RANDOMIZED MODULATION SCHEMES FOR POWER CONVERTERS GOVERNED BY MARKOV CHAINS

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Abstract

Randomized modulation of switching in power converters holds promise in reducing input filtering requirements and in reducing acoustic noise in motor drive applications. This paper is devoted to issues in analysis and synthesis of randomized modulation schemes based on finite Markov chains. The main advantage of this novel type of randomized modulation is the availability of an explicit control of time–domain performance, in addition to the possibility of shaping the power spectra of signals of interest. Numerical (Monte Carlo) and experimental verifications for our power spectral formulas are presented. We also formulate representative narrow– and wide–band synthesis problems in randomized modulation, and solve them numerically. Our results suggest that randomized modulation is very effective in satisfying narrow–band constraints, but has limited effectiveness in meeting wide–band signal power constraints.

Keywords: power electronics, power converters, randomized modulation, Markov chains

1. Introduction

Switching power converters are designed to convert electrical power from one form to another at high efficiency. The high efficiency is obtained by using only switching devices, energy storage elements and transformers (all of which are ideally lossless), and relying on appropriate modulation of the switches to convert the available AC or DC voltage/current waveforms of the power source into (approximately) the AC or DC waveforms required by the load.

Converter waveforms that are periodic have spectral components only at integer multiples of the fundamental frequency. The allowable harmonic content of some of these waveforms is often constrained. Thus stringent filtering requirements may be imposed on the power converter. A significant part of a power converter's volume and weight can thus be due to an input or output filter. Similar requirements hold for acoustic noise control in motor applications. Harmonic components of the motor voltages and currents may excite mechanical resonances, leading to increased acoustic noise and to possible torque pulsations. Solutions to these problems include either a costly mechanical redesign, or an increase in the switching frequency of the power converter supplying the motor, which in turn increases the switching power losses.

As the use of pulse–width modulation (PWM) technology and microprocessors in power converters matured during the early eighties, new methods became available to address the effects of acoustic noise in DC/AC converters supplying motors, and the effects of electromagnetic interference (EMI). While most of the engineering effort was directed towards the optimization of deterministic PWM waveforms ("programmed switching"), an alternative in the form of randomized modulation for DC/AC conversion was offered in [11]. The same idea has been pursued in a DC/DC setup in [10], and in numerous references afterwards, for example [3, 4].

All prior results with random modulation, with the exception of [7], are based on schemes in which successive randomizations of the switching pulse train are statistically independent and governed by invariant probabilistic rules. We denote such schemes as

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stationary. While these implementations tend to be very successful in achieving certain kinds of spectral shaping in the frequency domain, they offer no guarantee or even description of the time–domain performance that accompanies the switching. This is objectionable in many cases, for example when accumulated deviations of the randomized switching waveform from the nominal (deterministic) waveform give rise to inadmissible variations in related currents and voltages.

In this paper we describe a generalization of the class of stationary randomized modulation schemes [7],[8] that enables explicit control of the time–domain performance of randomized switching, in addition to spectral shaping in the frequency domain. In this technique, the switching signal $q(t)$ comprises a sequence of distinct waveform segments, chosen in sequence according to a Markovian model. In developing an analysis approach for this class of randomized signals, we present previous results from communication theory that are not well known outside that community, and develop some new results as well. We also pose and solve numerically certain synthesis problems that are formulated to assess the effectiveness of randomized modulation in achieving various performance specifications in the frequency domain. This paper is an abridged version of [9] where a more detailed exposition can be found.

2. Autocorrelation and Power Spectrum

The **time–average autocorrelation** [6] of a random process $x(t)$ is defined as

$$R_x(\tau) = \lim_{W\to\infty} \frac{1}{2W} \int_{-W}^{W} E[x(t)x(\tau+t)]dt, \qquad (1)$$

where the expectation $E[\bullet]$ is taken over the whole ensemble, $[\bullet]$. The process is termed quasi–stationary [6] if this limit (and a similar one for $E[x(t)]$) exist. This definition is applicable to deterministic signals as well, since for deterministic signals the ensemble consists of a single member. The (mean or average) **power density spectrum** $S_x(f)$ is then defined as the Fourier transform of $R_x(\tau)$:

