2014 International Microwave Symposium, June 2014 (to appear).

Transmission Line Resistance Compression Networks for Microwave Rectifiers

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***Abstract*—This work presents a development of multi-way transmission-line resistance compression networks (TLRCNs) and their application to rf-to-dc conversion. We derive analytical expressions for the behavior of TLRCNs, and describe two design methodologies applicable to both single- and multi-stage implementations. A 2.45-GHz 4-way TLRCN network is implemented and applied to create a resistance-compressed rectifier system that has narrow-range resistive input characteristics over a 10-dB power range. It is demonstrated to improve the impedance match to mostly-resistive but variable input impedance class-E rectifiers over a 10-dB power range. The resulting TLRCN plus rectifier system has >50% rf-to-dc conversion efficiency over a >10-dB input power range at 2.45 GHz (peak efficiency 70%), and SWR <1.1 over a 7.7-dB range.**

***Index Terms*—transmission lines, impedance matching, matching networks, resonant rectifiers, rectennas, wireless power transfer**

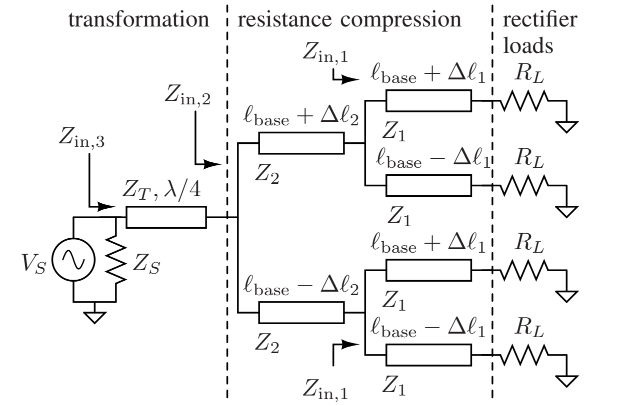

Fig. 1. Four-way TLRCN with quarter-wave impedance transformation stage. Each pair of transmission line lengths can be expressed in terms of a base line length plus and minus a delta length. The rectifiers are represented here as resistors with equal (but varying) value R_L.

I. Introduction

In numerous applications it is desirable to implement microwave rectification to capture rf energy and convert it to dc power. Such applications include energy recovery from terminations such as isolation ports [1], dc-dc conversion [2], and wireless power transfer [3]. In many such applications, it is desirable for the rf impedance at the rectifier input to appear constant and resistive across power level (e.g., such that impedance matching or isolation can be maintained, reflected power can be minimized, etc.). In this paper, we focus on a means for maintaining near-constant input impedance in microwave-to-dc rectifier systems. This can be a challenge because at different incident power levels, the input impedance to an rf rectifier typically varies [1], [2], [4].

We explore the use of resistance compression networks (RCNs) to interface between an rf input and a plurality of rectifiers, minimizing the effective input impedance variation and acting as an impedance transformation stage. RCNs provide reduced impedance variation at the rf input (as compared to the rectifier inputs), and in discrete implementations have been applied to dc-dc resonant converters [4] and in isolation-port energy recovery in outphasing systems [1]. At microwave frequencies, however, such discrete RCN networks have limitations relating to the component non-idealities and interconnect parasitics. Recently, initial work has explored the use of transmission-line resistance compression networks (TLRCNs) which are based on transmission-line sections and are suitable for microwave frequencies. A two-way TLRCN based on a pair of transmission-line sections (asymmetric about a 90-degree base length) was proposed in [5], [6] for energy recovery in outphasing power amplifiers.

