Feedback Control of Paralleled Symmetric Systems, with Applications to Nonlinear Dynamics of Paralleled Power Converters

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Abstract - **This paper addresses the dynamic analysis of a class of paralleled symmetric systems having nonlinear subsystem coupling and external feedback. A transformation is introduced that allows the stability of the full system to be assessed through examination of only two low-order subsystems. The paper also considers the use of the new results for the analysis of paralleled power converter systems with active current sharing. It is shown that the analysis of paralleled converter systems can be simplified by this approach even when challenging nonlinearities appear in the current-sharing control loops.**

I. Introduction

Systems comprising identical subsystems coupled identically to one another - termed *symmetric* systems [1,2] - are of interest in several applications. Our particular application involves active current sharing among paralleled switching power converters that supply a common load whose voltage is regulated, [3], see Fig. 1. As will become clear shortly, this application corresponds to a case in which the outputs of the symmetrically coupled subsystems are summed to form the overall system output, which is then fed back through some dynamics to generate a common input to the subsystems. In other words, the symmetrically-coupled subsystems are *paralleled*, and then have a feedback loop closed around them.

The key to studying the open-loop dynamics of a *linear* (though possibly time varying) symmetric system comprising N identical subsystems, each described by an n^{th} order state vector, is a transformation to a new set of variables, namely (i) an n^{th} order vector that is the *average* of the *N* subsystem state vectors, and (ii) *N*-1 vectors of order *n*, describing the *deviations* of *N*-1 of the subsystem state vectors from the average vector (the deviation of the remaining state vector from the average vector then follows directly, since the sum of all *N* deviations has to be 0). The result of such a transformation is to create (i) a linear n^{th} order *common-mode subsystem* that describes the dynamics of the average state vector, and (ii) *N*-1 linear *differential-mode subsystems*, all *identical* to each other and of order only *n*, that describe the dynamics of the deviations from the average.

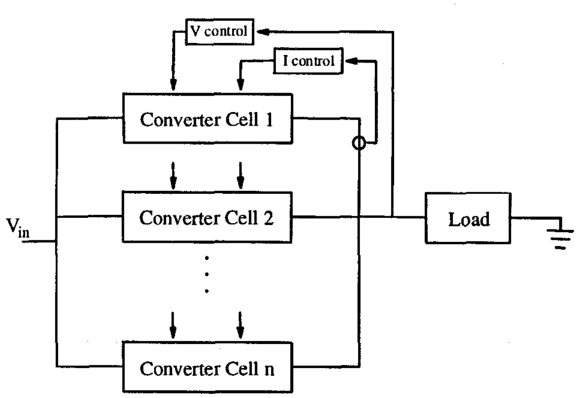

Fig. 1 A parallel converter system supplying a single load.

The fact that the differential-mode subsystems are identical allows one to study their dynamics by just studying that of a single representative; the fact that each of these subsystems is of order only *n* means that a significant order reduction is thereby obtained. For instance, stability analysis of the overall system now involves only assessing two subsystems (the common-mode, and a representative differential-mode subsystem) of order *n* each, rather than assessing the original system, whose order is *Nn*.

We also note that if each of the original symmetric subsystems is driven by a *common* signal, which may be exogenous or a function (possibly nonlinear and/or time varying) of the state variables, then this signal will, after the transformation, drive the common-mode subsystem but not the differential-mode subsystems. Thus one can first study the linear differential-mode dynamics, and then assess the effects of the differential-mode and other external signals on the common-mode.

A related observation is that when symmetrically-coupled subsystems are paralleled, the differential-mode subsystems end up being uncontrollable and unobservable, as is easily shown. The external feedback thus acts only on the common-mode dynamics. The study of the overall system then reduces to studying a representative differential-mode subsystem, and also studying the feedback loop comprising the common-mode dynamics and the feedback dynamics.