$$S_x(f) = \int_{-\infty}^{\infty} e^{-j2\pi f\tau} R_x(\tau)d\tau \qquad (2)$$

In cases of practical interest, $S_x(f)$ can have a continuous and an impulsive part [1]. The impulsive part of $S_x(f)$ is referred to as the **discrete** spectrum, and it is characterized entirely by the locations $f_1, f_2, \ldots$ of the impulses ("line frequencies", "harmonic frequencies") and by positive numbers $p_1, p_2, \ldots$ representing the strengths of the impulses (i.e. the signal power at the harmonic frequencies). Integrating $S_x(f)$ over a frequency range yields the signal power in that frequency range.

3. Modulation Based on Markov Chains

3.1. Concept of Switching Governed by Markov Chains

In this section we consider the class of randomized modulation schemes in which the switching signal $q(t)$ comprises a sequence of distinct waveform segments, chosen according to the state of a Markov chain. A switching waveform segment of length T_k is associated with the Markov chain being in the k-th state. Since the switching pattern in one cycle can be made dependent on the state of the underlying Markov chain, an additional degree of flexibility is available. State transition probabilities can be chosen so that large deviations from desired average steady-state behavior are discouraged or prevented altogether. Electronic circuits that implement switching based on a Markov chain are not necessarily more complicated than circuits employed in stationary modulation, as demonstrated by the realizations described in [7].

3.2. Power Spectra Generated by Ergodic Markov Chains

Ergodic Markov chains (i.e irreducible and aperiodic chains [5]) are considered in this section. Our goal is to analyze the continuous–time switching waveforms associated with an n–state discrete–time Markov chain. The chain is characterized by the $n \times n$ state transition matrix $P = [P_{k,l}]$. This matrix is a stochastic matrix, i.e. its rows sum to 1.

At a state transition from state k, a switching cycle of length T_k is generated. The switching function waveform $q(t)$ is a concatenation of such cycles. We assume throughout that the Markov chain is in steady state. The steady-state probabilities (also called invariant probabilities) of the chain can be found from [5]:

$$\Pi P = \Pi \qquad (3)$$

$$\sum_{k=1}^{n} \Pi_k = 1 \qquad (4)$$

The row vector Π is thus the normalized left eigenvector of the matrix P corresponding to the eigenvalue 1, and its existence follows from the assumed ergodicity of the underlying Markov chain. The corresponding right eigenvector is $\mathbf{1_n}$, an n-vector of ones. The entry Π_k can also be interpreted as the fraction of

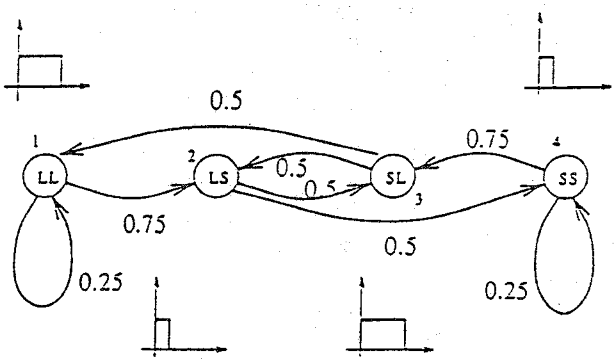

Figure 1: An example of switching governed by an ergodic Markov chain.

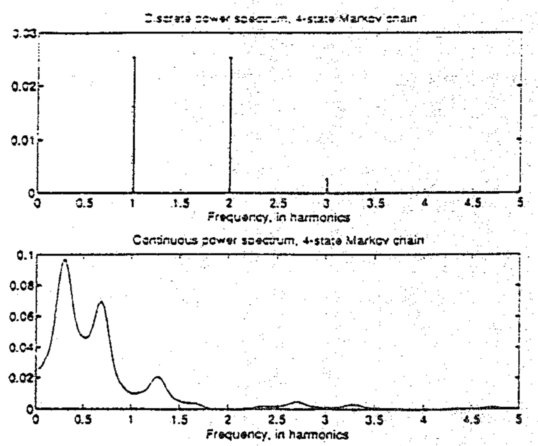

Figure 2: Calculated power spectrum of $q(t)$ for the Markov chain example of Section 3.2.1.

a (large) total number of state transitions that the chain spends in state k.