We present here both theory and design methodologies for TLRCNs and their application to microwave rectification. Balanced splitting of the input power to multiple loads along with small variation in driving-point impedance is achieved through a network consisting only of transmission line segments. We first show that additional base lengths (in addition to 90 degrees) can be employed for TLRCNs, enabling more flexible system design while minimizing losses. Second, we show that as with the related discrete-component resistance compression networks of [4], [7], multi-stage (or "multi-way") TLRCNs can be realized that achieve smaller input resistance variations than single-stage designs. In addition to providing improved input impedance characteristics, an advantage of such multi-way TLRCNs is that they distribute the input power equally among several rectifiers, which is particularly advantageous at frequencies where power-handling of rectifiers is limited and multiple devices must be used. Furthermore, we introduce two design methodologies for TLRCNs. These methods enable one to select the transmission line base lengths and characteristic impedances for two-way and multi-way designs as in Fig. 1.

II. Transmission Line RCN

A. Theoretical Development

Fig. 2 shows the basic single-stage TLRCN network section, loaded with identical resistive loads R_L, having an input port with driving point impedance Z_{in} that one desires to maintain

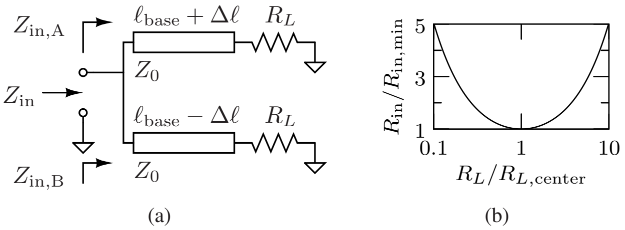

Fig. 2. Basic single-stage TLRCN and resistance compression behavior. (a) — topology, (b) — normalized input resistance vs. normalized load resistance for $Z_0 = R_{L,\text{center}}$, $\ell_{\text{base}} = \lambda/4$, and $\Delta\ell = \lambda/8$.

near a specified value. The transmission line branches have lengths that can be expressed in terms of a base length plus or minus a delta length (or a base angle plus/minus a delta angle):

$$\begin{aligned} \ell_1 &= \ell_{\text{base}} + \Delta\ell, \quad \ell_2 = \ell_{\text{base}} - \Delta\ell \\ \theta_1 &= \theta_{\text{base}} + \Delta\theta, \quad \theta_2 = \theta_{\text{base}} - \Delta\theta \end{aligned} \tag{1}$$

While there are multiple possibilities for base lengths, we first consider a base length ℓ_{base} of $\lambda/4$ (a quarter wavelength at the operating frequency). In this case, the branch input admittances at the operating frequency are:

$$\begin{aligned} Y_{\text{in,A}} &= \frac{1}{Z_0}\frac{Z_0 - jR_L\cot(\Delta\theta)}{R_L - jZ_0\cot(\Delta\theta)} \\ Y_{\text{in,B}} &= \frac{1}{Z_0}\frac{Z_0 + jR_L\cot(\Delta\theta)}{R_L + jZ_0\cot(\Delta\theta)} \end{aligned} \tag{2}$$

Because these admittances are complex conjugates, for identical load resistances the network will divide power entering the input port equally to both loads. In this case the input impedance seen at the input port at the operating frequency is resistive, and can be shown to be:

$$Z_{\text{in},\frac{\lambda}{4}} = \frac{|\cot(\Delta\theta)|}{2\left(1+\cot^2(\Delta\theta)\right)}\left(\frac{R_L}{|\cot(\Delta\theta)|} + \frac{Z_0^2}{\frac{R_L}{|\cot(\Delta\theta)|}}\right) \tag{3}$$

Alternatively, a base length of $\lambda/2$ can be chosen, in which case the input impedance of the single-stage TLRCN is:

$$Z_{\text{in},\frac{\lambda}{2}} = \frac{|\tan(\Delta\theta)|}{2\left(1+\tan^2(\Delta\theta)\right)}\left(\frac{R_L}{|\tan(\Delta\theta)|} + \frac{Z_0^2}{\frac{R_L}{|\tan(\Delta\theta)|}}\right) \tag{4}$$

Although a greater base length leads to longer total conduction paths in the combiner (and hence higher losses), a base length of $\lambda/2$ may be preferable in cases where the shorter ($\ell_{\text{base}} - \Delta\ell$) transmission line segment becomes impractically small. Furthermore, operation will be the same with any additional multiple of $\lambda/2$ added to the base length, and this degree of freedom may be useful for controlling the input impedance at one or more harmonic frequencies. In the experimental demonstrations in this work, we use a $\lambda/4$ base length.