With this basic picture in mind, we can turn to our specific application, which is represented schematically in Fig. 1. The converter output currents are summed and fed to the load. The linear "V control" feedback loops are aimed at

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regulating to zero the load-voltage deviations from a specified reference value. These loops may be absorbed into the dynamics of the individual symmetric subsystems, without destroying the symmetric structure or its linearity. The "I control" feedback loops are designed to reduce to zero the individual output current deviations from a reference current. This reference current may be the *average* current or the *maximum* of the individual output currents; in both cases, proper current sharing among the converters is enforced, provided the feedback control works as desired. When the current reference is the average current, the "I control" loops are linear, and can again be absorbed into the dynamics of the individual symmetric subsystems. However, when the maximum current is used as a reference, the "max" operation introduces a nonlinearity that prevents similar absorption into the individual symmetric subsystems while preserving symmetry and linearity. Nevertheless, since the nonlinear term affects each of the symmetric subsystems equally, it ends up leaving the differential-mode subsystems unaffected, and only drives the common-mode subsystem. The load dynamics, which relate variations in the output current (i.e. the sum of the subsystem output currents) to variations in the load voltage (i.e. the common "input" to the subsystems), constitute additional feedback around the common-mode subsystem. This special structure permits a complete analysis of our application.

Section II of the paper introduces the state-space structure of the class of systems to be analyzed, and displays the transformation into differential-mode and common-mode subsystems. Section III introduces simple averaged models for paralleled power converters with active current sharing, and demonstrates how the theoretical developments of Section II can be employed for stability analysis in this application. Section IV presents a circuit-based interpretation of these results for an example system, along with supporting simulation results, and Section V concludes the paper.

II. Analysis of Paralleled Symmetric Systems

Consider a paralleled symmetric system that has identical nonlinear couplings among the symmetric subsystems and is connected in closed loop with another system. The resulting system can be described in state-space form as:

$$\frac{d}{dt}v = A\cdot v + f \tag{1}$$

where

$$v = \begin{bmatrix} x_1 \\ x_1 \\ \vdots \\ x_N \\ \hline w \end{bmatrix},\quad f = \begin{bmatrix} f_1(x) \\ f_1(x) \\ \vdots \\ f_1(x) \\ \hline 0 \end{bmatrix},\ and$$

$$A = \left[\begin{array}{ccccc|c} A_1 & A_2 & A_2 & \cdots & A_2 & A_3 \\ A_2 & A_1 & A_2 & \cdots & A_2 & A_3 \\ \vdots & \vdots & \ddots & & \vdots & \vdots \\ A_2 & A_2 & A_2 & \cdots & A_1 & A_3 \\ \hline A_4 & A_4 & A_4 & \cdots & A_4 & A_5 \end{array}\right]. \tag{2}$$

Here, x_1 to x_N are the vectors of state variables of the symmetric subsystems and are denoted collectively by x; w is the state vector of the system in the feedback path; and f_1 is the nonlinear coupling shared by the symmetric subsystems. (We can also allow for additional external inputs; these are omitted for simplicity here, but our example in Section IV includes such inputs.) The large dimensionality of the system, along with the nonlinear couplings, would appear to make dynamic analysis challenging, but the particular structure of (2) actually allows dramatic simplification.

We now introduce a transformation

$$\hat{v} = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \\ - \\ w \end{bmatrix} = \begin{bmatrix} T_L & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \\ - \\ w \end{bmatrix} = Tv \tag{3}$$

where T_L is Lunze's transformation for linear symmetrically-coupled systems [1]:

$$T_L = \frac{1}{N}\begin{bmatrix} (N-1)I & -I & \cdots & -I & -I \\ -I & (N-1)I & \cdots & -I & -I \\ \vdots & \vdots & & \vdots & \vdots \\ -I & -I & \cdots & (N-1)I & -I \\ I & I & \cdots & I & I \end{bmatrix} \tag{4}$$

