APPENDIX A
OPTIMIZATION OF MAGNETIC-CORE INDUCTORS

In Section III, different magnetic materials are compared and evaluated with the assumption that optimum magnetic-core inductors made in all these materials will have the same relative dimensions as the coreless design on which they are based. However, magnetic-core inductors may have their own relative optimum dimensions for the maximum quality factor Q_L or the minimum size for different materials, thus the methods proposed in Section III may not be a fair comparison. That is, we need to establish whether or not the best *shape* for an inductor changes significantly with scale or material characteristics.

As will be seen, the results are quite reasonable and the approaches of Section III lead to near optimum designs under a wide range of conditions. We consider one optimization case in this paper. We assume a magnetic-core toroidal inductor's d_o and h are restricted to be constant (e.g., as stipulated by the specification of a power electronics circuit), and we optimize d_i to get the maximum quality factor Q_L. In the optimization, make the assumption that core losses dominate and neglect copper loss. We do take into account the fact that the flux density inside the core is not uniform when calculating core loss.

The total core loss P_0 is calculated without the approximation of uniform flux. From (6),

$$B_{pk}(r) = \frac{\mu_0 \mu_r N I_{pk}}{2\pi r} = \frac{I_{pk}}{r}\sqrt{\frac{\mu_0 \mu_r L}{2\pi h \ln\left(\frac{d_o}{d_i}\right)}} \quad (28)$$

Where r specifies a radius from the center of the core ($\frac{d_i}{2} < r < \frac{d_o}{2}$).

$$
\begin{aligned}
P_0(d_i) &= \int_{\frac{d_i}{2}}^{\frac{d_o}{2}} P_V \, \mathrm{d}V = \int_{\frac{d_i}{2}}^{\frac{d_o}{2}} K B_{pk}(r)^\beta 2\pi r h \, \mathrm{d}r \\
&= 2\pi h K \left[I_{pk}\sqrt{\frac{\mu_0 \mu_r L}{2\pi h \ln\left(\frac{d_o}{d_i}\right)}} \right]^\beta \int_{\frac{d_i}{2}}^{\frac{d_o}{2}} r^{1-\beta} \, \mathrm{d}r \quad (29)
\end{aligned}
$$

If $\beta \neq 2$,

$$
P_0(d_i) = \frac{2\pi h K}{2 - \beta}\left[I_{pk}\sqrt{\frac{\mu_0 \mu_r L}{2\pi h \ln\left(\frac{d_o}{d_i}\right)}} \right]^\beta \left[\left(\frac{d_o}{2}\right)^{2-\beta} - \left(\frac{d_i}{2}\right)^{2-\beta} \right] \quad (30)
$$

If $\beta = 2$,

$$P_0(d_i) = \mu_0 \mu_r K L I_{pk}^2 \quad (31)$$

From (30) and (31), let $d_i = 0.5d_o$ and we normalize the total core loss $P_0(d_i)$ by the total loss P_0 at $d_i = 0.5d_o$. If $\beta \neq 2$,

$$\frac{P_0(d_i)}{P_0(0.5d_o)} = \left[\frac{\ln 2}{\ln\left(\frac{d_o}{d_i}\right)} \right]^{0.5\beta} \left[\frac{1 - \left(\frac{d_i}{d_o}\right)^{2-\beta}}{1 - 0.5^{2-\beta}} \right] \quad (32)$$
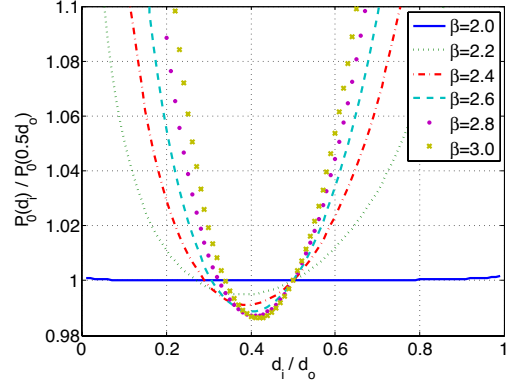

Fig. 10. Plot of core power loss dissipation in a rectangular cross-section toroidal core as a function of $\frac{d_i}{d_o}$, normalized to that with $\frac{d_i}{d_o} = 0.5$. Results are parameterized in Steinmetz parameter β. It can be seen that over a wide range of β, $\frac{d_i}{d_o} = 0.5$ is very close to the optimum, and that results are not highly sensitive to $\frac{d_o}{d_i}$.

If $\beta = 2$,

$$\frac{P_0(d_i)}{P_0(0.5d_o)} = 1 \quad (33)$$

In (32), $\frac{P_0(d_i)}{P_0(0.5d_o)}$ only depends on the ratio of $\frac{d_o}{d_i}$ and Steinmetz parameter β. We plot P_0 as a function of $\frac{d_o}{d_i}$ for different β in Fig. 10. From Fig. 10, we can see that the optimum d_i is around $0.4d_o$, with an exact value that depends β. When d_i varies between $0.22d_o$ and $0.64d_o$, the total core loss P is very flat and the deviation from the minimum core loss is less than 10%. We choose $d_i = 0.5d_o$ instead of $d_i = 0.4d_o$ for the following considerations: firstly, $d_i = 0.5d_o$ is a more typical dimension ratio for commercial magnetic cores (e.g., see Table II); secondly, as shown in [5], the error due to the assumption of average flux density is less than 10% if $d_i \leq 0.5d_o$. The error in assuming that the optimum inside diameter is $d_i = 0.5d_o$ is lower than 2% for a wide range of β values. So we can think $d_i = 0.5d_o$ as the nearly-optimum dimension for a wide range of magnetic materials. We can thus compare and evaluate different magnetic materials under the same dimensions and our assumption in Section III is correct. In Section III-D, we also use the same assumption to investigate (9), (11) and (17).

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