is not able to receive data from other transmitting nodes in the same time slot. This requires a node to be back-filled with the information that it is missing. We consider single-packet reception, (for which network coding alone yields no gain), multi-packet reception without network coding, and combined multi-packet reception and network coding. We show that the initial transmissions and the back-filling process can be greatly sped up through a combination of network coding and multi-packet reception. In particular, we show that MPR capability of 2 will not reduce the total delivery time for packets within a data network unless network coding is used. We also demonstrate that a combination of network coding and multi-packet reception can reduce this time by a factor of m , where m is the MPR capability of the system. We demonstrate that network coding can substitute for some degree of MPR and achieve the same, or almost the same, performance as the higher degree of MPR without network coding. In effect, network coding allows almost double the traffic for a given level of MPR.

I. INTRODUCTION

In networks, multi-user interference is an important limitation. In order to alleviate it, the use of multi-packet reception (MPR) has been proposed [1], [2]. MPR may be implemented in a variety of ways, ranging from orthogonal signaling schemes such as OFDM to spread spectrum techniques such as frequency hopping or DS-CDMA. Such systems allow a limited number of packets to be simultaneously received at a node with MPR capability. In this paper we analyze the effects on network performance of a range of MPR capabilities, without restricting ourselves

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to specific physical layer implementations. Multi-packet reception can help when used with wireless MAC protocols (e.g. ALOHA), in which conventional models consider the transmission of multiple packets to be a collision and the throughput can be optimized through different back-off mechanisms [3]. There has been a renewed interest in capture schemes as explained in [4], [5], and [6], the simplest form of which can be used to capture the packet from the signal that has higher energy at the receiver.

It has been shown, in [7] and [8], that network coding can be used to improve throughput by allowing mixing of data at intermediate nodes in a network. If network coding and MPR are utilized together, we expect a higher throughput because MPR at a node enables combination of diverse packets which can be coded together and consequently, each transmission can on average disseminate more information to the network. The main focus of our paper is to show when and where we should combine network coding with multi-packet reception to improve performance.

The paper is organized as follows. In Section II, the network model and parameters are introduced. In Section III, we characterize single-packet reception. In Section IV, multi-packet reception without network coding is described. In Section V, we present the gains associated with the combination of network coding and multi-packet reception. Finally, in Section VI, we conclude the paper with a brief comparison of the results.

II. NETWORK MODEL AND PARAMETERS

Consider a wireless network represented by a set of N nodes. We assume that all nodes are within transmission range of each other. That is to say, given that node i is transmitting, any node $j \neq i$ in the network can receive the transmitted packets if j itself is not transmitting. Note that our model preserves the half-duplex constraint. Within this network, there is a set of J nodes ($J \leq N$) that have packets for transmission. Each of these J nodes has k packets and transmission occurs in time slots.

Let us define the MPR coefficient, m , to denote the maximum number of simultaneous receptions that is pos-

sible per node per time slot. For each node i , we define $T(i)$ to be the first time slot in which it has received every packet transmitted by the J transmitting nodes. Two time variables are defined to measure the performance of the system. First is the total time T_{tot} required for dissemination of information to all nodes; this time is a measure of delay performance of the last node that acquires all the packets. Second is the average time T_{avg} for dissemination of information; this time measures the average delay performance of the system. In Sections IV, V, and VI we introduce two auxiliary time variables: T_1 which is the number of time slots until each of the J transmitting nodes has transmitted its packets and T_2 to denote the number of time slots to complete the back-filling.

To avoid trivialities, two assumptions are made regarding the size of the network. First, $N - J \geq m$ which states that the number of non-transmitting nodes is lower bounded by the MPR coefficient. Second, $N \geq 2m$ which ensures that the number of nodes in the network is at least twice the MPR capability of the system. In summary:

- N : Number of nodes in the network.
- J : Number of transmitting nodes within the network.
- k : Number of packets to be transmitted per node.
- m : Number of possible receptions per node per time slot.
- $T(i)$: The first time slot in which node i has received every packet.
- T_1 : Number of time slots until each of the J transmitting nodes has transmitted its packets.
- T_2 : Number of time slots to complete the back-filling process.
- T_{tot} : Total time required for dissemination of information (in time slots).
- T_{avg} : Average time for a node to receive the last packet of the transmitted information (in time slots).
- NC, \overline{NC} : Denote network coding, or lack of it, respectively.
- MPR, \overline{MPR} : Denote multi-packet reception, or lack of it, respectively.
- $N - J \geq m$
- $N \geq 2m$