The effect of T_L is to replace the x_i for $i = 1$ to N-1 by their deviations $\hat{x}_i$ from their average $\hat{x}_N$, and to replace x_N by $\hat{x}_N$. Thus, $\hat{x}_i$ for $i = 1$ to N-1 become differential-mode state variables, and $\hat{x}_N$ comprises the common-mode state variables. Applying the transformation (3), (4) to the system (1), (2) results in the following transformed system:

$$\frac{d}{dt}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_{N-1} \\ \hline \hat{x}_N \\ w \end{bmatrix} = \left[\begin{array}{cccc|cc} A_d & 0 & \cdots & 0 & 0 & 0 \\ 0 & A_d & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & A_d & 0 & 0 \\ \hline 0 & 0 & \cdots & 0 & A_c & A_3 \\ 0 & 0 & \cdots & 0 & NA_4 & A_5 \end{array}\right] \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_{N-1} \\ \hline \hat{x}_N \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \hline f_1(x) \\ 0 \end{bmatrix}$$

where

$$A_d = A_1 - A_2 \quad , \qquad A_c = A_1 + (N-1)A_2 \ . \qquad (6)$$

The transformation thus results in a simple block-diagonal structure for the transformed A matrix, revealing a set of N-1 identical, decoupled differential-mode subsystems, and a single common-mode subsystem. In addition, the transformation isolates the nonlinearity of the original system into the common-mode subsystem.

Stability assessment is much easier in the transformed system than in the original system. The differential-mode subsystems are entirely linear and have identical dynamics, so their stability can be assessed by examining the eigenvalues of A_d. The common-mode subsystem remains nonlinear, but can be tractable.

III. Application to Paralleled Power Converters

Power converters are often operated in parallel to create a single large power converter system, as illustrated in Fig. 1. This is done to achieve higher reliability and availability through redundancy, and to permit the use of individual converters of smaller rating, which can be manufactured more efficiently than large converters [4]. A crucial issue in such paralleled power converter architectures is that of current sharing, which must be maintained in order to realize many of the benefits of a paralleled architecture [3-17]. The favored approach in present practice is to impose current sharing through the use of an explicit control loop based on a feedback signal common to all of the converters. Often, the nature of the feedback signal results in nonlinear current-sharing dynamics, which can be hard to analyze. The high dimensionality of the system (especially when a large number of paralleled converters are employed) further complicates system analysis and control. The theoretical results of Section II can be employed to simplify the analysis of such systems.

Consider an averaged model [18] for a current-mode-controlled power converter, Fig. 2. The output voltage v_{out} of the converter is compared to a reference voltage v_{ref}, and the error between the two is used by the voltage control compensator to generate a reference current i_{ref} for the converter. Based on this current reference, the current-controlled power stage generates an average output current i_{out} into the filter/load combination. If a voltage-loop compensator $G_c(s)$ of the form

$$G_c(s) = \frac{\kappa_c}{\tau_c s + 1} \qquad (7)$$

is used, and assuming the current-controlled power stage gain may be well approximated as $H(s) \approx 1$, then we can form a circuit representation of the averaged model as shown in Fig. 3, where $L = \tau_c / \kappa_c$, and $R = 1/\kappa_c$. This makes physical sense, since the averaged model of the converter is a voltage source in series with an output impedance whose nature is determined by the voltage control loop. (The R_o, C_o combination is taken as the load for specificity, but other loads can be treated similarly.)

In cases where multiple converters are paralleled, as illustrated in Fig. 4, one would not expect the converters to share current equally, given that the voltage references of the different converters will be slightly different. Furthermore, the desire for good output voltage regulation generally causes the output resistances of the converters to be kept small, meaning that even slight imbalances in the voltage references will cause large current imbalances. To address this, many paralleled converter systems employ an active current-sharing control system in which each cell adjusts its voltage reference by a small amount about a base value in order to maintain current sharing with the other cells. Typically, each converter compares its own output current to some function (such as the average, maximum, minimum, rms) of all the output currents, and uses this error signal to generate an adjustment (Δv_{ref}) to the base reference (v_{base}). For example, one approach that is easily implemented is to have each converter employ the following current-sharing control law:

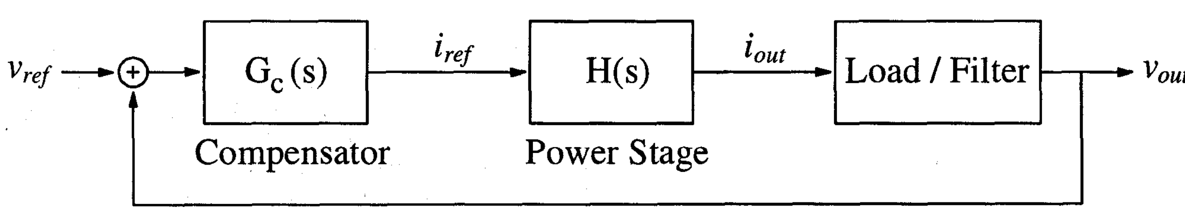

Fig. 2 Block diagram of the averaged model for a current-mode-controlled power converter.

$$\frac{d\Delta v_{ref,j}}{dt} = k_1\left[i_{max} - i_{out,j}\right] - k_2 \Delta v_{ref,j} \tag{8}$$

where

$$i_{max} = \max_{l}\{i_{out,l}\} \ . \tag{9}$$

The k_1 term adjusts the voltage reference to achieve current sharing, while the k_2 (decay) term keeps the reference voltage adjustment from wandering. It should be pointed out that while this control law is conceptually simple, popular, and particularly easy to implement, it is challenging from an analytical point of view because the *max(·)* function is not differentiable everywhere, thus limiting the usefulness of linearization-based analysis methods [3,17].

The state-space structures of **(1)**, **(2)** are useful for describing such paralleled converter systems. The vectors x_j describe the states of the individual converters, while the vector w describes the state of the load/filter combination. The entries of matrix A contain the description of the output voltage control law, while entries of A along with the coupling function f_1 contain the description of the current-sharing control law. Consider a system with the voltage control law **(7)**, the current-sharing control law **(8)**, and a capacitive output filter in parallel with a resistive load. If we choose the subsystem state vectors as $x_j^T = [i_{out,j}, \Delta v_{ref,j}]$, $w^T = [v_{out}]$, then the subsystem matrices are:

$$A_1 = \begin{bmatrix} -R/L & 1/L \\ -k_1 & -k_2 \end{bmatrix} , \quad A_2 = 0 \ , \quad A_3 = \begin{bmatrix} -1/L \\ 0 \end{bmatrix} ,$$
$$A_4 = \left[1/C_o \ \ 0\right] \ , \quad A_5 = \left[-1/(R_o C_o)\right] \tag{10}$$

and the nonlinear coupling function is

$$f_1(x) = \begin{bmatrix} 0 \\ k_1 \max_{l}(i_{out,l}) \end{bmatrix} \ . \tag{11}$$

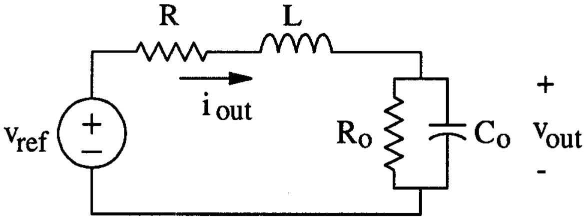

Fig. 3 Averaged circuit model for a current-mode-controlled power converter.

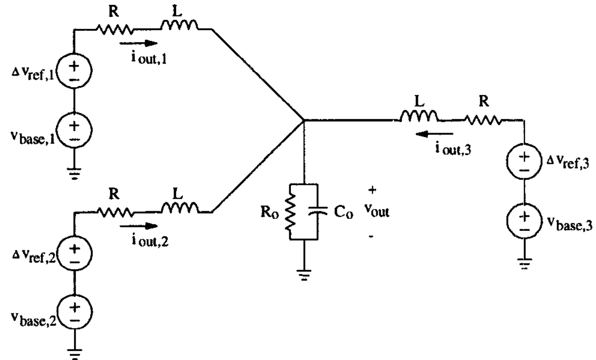

Fig. 4 Averaged circuit model for a parallel converter system with three converters.

