

Fig. 8. Growth factor of instability as a function of perturbation wavelength. Fast- and slow- modes occur at wavelengths above their respective critical wavelengths  $\lambda_f, \lambda_s$ . Inset is a sketch of coaxial cylinder with radius  $R = 2r$  and equal viscosities.

we present a generalized analytical linear theory for multi-fluid cylindrical structures, and show that this dimensional analysis is consistent with the exact asymptotic result.

#### B. Linear theory of concentric cylindrical shells with equal viscosities

A linear theory of capillary instability for a co-axial cylinder with equal viscosities is provided in the literature by Stone and Brenner [8]. The growth rate ( $\sigma$ ) for a wave vector  $k = 2\pi/\lambda$  is a solution of the following quadratic equations

$$\left\{ \sigma - \frac{k^2 \gamma_1}{r\eta} [1 - (rk)^2] \Lambda(r, r) \right\} \times \left\{ \sigma - \frac{k^2 \gamma_2}{R\eta} [1 - (Rk)^2] \Lambda(R, R) \right\} = \frac{k^4 \gamma_1 \gamma_2}{rR\eta^2} [1 - (rk)^2] [1 - (Rk)^2] \Lambda(r, R)^2, \quad (15)$$

where  $r$  and  $R$  are the radii of the unperturbed interfaces I and II,  $\gamma_1$  and  $\gamma_2$  are the interfacial tensions, and  $\eta$  is viscosity.  $\Lambda(a, b)$ , where  $a \leq b$ , is associated with the modified Bessel function,

$$\Lambda(a, b) = \int_0^\infty \frac{s J_1(sa) J_1(sb)}{(s^2 + k^2)^2} ds - \frac{1}{2k} \frac{d}{dk} [I_1(ak) K_1(bk)]. \quad (16)$$

For the case of  $\gamma_1 = \gamma_2 = \gamma$ , the growth rate has the following formula,



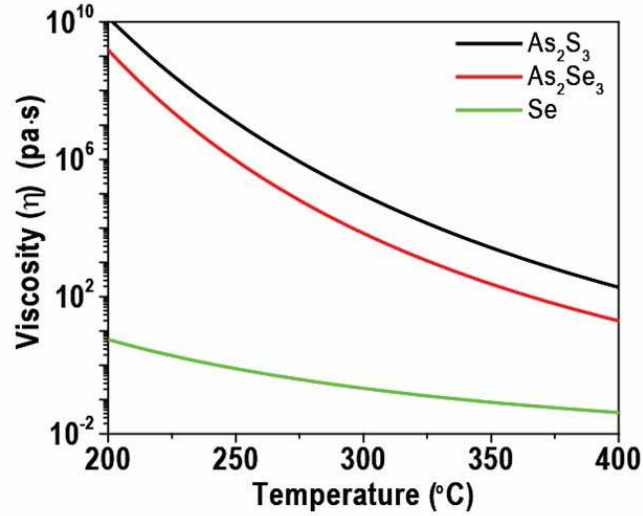


Fig. 9. Temperature-dependent viscosity for various chalcogenide glasses. Typical temperature during fiber drawing for glass Se,  $\text{As}_2\text{Se}_3$ ,  $\text{As}_2\text{S}_3$  is around 220, 260, 300 °C with the corresponding viscosities of  $10$ ,  $10^5$ ,  $10^5$  Pa · s, respectively.

$$\sigma(\lambda) = \frac{\gamma}{\eta r} \Psi(\lambda, R/r), \quad (17)$$

where the growth factor of  $\Psi(\lambda, R/r)$  in Eq. (17) is a complicated function of instability wavelength [8]. The instability time scale  $\tau \sim \sigma^{-1} \sim \eta r / \gamma$  is scaled with radius. For the case of  $R = 2r$ , this growth factor is calculated in Fig. 8. A positive growth factor indicates a positive growth rate ( $\sigma > 0$ ), for which any perturbation is exponentially amplified with time. Instability occurs at long wavelengths above a certain critical wavelength. Two critical wavelengths exist for the co-axial cylinder shell. One is a short critical wavelength  $\lambda_f = 2\pi r$  for a faster-growth mode (red line). The other is a long critical wavelength  $\lambda_s = 2\pi R$  for slower-growth mode (blue line).

### C. Viscosity of Materials During Thermal Drawing

Our chosen materials include chalcogenide glasses (Se,  $\text{As}_2\text{Se}_3$ , and  $\text{As}_2\text{S}_3$ ) and thermoplastic polymers (PES, PEI, and PSU). The viscosity of chalcogenide glass-forming melts depends on temperature and is calculated from an empirical Arrhenius formula [50],

$$\log \eta = \log \eta_0 + C \frac{\exp(D/T)}{2.3RT} - 1, \quad (18)$$

where  $R$  is the ideal gas constant,  $T$  is the temperature in Kelvin, and  $\eta$  is viscosity in Pa · s. The parameters of  $\log \eta_0$ ,  $C$ , and  $D$  for our materials are listed below:  $-2.0, 6651, 770.82$  for Se,

−3.09, 18877.8, 875.56 for  $\text{As}_2\text{Se}_3$ , and −3.62, 33744, 650.8 for  $\text{As}_2\text{S}_3$  [51]. These viscosities over a wide temperature range are plotted in Fig. 9. The typical temperature during a fiber drawing for Se,  $\text{As}_2\text{Se}_3$ , or  $\text{As}_2\text{S}_3$  films is around 220, 260, or 300 °C, respectively, with the corresponding viscosities of 10,  $10^5$ , or  $10^5$  Pa · s, respectively.

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