III. SINGLE PACKET RECEPTION ($m = 1$)

Since $m = 1$, all transmitting nodes will take turns transmitting their packet. Thus for $k = 1$:

$$T_{tot}^{\overline{MPR}} = J \quad (1)$$

More generally, if each of the J transmitting nodes has $k > 1$ packets to transmit, the total time will be increased by a factor of k and:

$$T_{tot}^{\overline{MPR}} = Jk \quad (2)$$

Consider the case where each of the J transmitting nodes successively transmits all of its k packets. It can be seen that the last transmitting node will have every packet after $(J-1)k$ transmissions and every other node will have it after Jk transmissions. Thus:

$$\begin{aligned} T_{avg}^{\overline{MPR}} &= \frac{1}{N} \sum_{i=1}^N T(i) \\ &= Jk - \frac{k}{N} \end{aligned} \quad (3)$$

Table I demonstrates how a typical transmission takes place when $J = 5$, and $N = 8$ without MPR. Each node is denoted by a letter, A through H , and time slots are enumerated by t_1 to t_5 . During each time slot, the transmitting node is explicitly marked with an arrow. For example $X_A \rightarrow$, shows that node A transmitted during t_1 and its packet was received by every other node during the same time slot as denoted by X_A in the other rows.

Node (i)	t_1	t_2	t_3	t_4	t_5	$T(i)$
A	$X_A \rightarrow$	X_B	X_C	X_D	X_E	5
B	X_A	$X_B \rightarrow$	X_C	X_D	X_E	5
C	X_A	X_B	$X_C \rightarrow$	X_D	X_E	5
D	X_A	X_B	X_C	$X_D \rightarrow$	X_E	5
E	X_A	X_B	X_C	X_D	$X_E \rightarrow$	4
F	X_A	X_B	X_C	X_D	X_E	5
G	X_A	X_B	X_C	X_D	X_E	5
H	X_A	X_B	X_C	X_D	X_E	5

TABLE I
TRANSMISSION SCHEDULE OF A NETWORK WITH $N = 8$, $J = 5$, AND $m = 1$

Note that since MPR is not present, coding cannot help. In other words, if each node codes its packets before transmission, there will be no reduction in the total transmission time or the average transmission time. Hence:

$$\begin{aligned} T_{avg}^{\overline{MPR}, \overline{NC}} &= T_{avg}^{\overline{MPR}, NC} \\ T_{tot}^{\overline{MPR}, \overline{NC}} &= T_{tot}^{\overline{MPR}, NC} \end{aligned}$$

IV. MULTI-PACKET RECEPTION WITHOUT NETWORK CODING

Consider J transmitting nodes, each with one packet to be distributed to every other node. Since receivers are limited by m receptions per time slot, the set of transmitting nodes is partitioned into groups of m nodes such that all nodes in a given group can transmit simultaneously. If J is not a multiple of m , the last group will have $J \bmod m$ nodes. Let T_1 be the number of time slots it takes until

each of the J transmitting nodes has transmitted its packet. There will be $\lceil \frac{J}{m} \rceil$ such groups, thus:

$$T_1 = \left\lceil \frac{J}{m} \right\rceil \quad (4)$$

Because of half-duplex constraints, the J transmitting nodes need to be back-filled. Note that transmissions occur in distinct groups of m nodes, and each transmitting node has missed a maximum of $m - 1$ packets during its transmission slot. Since $N - J \geq m$ in our model, the network will back-fill the previously defined groups consecutively by utilizing m of the non-transmitting nodes. Each group can be back-filled in one time slot and back-filling will take the same number of slots as T_1 . Let T_2 denote the number of slots to complete the back-filling:

$$T_2 = \left\lceil \frac{J}{m} \right\rceil u(J - 1) \quad (5)$$

where $u(J)$ is the unit step function defined as:

$$u(J) = \begin{cases} 0 & : J \leq 0 \\ 1 & : J > 0 \end{cases}$$

Table II shows a sample transmission schedule for $N = 8$, $J = 5$, and $m = 2$. We note that this sample strategy does not fully utilize the MPR since node E transmits alone. In contrast, our scheduling discussed in Part B below more fully utilizes MPR.