The direct analysis of a parallel power converter system of this type is challenging due to both the large number of interdependent state variables (especially when many converters are paralleled) and the nature of the nonlinear coupling among subsystems. However, applying the transformation **(3)**, **(4)** results in dramatic simplification of the problem. In an N-converter system, the transformation yields N-1 decoupled, linear (differential-mode) subsystems of low order (whose dynamics are easily determined), and a single nonlinear (common-mode) subsystem of low order. Furthermore, the analysis of the nonlinear subsystem is extremely simple in this case because the nonlinear term turns out to be only a function of the differential-mode variables, whose behavior is known from the linear subsystem analysis. We carry out the analysis in the next section, using circuit realizations of the transformed system equations to make analysis transparent for readers who would rather think with circuits than differential equations!

IV. Circuit Representations of the Transformed Model

A straightforward circuit interpretation of the transformed system provides insight into the transformation to differential-mode and common-mode subsystems. We again consider the 3-converter parallel system modeled in Fig. 4, with the nonlinear current-sharing control law of **(8)**, **(9)**. One possible circuit realization of the transformed system is shown in Figs. 5 and 6, where $R_x = R_y = k_1 / k_2$, $C_x = C_y = 1 / k_1$. As can be seen in Fig. 5, there are two identical, decoupled differential-mode circuits, which are linear and whose stable dynamics are easily computed. Figure 6 shows the circuit realization of the common-mode subsystem; the transformed version of the load system and common-mode elements of the paralleled converters are easily identified. The effects of the nonlinear current-sharing control loop are simply captured by the controlled current source $\hat{f}$, whose value is defined as

$$\hat{f} = \max(i_{x1} \ , \ i_{x2} \ , \ -i_{x1} - i_{x2}) \ . \tag{12}$$

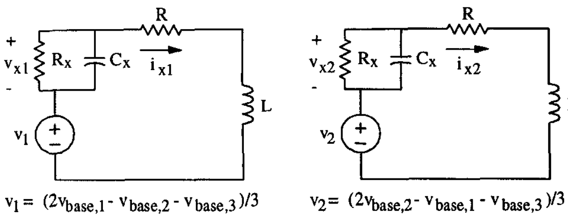

Fig. 5 Differential-mode subsystem circuit model for a three-cell parallel converter system.

The advantage of the transformation with respect to the nonlinearity is apparent; the differential-mode circuit variables feed into the nonlinearity in the common-mode circuit, but the resulting common-mode circuit values do not feed back into the differential-mode circuit, greatly simplifying the analysis. Because the differential-mode circuits are stable, and we know the nature of the nonlinearity **(12)**, we know that the variables in the common-mode portion of the circuit decay exponentially as well (although "max" is not differentiable, it is Lipschitz, which is all we need).

To illustrate these results, we have developed simulations of both an example three-cell paralleled converter system (Fig. 4) and the equivalent transformed system (Figs. 5,6). The averaged model for each of the three cells has parameters $k_1 = 5$, $k_2 = 2$, $L = 0.03$ H, R = 4 Ω, and the load parameters are $R_o = 1$ Ω, $C_o = 0.05$ F. Also, we assume that the base reference voltages for the three cells ($v_{base,j}$) are 0 V, 0.1 V, and 0.05 V, and that the initial conditions of the reference adjustments ($\Delta v_{ref,j}$) are 0.5 V, 0.2 V, and 0 V, respectively. Figure 7 shows the simulated response of the three-cell system, while Fig. 8 shows the simulated response of the equivalent transformed system. Appropriate combinations of the transformed system variables exactly match the response of the original system (so we omit any graphical comparison). This fact confirms the validity of the transformation and the circuit-based interpretation of the transformed system.