The characteristics in (3) and (4) clearly realize resistance compression, as the input impedance varies resistively over a small range when the load resistances vary together over a wide range. When a base length of $\lambda/4$ is chosen, for example, the network provides "balanced" compression for a range of load resistances having a geometric mean of $R_{L,\text{center}}$:

$$R_{L,\text{center}} = Z_0 \cdot |\cot(\Delta\theta)| \tag{5}$$

At this load resistance value $R_{L,\text{center}}$, the input resistance has minimum value $R_{\text{in,min}}$,

$$R_{\text{in,min}} = Z_0\frac{|cot(\Delta\theta)|}{1+\cot^2(\Delta\theta)} \tag{6}$$

and the ratio of the maximum to minimum input resistance is:

$$\frac{R_{\text{in,max}}}{R_{\text{in,min}}} = \frac{1}{2}\left(\sqrt{\frac{R_{L,\text{max}}}{R_{L,\text{min}}}} + \sqrt{\frac{R_{L,\text{min}}}{R_{L,\text{max}}}}\right) \tag{7}$$

For example, a 10:1 range of load resistance is compressed by a single-stage TLRCN to a 1.74:1 ratio in input resistance, and a 100:1 range in load resistance is compressed to only a 5.05:1 ratio of input resistance (Fig. 2b).

Other base lengths likewise result in equal power transfer to the two loads and resistance compression. For example, when a base length of $\ell_{\text{base}} = \lambda/2$ ($\theta_{\text{base}} = \pi$ radians) is chosen, the input resistance characteristics are the same as in (5)–(7) but with $\cot(\Delta\theta)$ replaced with $-\tan(\Delta\theta)$.

B. Design Procedure

In this section we present two approaches to TLRCN design, each providing particular benefits. In the first approach, a base length (here, $\lambda/4$) is chosen, and the delta length is chosen a priori to be $\Delta\ell = \lambda/8$ ($\Delta\theta = \pi/4$ radians). The characteristic impedance Z_0 is chosen equal to the geometric mean of the maximum and minimum load resistances of interest, $R_{L,\text{center}}$. As a result of this choice, the load resistances R_L vary geometrically about the transmission line characteristic impedance, which helps reduce the transmission line reflection and loss. In cases where the resulting input resistance (range given by (6), (7)) is not at the desired value, an additional impedance transformation stage may be placed at the input of the TLRCN, for example by including an additional quarter-wave line at the input to the compression stage as in Fig 1.

The second design approach directly realizes both resistance compression and a desired specified input resistance using the topology in Fig. 2 by choosing both the differential length and the characteristic impedance of the transmission lines. As in the first approach, the center resistance $R_{L,\text{center}}$ is selected as the geometric mean of the maximum and minimum load resistances of interest, $R_{L,\text{max}}$ and $R_{L,\text{min}}$. One then defines a minimum input resistance $R_{\text{in,min}}$ to be realized by the resistance compression network. This minimum input resistance is selected based on the desired resistance $R_{\text{in,desired}}$ seen at the input of the TLRCN. For example, one may choose $R_{\text{in,min}} = R_{\text{in,desired}}$ (recognizing that the actual input resistance will be equal or greater than this desired value). Alternatively, one may choose the median value of the input resistance that occurs over the expected range of load resistances to match the desired input resistance. In this case,