Node (i)	t_1	t_2	t_3	t_4	t_5	$T(i)$
A	$x_A \rightarrow$	x_C, x_D	x_E	x_B		4
B	$x_B \rightarrow$	x_C, x_D	x_E	x_A		4
C	x_A, x_B	$x_C \rightarrow$	x_E		x_D	5
D	x_A, x_B	$x_D \rightarrow$	x_E		x_C	5
E	x_A, x_B	x_C, x_D	$x_E \rightarrow$			2
F	x_A, x_B	x_C, x_D	x_E	$x_A \rightarrow$	$x_C \rightarrow$	3
G	x_A, x_B	x_C, x_D	x_E	$x_B \rightarrow$	$x_D \rightarrow$	3
H	x_A, x_B	x_C, x_D	x_E			3

TABLE II
TRANSMISSION SCHEDULE OF A NETWORK WITH $N = 8$, $J = 5$, AND $m = 2$ WITHOUT NETWORK CODING

Now consider the case where each transmitting node has k packets. We will present the results for two separate cases, namely for $J \leq m$ and $J > m$.

A. $J \leq m$

When the number of transmitting nodes is less than the MPR coefficient, we are not able to fully utilize the MPR

capability of the system and the initial transmissions will occur in groups of J nodes per time slot and:

$$T_1 = k \quad (6)$$

As discussed previously, $N - J \geq m$ and we will back-fill the J transmitting nodes by m of the non-transmitting nodes. Thus:

$$T_2 = \left\lceil \frac{Jk}{m} \right\rceil u(J - 1) \quad (7)$$

Thus, the total completion time is:

$$\begin{aligned} T_{tot}^{MPR, \overline{NC}} &= T_1 + T_2 \\ &= k + \left\lceil \frac{Jk}{m} \right\rceil u(J - 1) \end{aligned} \quad (8)$$

The average completion time can be upper bounded by noting that the $N - J$ non-transmitting nodes will have all the data by T_1 and the J transmitting nodes will have every packet in at most $T_1 + T_2$ time slots. Thus:

$$\begin{aligned} T_{avg}^{MPR, \overline{NC}} &\leq \frac{(N - J)T_1 + J(T_1 + T_2)}{N} \\ &\leq k + \frac{J}{N} \left\lceil \frac{Jk}{m} \right\rceil u(J - 1) \end{aligned} \quad (9)$$

B. $J > m$

In this case, the optimal strategy is to transmit m new packets in each time slot. This is achieved by using all possible combinations of m transmitting nodes. In other words, there are $\binom{J}{m}$ distinct groups of m out of J nodes. Any given node is present in $\binom{J-1}{m-1}$ groups and is excluded from $\binom{J-1}{m}$ groups.

Assume that k is an integral multiple of $\binom{J-1}{m-1}$. Then each node transmits $k / \binom{J-1}{m-1}$ packets during each transmission round, and T_1 can be obtained as:

$$T_1 = \binom{J}{m} \frac{k}{\binom{J-1}{m-1}} = \frac{Jk}{m} \quad (10)$$

Recall that the J transmitting nodes need to be back-filled because of half-duplex constraints. Since the transmitting groups during time T_1 are distinct and known to other nodes, we can back-fill the groups consecutively. Let T_2 denote the back-filling time:

$$T_2 = \frac{Jk}{m} \quad (11)$$

Thus, the total completion time is:

$$T_{tot}^{MPR, \overline{NC}} = T_1 + T_2 = 2 \left(\frac{Jk}{m} \right) \quad (12)$$

The average completion time can be upper bounded following the same argument used in part A. Thus:

$$\begin{aligned} T_{avg}^{MPR,NC} &\leq \frac{(N-J)T_1 + J(T_1 + T_2)}{N} \\ &\leq \frac{Jk}{m} \left(1 + \frac{J}{N}\right) \end{aligned} \quad (13)$$

V. MULTI-PACKET RECEPTION WITH NETWORK CODING

Following the model presented in Section IV, we will introduce network coding as an instrument to reduce the total transmission time. We present a strategy that utilizes network coding only in back-filling. Table III illustrates an example in detail; this example, like that in Table II, is sub-optimal since node E transmits alone and hence the MPR capability is not fully leveraged. However, our scheduling in Part B more fully utilizes the MPR. Coefficients α_1 through α_5 are used as coding coefficients and are chosen according to the size of the required finite field.