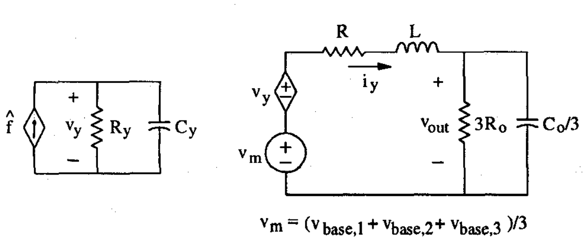

Fig. 6 Common-mode subsystem circuit model for a three-cell parallel converter system.

V. Conclusions

In this paper, we have addressed the dynamic analysis of a class of paralleled symmetric systems having nonlinear subsystem coupling and external feedback. It has been demonstrated that a transformation into differential-mode and common-mode subsystems permits a rather complete understanding of the dynamic behavior of the whole system. With this method, one only needs to study two low-order subsystems to assess the stability of the full system. The paper also considers the use of the new results for the analysis of paralleled power converters with nonlinear active current-sharing control. In addition to representing an important use of our results, this application allows us to develop a circuit-based interpretation of the transformation. It is shown that analysis of paralleled converter systems can be simplified by this approach, even when challenging nonlinearities appear in the current-sharing control loops.

Acknowledgments

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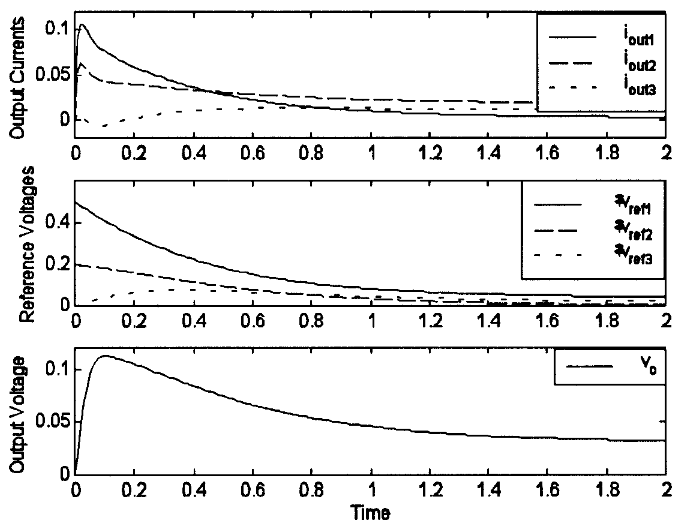

Fig. 7 Simulation of 3-cell paralleled converter system.

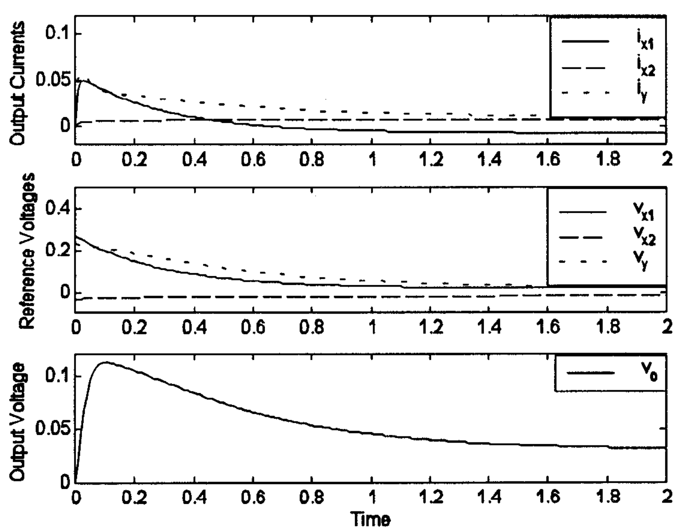

Fig. 8 Simulation of Transformed 3-cell system.