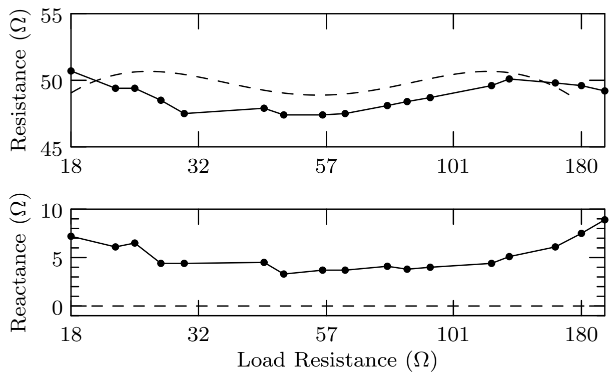

Fig. 3. Theoretical (dashed) and measured (solid) input resistance and reactance to the two-stage TLRCN when it is terminated with resistive loads. The phase of the input impedance remains within about 4 degrees over the entire 18–170 Ω operating range.

$R_{\mathrm{in,min}}$ is selected as:

$$R_{\mathrm{in,min}} = \frac{2 \cdot R_{\mathrm{in,desired}}}{1 + \frac{1}{2}\sqrt{\frac{R_{L,\max}}{R_{L,\min}}} + \frac{1}{2}\sqrt{\frac{R_{L,\min}}{R_{L,\max}}}} \tag{8}$$

Based on selecting $R_{L,\mathrm{center}}$ and $R_{\mathrm{in,min}}$, the characteristic impedance Z_0 and value for $\Delta\theta$ can be directly chosen as follows (from (6) and (5), and for $\ell_{\mathrm{base}} = \lambda/4$):

$$\cot(\Delta\theta) = \sqrt{\frac{R_{L,\mathrm{center}}}{R_{\mathrm{in,min}}} - 1} \tag{9}$$

$$Z_0 = \frac{R_{L,\mathrm{center}}}{\sqrt{\frac{R_{L,\mathrm{center}}}{R_{\mathrm{in,min}}} - 1}} \tag{10}$$

An advantage of this design approach as compared to the first one is that, within practical limits, it may not require an additional impedance transformation stage and thus results in a more compact structure. However, as $R_{\mathrm{in,min}}$ approaches $R_{L,\mathrm{center}}$, the characteristic impedance of the transmission lines can grow impractically large.

Multi-way TLRCNs can be synthesized by cascading single-level TLRCN networks in a tree structure (as seen in Fig. 1). If the first design methodology is chosen (in which $\Delta\theta$ is chosen a priori to be $\pi/4$), then the multi-stage TLRCN can be optimized in terms of minimizing the peak deviation from a median input resistance using a technique akin to that described in [7]. This technique, with $\ell_{\mathrm{base}} = \lambda/4$ and with an impedance transformation stage at the input to the TLRCN, was used in the example design in this work. Alternatively, both the characteristic impedances and differential lengths of each stage can be optimized as in the second design approach.

III. Implementation and Measurements

A. TLRCN

For our example implementation for operation at 2.45 GHz, we assume that the resistive loads will vary over a 18-170 Ω range, corresponding to four tuned rectifiers acting as variable and approximately resistive loads. We choose a base transmission line length $\ell_{\mathrm{base}} = \lambda/4$ and $\Delta\ell = \lambda/8$, and use the four-way TLRCN with quarter-wave impedance transformation stage shown in Fig. 1. In the first compression stage, we choose $Z_1 = \sqrt{18 \cdot 170} = 55\ \Omega$, so that this stage compresses the resistances presented by the loads into a range of $27.5\ \Omega \leq Z_{\mathrm{in},1} \leq 46.7\ \Omega$. This resistance range acts as the load for the second stage of compression. Again following the first design approach from Section II-B, the characteristic impedance for the second stage is chosen as $Z_2 = \sqrt{27.5 \cdot 46.7} = 36\ \Omega$ (and $\Delta\ell = \lambda/8$). The compressed input resistance $Z_{\mathrm{in},3}$ (see Fig. 1) varies over a 18–18.55 Ω range. The transformation stage, comprising a quarter-wave line with characteristic impedance $Z_T = 30.2\ \Omega$, transforms this impedance into the desired overall input impedance of approximately 50 Ω. The calculated input impedance when R_L varies over a 18–170 Ω range is shown in Fig. 3.