Due to its technical nature, for a detailed derivation of spectral formulas the reader is referred to [9]. The end result for the continuous power spectrum is

$$S_{cx}(f) = U(f)^H[\Theta F(f) + (\Theta F(f))^H - \Theta]U(f) \quad (5)$$

while the final result for the intensities of the impulses ("lines" in the discrete spectrum) is

$$S_{dy}(\frac{k}{T}) = \frac{1}{T^2} \mid \Pi U(\frac{k}{T}) \mid^2 \quad (6)$$

We use $U(f)$ to denote the vector of Fourier transforms of the waveform segments associated with states of the Markov chain; Θ is a diagonal matrix with steady-state probabilities Π; $F(f) = \sum_{m=1}^{\infty}(\hat{Q}(f))^m$ where $\hat{Q}(f)$ has entries $\hat{Q}_{k,l}(f) = [P_{k,l}e^{-j2\pi T_k f}]$.

3.2.1 An Example of Switching Governed by a Markov Chain:

Consider a scheme for DC/DC converters. Suppose we have two kinds of duty ratios D available: long, L, D=0.75; and short, S, D=0.25. The duty ratios have the desired average of 0.5, but we want to discourage long sequences of pulses of the same kind, thus preventing ripple buildup. We introduce a 4 state Markov chain, corresponding to the following policy. The controller observes the last two pulses and if they are SL or LS, then either of the pulses is fired with probability 0.5 for the next cycle. If the pair observed is LL, then an S pulse is applied with probability 0.75 (and an L pulse with probability 0.25). If the pair observed is SS, then an L pulse is applied with probability 0.75 (and an S pulse with probability 0.25). The chain is shown schematically in Fig. 1.

The theoretical discrete and continuous spectra corresponding to our example are shown in Fig. 2, where unit frequency corresponds to the switching frequency. The measured power spectrum in the same case is shown in Fig. 3. The circuit used for experimental verification was a down (or buck) converter, without the output capacitor, and the nominal switching frequency was 10 kHz. Our experimental experience is that randomized modulation schemes based on Markov chains are not more difficult to implement than the schemes reported earlier in the literature. For example, only a two-bit random number generator and a state counter are needed to implement the Markov chain of this example.

The results can be compared with deterministic switching at a constant duty ratio of 0.5, in which case only the discrete spectrum exists, with a first harmonic of $\frac{1}{\pi^2} = 0.1013$, for example (and subsequent odd harmonics reduced by $\frac{1}{n^2}$).

Another meaningful comparison is with a randomized PWM scheme in which a random choice is made at each trial between duty ratios of 0.25 and 0.75, independently of previous outcomes. Results for randomized PWM can be found for example in [7]. While the two schemes are quite similar in terms of their power spectra, their time–domain performance is very different. As an example, let us consider the event "five successive long (L) pulses" in both schemes. This event could be of interest, since it is associated with a fairly large net buildup of the local duty–ratio. In the case of independent randomized PWM, probability of "5 L in a row" is $(1/2)^5 = 0.03125$ [7]. In the case of the modulation based on the Markov chain from the example, the probability of the same event equals $0.2 \times (1/4)^3 = 0.003125$ [7], i.e. it is reduced ten times. These results have been verified both in simulations and in an actual circuit implementation.

3.3. Periodic Markov Chains

The case of pulse trains specified by a class of periodic Markov chains is considered in this section.

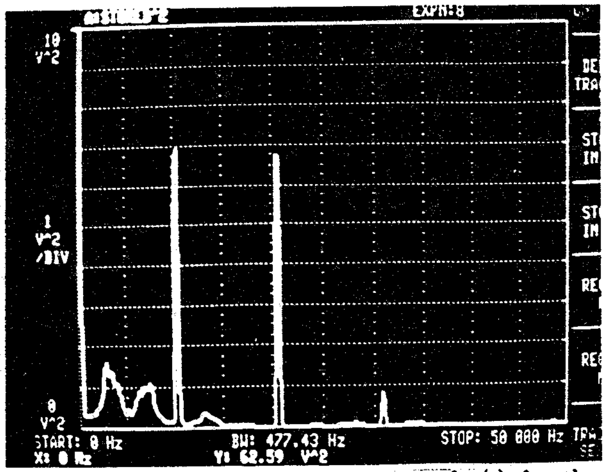

Figure 3: Measured power spectrum of $q(t)$ for the Markov chain example.