Node (i)	t_1	t_2	t_3	t_4	$T(i)$
A	$x_A \rightarrow$	x_C, x_D	x_E	x_B	4
B	$x_B \rightarrow$	x_C, x_D	x_E	x_A	4
C	x_A, x_B	$x_C \rightarrow$	x_E	x_D	4
D	x_A, x_B	$x_D \rightarrow$	x_E	x_C	4
E	x_A, x_B	x_C, x_D	$x_E \rightarrow$		2
F	x_A, x_B	x_C, x_D	x_E	$\alpha_1 x_A + \alpha_2 x_B + \alpha_3 x_C + \alpha_4 x_D + \alpha_5 x_E \rightarrow$	3
G	x_A, x_B	x_C, x_D	x_E		3
H	x_A, x_B	x_C, x_D	x_E		3

TABLE III
TRANSMISSION SCHEDULE OF A NETWORK WITH $N = 8$, $J = 5$, AND $m = 2$ WITH NETWORK CODING

As before, let us analyze the problem separately for $J \leq m$ and $J > m$.

A. $J \leq m$

Since we only use network coding in backfilling, T_1 is the same as the case of MPR with no network coding:

$$T_1 = k \quad (14)$$

Recall that $N - J \geq m$, so we can find at least m nodes that did not participate in any of the previous transmissions and have received each of the Jk transmitted packets. The back-filling is accomplished by having each of these m nodes transmit a linear combination of all the packets that it has received so far. Every time these m nodes transmit, m independent coded packets are sent out and, as a result, the original J transmitting nodes will get m

new degrees of freedom in each time slot and back-filling will be completed in $\lceil (J-1)k/m \rceil$ time slots. Thus:

$$T_2 = \left\lceil \frac{(J-1)k}{m} \right\rceil \quad (15)$$

The total transmission time for this strategy is:

$$\begin{aligned} T_{tot}^{MPR,NC} &= T_1 + T_2 \\ &= k + \left\lceil \frac{(J-1)k}{m} \right\rceil \end{aligned} \quad (16)$$

To calculate the average completion time, recall that all non-transmitting nodes will have every packet by T_1 and the J transmitting nodes will have the data after $T_1 + T_2$ time slots. Thus:

$$\begin{aligned} T_{avg}^{MPR,NC} &= \frac{(N-J)T_1 + J(T_1 + T_2)}{N} \\ &= k + \frac{J}{N} \left\lceil \frac{(J-1)k}{m} \right\rceil \end{aligned} \quad (17)$$

B. $J > m$

As in Section IV, let us assume that k is a multiple of $\binom{J-1}{m-1}$ and the transmitting nodes will send $k/\binom{J-1}{m-1}$ uncoded packets during the first $\binom{J}{m}$ time slots, therefore T_1 will have the same value as in Section IV:

$$T_1 = \binom{J}{m} \frac{k}{\binom{J-1}{m-1}} = \frac{Jk}{m} \quad (18)$$

During each transmission, exactly m nodes transmitted simultaneously and any given node within a transmitting group was unable to receive the packets from the other $m-1$ nodes. Since each node transmitted k packets, it is missing $(m-1)k$ packets in total. Following the same reasoning used in part A of this section, the back-filling will be completed in $(m-1)k/m$ time slots. Thus:

$$T_2 = \frac{(m-1)k}{m} \quad (19)$$

Thus, the total completion time is:

$$\begin{aligned} T_{tot}^{MPR,NC} &= T_1 + T_2 \\ &= \frac{k}{m} (J + m - 1) \end{aligned} \quad (20)$$

The average completion time can be calculated as:

$$\begin{aligned} T_{avg}^{MPR,NC} &= \frac{(N-J)T_1 + J(T_1 + T_2)}{N} \\ &= \frac{Jk}{m} \left(1 + \frac{m-1}{N}\right) \end{aligned} \quad (21)$$

This strategy demonstrates how network coding can be used with MPR to reduce the total and average transmission times.

VI. COMPARISON

In practical networks the number of transmitting nodes is usually much greater than the MPR capability of the system, hence we will compare the results of Sections III, IV, and V for the case where $J > m$. Let us revisit the results when each transmitting node has k packets.

Without MPR:

$$T_{tot}^{\overline{MPR,NC}} = T_{tot}^{\overline{MPR,NC}} = Jk \quad (22)$$

$$T_{avg}^{\overline{MPR,NC}} = T_{avg}^{\overline{MPR,NC}} = Jk - \frac{k}{N} \quad (23)$$

When MPR of m is used without network coding:

$$T_{tot}^{MPR,\overline{NC}} = 2 \left(\frac{Jk}{m} \right) \quad (24)$$

$$T_{avg}^{MPR,\overline{NC}} \leq \frac{Jk}{m} \left(1 + \frac{J}{N} \right) \quad (25)$$

When MPR is complemented with network coding:

$$T_{tot}^{MPR,NC} = \frac{k}{m} (J + m - 1) \quad (26)$$

$$T_{avg}^{MPR,NC} = \frac{Jk}{m} \left(1 + \frac{m-1}{N} \right) \quad (27)$$

Comparing (22) and (24), we can see that MPR reduces T_{tot} by a factor of $m/2$. Note that when $m = 2$, the total transmission time remains unchanged. This is depicted in Fig. 1 where lack of network coding is represented by dashed lines. Notice that the dashed lines do not change between $m = 1$ and 2. We will later compare this result with the case that combines MPR with network coding.

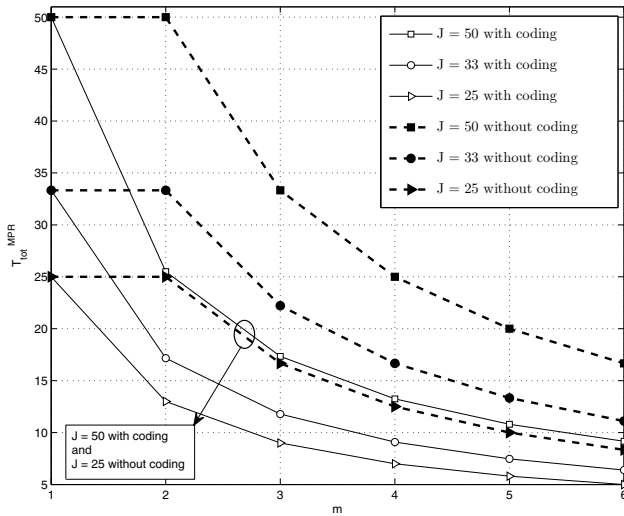


Fig. 1. T_{tot} as a function of m for different values of J and $k = 1$. Solid lines denote use of network coding and dashed lines are for the case without coding.

To see the advantage of network coding, compare (24) to (26). The total transmission time is reduced by a factor of $2J/(J + m - 1)$ which becomes arbitrarily close to two with increasing J . Let us revisit Fig. 1. An ellipse in the figure points out the close proximity of two lines: one representing $J = 50$ with coding and the other $J = 25$ without coding. Coding yields the same total dissemination time as a network with no coding and only half the traffic. Note that if network coding is used, we can reduce the total transmission time even when $m = 2$, which was not achievable without network coding. While Fig. 1 presents the value of T_{tot} for $k = 1$ packet per transmitting node, if $k > 1$ and k satisfies the integrality constraint discussed in Sections IV and V, the value of T_{tot} shown in this figure would be multiplied by k . We will follow the same approach in calculations of T_{tot} in Fig. 2 and Fig. 3.

In Fig. 2, we show that when m is fixed, T_{tot} grows linearly with the number of nodes if the ratio J/N is kept constant. As shown in the figure, the total transmission time of a network with $J = \lceil N/2 \rceil$ transmitting nodes that uses network coding is only slightly higher than that of a network with $J = \lceil N/4 \rceil$ transmitting nodes that does not use coding.

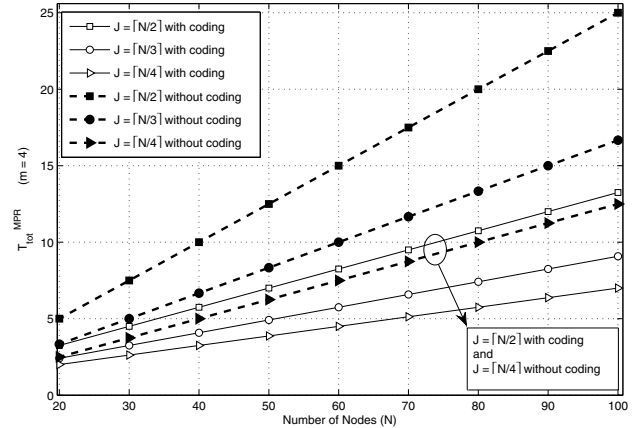


Fig. 2. T_{tot} as a function of N for different ratios of J/N when $m = 4$ and $k = 1$. Solid lines denote use of network coding and dashed lines are for the case without coding.