A related result for the special case of synchronous Markov chains is given in [2]. It is assumed that the state of the chain goes through a sequence of $\hat{N}$ classes of states C_l, occupying a state in each class for an average time $\tilde{T}_l$, $l = 1, \cdots, \hat{N}$. In the power electronic setup, periodic Markov chains are of interest in randomized modulation for DC/AC applications, where the basic (reference) on–off pattern changes from one cycle to the next in a deterministic fashion. This pattern is further dithered in each cycle using a set of dependent (Markovian) trials in order to satisfy time-domain constraints (for example to control the local time-average, or "ripple", of waveforms of interest).

The conditioning used in the derivation of the power spectrum formula in the previous section has to be adjusted in the following way. The contribution that states of the Markov chain belonging to the class C_k make to the time-averaged autocorrelation (1) is scaled by $\tilde{T}_k / \sum_{l=1}^{\hat{N}} \tilde{T}_l$, where $\tilde{T}_l$ is the expected time spent in the class C_l, before a transition into the class C_{l+1} (we evaluate these quantities later).

Let P_1 denote the product of submatrices of P in the following order $P_1 = P_{\hat{N}1} \cdots P_{23} P_{12}$, and let Π^1 denote the vector of the steady-state probabilities, conditional on the system being in class C_1. Then

$$\Pi^1 = \Pi^1 \; P_1 \tag{7}$$

and the average time spent in class C_1 is $\tilde{T}_1 = \sum_k \Pi^1_k T_k$, where the summation is taken over all states in class C_1.

Let $\tilde{T} = \sum_{l=1}^{\hat{N}} \tilde{T}_l$, and let $\Theta_k = diag(\Pi^k)$. When we add the contributions of all classes to the average power spectrum (scaled by the relative average duration of each class, as explained before), the result can be written in the following compact form

$$\begin{aligned} S_x(f) &= \frac{1}{\tilde{T}}[\sum_{k=1}^{\hat{N}} \frac{\tilde{T}_k}{\tilde{T}} U_k^H(f) \Theta_k U_k(f) + 2Re(1_{\hat{N}}^T S_c 1_{\hat{N}})] \\ &\quad + \frac{1}{\tilde{T}^2} Re(1_{\hat{N}}^T S_d 1_{\hat{N}}) \sum_{l=-\infty}^{\infty} \delta(f - \frac{l}{\hat{T}}) \end{aligned} \tag{8}$$

where $\hat{T}$ is the greatest common denominator of all waveform durations, $1_{\hat{N}}$ is an $\hat{N} \times 1$ vector of ones and U_k is the vector of Fourier transforms of waveforms assigned to states in class C_k. A circular indexing scheme (i.e. modulo $\hat{N}$) is used in this subsection.

The matrix S_c has a Toeplitz structure, with (k,l)–th entry

$$S_{c\,k,l}(f) = \frac{\tilde{T}_k}{\tilde{T}} U_k^H(f)(I - \Lambda_k(f))^{-1} \Lambda_{k,l}(f) U_l(f) \tag{9}$$

where Λ_k is a product of $\hat{N}$ matrices

$$\Lambda_k = Q_{k-1,k} \cdots Q_{k,k+1} \tag{10}$$

and

$$\Lambda_{k,l} = Q_{l-1,l} \cdots Q_{k,k+1} \tag{11}$$

with no repetitions allowed in $\Lambda_{k,l}$, so that the number of matrices forming $\Lambda_{k,l}$ is $\hat{N} - \mid k - l \mid$. Also

$$S_{d\,k,l}(f = \frac{l}{\hat{T}}) = \frac{\tilde{T}_k}{\tilde{T}} U_k^H(f = \frac{l}{\hat{T}}) \Pi^k (\Pi^l)^T U_l(f = \frac{l}{\hat{T}}) \tag{12}$$

The result (8) appears to be novel [7], and it is will be now verified via an example.