Fig. 3 shows that for a given value of m , the total time T_{tot} increases linearly with the number of transmitting nodes J . It is interesting to note that the lines in the figure cross one another when $J \in [1, 4]$. This occurs because $J \leq m$ in this range, and the behavior is governed by equations (8) and (16) where we have assumed that k is divisible by m to allow scalability for the results. If the traffic comes from a fixed number of nodes J , the total transmission time T_{tot} will not be affected by an increase in N .

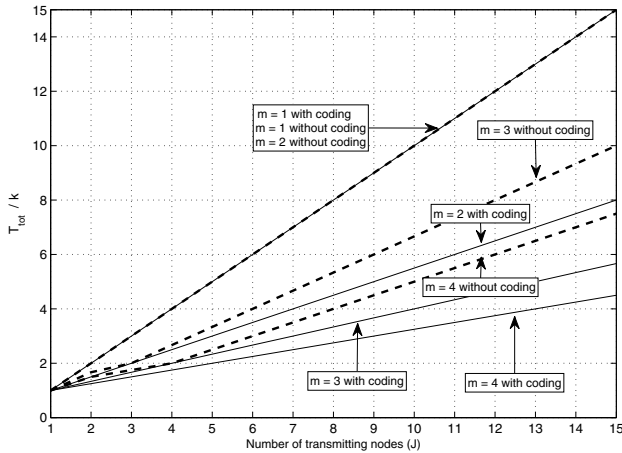


Fig. 3. T_{tot}/k as a function of J for different values of m .

Finally, Fig. 4 illustrates how the number of packets k at each transmitting node affects T_{tot} for a network with $N \geq 9$ and $J = 5$. Notice the similarity between Fig. 3 and Fig. 4. In essence T_{tot} increases linearly with the number of packets k . Again, notice that total transmission time of $m = 2$ with coding and $m = 4$ without coding are close to each other. The amount of time saved by adding network coding to a fixed level m of MPR is seen to increase with the number of packets k . The small discontinuities seen on each line are caused by the integrality constraint on k .

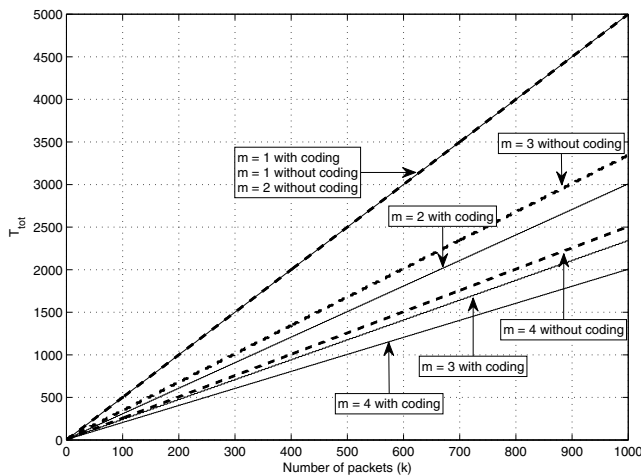


Fig. 4. T_{tot} as a function of k for different values of m when $N \geq 9$ and $J = 5$.

VII. CONCLUSION

In conclusion, we have shown that MPR can reduce the total time for a file transfer by as much as a factor of $\frac{m}{2}$

without network coding. It is important to note that a two-fold MPR capability will not reduce the total dissemination time without network coding and is thus ineffective. We have also shown that no gain can be obtained, if network coding is used without MPR. We finally argued that the combination of network coding and MPR can reduce the total transfer time by as much as a factor of m . We also showed that a MPR equipped network that uses coding behaves similarly to an equivalent network that has half of that traffic and does not use network coding. Our work demonstrates a number of significant gains that do not scale with N , in contrast to previous works such as [9] and [10], in which the network is analyzed through scaling laws, and hence would not show such gains. Erasures in these networks are currently under investigation, and we expect an even greater gain from the combined usage of network coding and MPR when erasures are present and must be handled.

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