3.3.1 An Example of Switching Governed by a Periodic Markov Chain:

In this example we consider a simplification of a switching scheme applicable for DC/AC converters. The goal is to generate a switching function in which blocks of pulses have duty ratios

$$[0.5, \;\; 0.75, \;\; 0.5, \;\; 0.25]$$

It is also desirable to prevent large deviations of the "local" average from values in the corresponding conventional (deterministic) pulse train. The periodic Markov chain shown in Fig.4, with 8 states divided into 4 classes, is an example of a solution to such a design problem. We analyze this chain using (8), and in Fig.5 we compare the theoretical predictions (dotted line) with results obtained via Monte Carlo simulations (solid line). The agreement between the two is quite satisfactory. An application of (8) for Markov chains with many more classes of states could become computationally intensive. This is not a major drawback, however, due to the off-line character of the calculation. In Fig. 6 we show the experimentally observed power spectrum for the same periodic Markov chain.

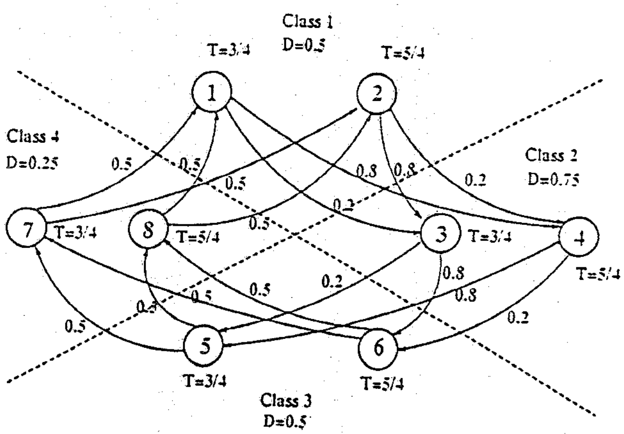

Figure 4: Periodic Markov chain with 8 states and 4 classes.

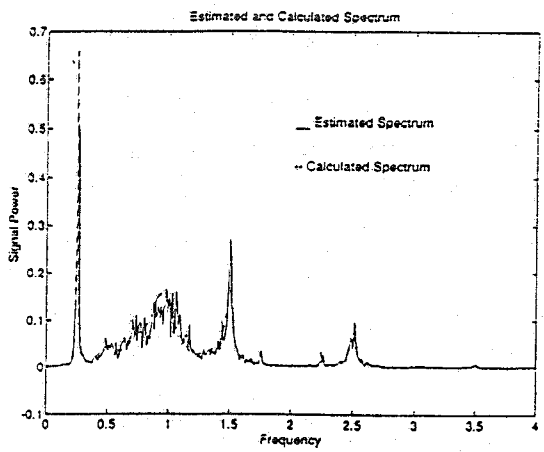

Figure 5: Estimated and calculated spectrum for the periodic Markov chain with 8 states and 4 classes.

4. Synthesis Problems

In this section the goal is to explore how effective randomized modulation is in achieving various performance specifications in the frequency domain. Desirable properties of power spectra are dependent on the particular application. Requirements of particular interest in practice are the following:

- Minimization of one or multiple, possibly weighted, discrete harmonics. This criterion corresponds to cases where the narrow–band characteristics corresponding to discrete harmonics are particularly harmful, as for example in acoustic noise, or in narrow–band interference.
- Minimization of signal power (integral of the power spectrum) in a frequency segment that is of the order of an integral multiple of the switching frequency. This criterion corresponds to wide–band constraints in military specifications, and it could be of interest for EMI problems.

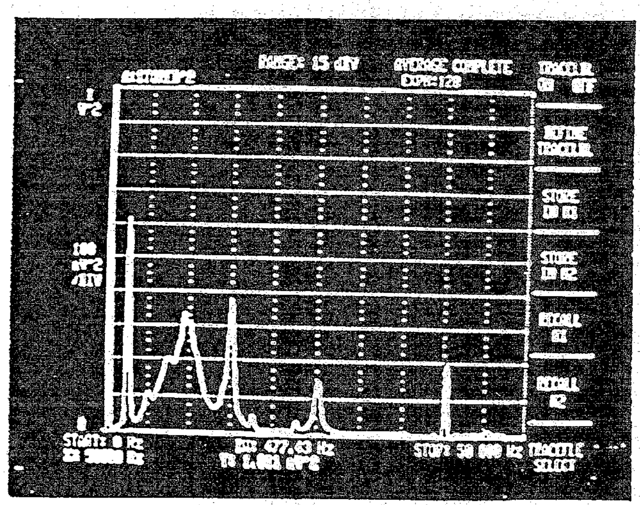

Figure 6: Measured power spectrum for the periodic Markov chain with 8 states and 4 classes.

A typical narrow–band optimization criterion is a weighted sum of discrete harmonic intensities between the l-th and L-th harmonics, and is denoted as $J_{l,L}^{NB}$. A reasonable wide–band optimization criterion, used for illustration in this section, corresponds to the minimization of the signal power in the frequency segment $[0,\ 1.5]$, where the average switching frequency is 1.

The optimization process has to address two related issues:

1. Design of an n-state Markov chain (possibly periodic), which reduces to the specification of a stochastic matrix P;
2. Choice of n 0–1 functions, each supported on $[0, T_k]$, that correspond to distinct cycles of the switching function.

While the criterion functions are defined in the frequency domain, the design is performed in the time–domain. This makes the optimization problem difficult, and we present and comment on numerical results.

Synthesis problems for randomized modulation governed by Markov chains will be illustrated on the example from section 3.2.1. The transition probabilities $p_{1,2}$ from state 1 to state 2, and $p_{2,3}$ from state 2 to state 3 will be optimized, while the symmetry of the chain is preserved. Thus the transition matrix is

$$P = \begin{bmatrix} p_{1,2} & 1-p_{1,2} & 0 & 0 \\ 0 & 0 & p_{2,3} & 1-p_{2,3} \\ 1-p_{2,3} & p_{2,3} & 0 & 0 \\ 0 & 0 & 1-p_{1,2} & p_{1,2} \end{bmatrix} \quad (13)$$

The duty ratios of the short and long pulses are also made variable, with the same average value $D = 0.5$ as in the original example. The optimized narrow–band criteria are shown in Table 1. Thus a considerable improvement in criterion value is attained as a consequence of optimization. In the table randomized PWM corresponds to independent, equally likely trials with $D_1 = 0.25$, $D_2 = 0.75$, the standard Markov chain is characterized with probabilities and duty ratios $p_{1,2} = .75, p_{2,3} = .5, D_1 = 0.25$, $D_2 = 0.75$, while the optimal chain is $p_{1,2} = .75, p_{2,3} = .5, D_1 = 0.05$, $D_2 = 0.95$ It is evident that switching based on a

Table 1: Narrow–band optimization, Markov chain example, criterion values ($\times 10^{-4}$).

Modulation	J_1^{NB}	$J_{1,41}^{NB}$
Randomized PWM	253	613
Standard Markov Chain	253	613
Optimal Markov Chain	0	11.5

Markov chain is not particularly effective in reducing wide–band signal energy, as illustrated in Table 2. In this table randomized PWM and the standard chain are as before, while the optimal chain is given by $p_{1,2} = .05, p_{2,3} = .95, D_1 = 0.22$, $D_2 = 0.77$. The

Table 2: Wide–band optimization, Markov chain example, criterion values ($\times 10^{-4}$).

Modulation	J_1^{WB}
Randomized PWM	800
Standard Markov Chain	806
Optimal Markov Chain	786

purpose of our optimization procedures is to point out salient capabilities of the randomized modulation governed by Markov chains, and consequently our searches were performed over granular grids.

5. Conclusions

In this paper we have presented analysis and synthesis results for randomized modulation strategies governed by Markov chains, suitable for different classes of power converters. Random modulation switching schemes governed by Markov chains that are applicable to DC/DC and DC/AC converters have been described and analyzed. Our spectral formulas for periodic Markov chains are believed to be novel. Synthesis problems in randomized modulation have also been considered, where both optimization criteria and numerical results are described. It is shown that randomized pulse modulation can be very efficient in reducing the size of discrete harmonics and in satisfying narrow–band constraints, but is much less effective in dealing with wide–band requirements.

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