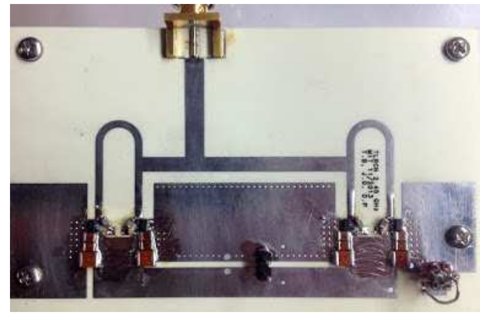

Fig. 4. Photograph of the prototype 2.45-GHz TLRCN and rectifiers.

The four-way TLRCN, shown in Fig. 4 (with rectifiers), was fabricated on a 30-mil thick RO4350 substrate and characterized by resistively terminating its four load ports over the range 18-200 Ω. The measured input impedance is shown in Fig. 3. The resistance compression effect is evident — when the load is varied over a 11:1 range, the input resistance varies over only a 1.07:1 range.

B. Class-E Resonant Rectifier

Class-E rectifiers can be designed to have near-resistive input impedance over a wide range of power levels, and are therefore useful for this application [8]. The 2.45 GHz rectifier design, including trim components used in the prototype system to counteract the parasitic effects of layout and passive components, is shown in Fig. 6, with component values given in Table I. The inductor $L_r = 2.3$ nH is chosen to resonate with the Avago HBAT540B diode's parasitic shunt capacitance, C_D, following the methodology in [8].

The rectifier design was first characterized individually by using a through power meter to record total input power from the driving power amplifier, and the SWR resulting from power-level-dependent impedance mismatch at the rectifier input. The trim inductor $L_{\mathrm{trim}} = 2.7$ nH was experimentally selected to minimize SWR when the input power was at the value expected to produce a 50-Ω input resistance. Due to a limited range of available inductor values (and physical layout

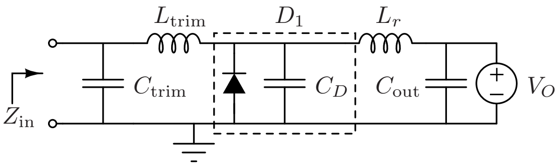

Fig. 5. Schematic of the resonant class-E rectifier used as a load for the TLRCN demonstration.

TABLE I
2.45 GHz Class-E rectifier component values

Component	Value	Mfr. Part #	Mfr.
D_1	–	HBAT540B	Avago
L_r	2.2 nH	0402HP-2N2X	CoilCraft
L_{trim}	2.7 nH	0402HP-2N7X	
C_{trim}	0.7 pF	600F 0R7JT	ATC
	27 pF ||	600L 270JT	
C_{out}	10 pF ||	MC12FA241J-F	CDE
	100 pF	MC08CA100D-F	

constraints), a shunt capacitor $C_{\text{trim}} = 0.7$ pF was included and found experimentally to minimize SWR at 27 dBm input power. This LC network thus trims out the capacitive component of the input impedance owing to the rectifier's inherent input characteristics and effects of the inductive via connections to ground. The resulting efficiency and SWR measurements of a single rectifier when $V_O = 9$ V are shown in Fig. 6 (dashed lines). Despite the trimming of input reactance, the rectifier has higher SWR than expected due to resistive load mismatch alone, and thus must have some input reactance.

C. System Measurements

In order to demonstrate the benefits of this approach in a rectenna the TLRCN was terminated with four rectifiers as shown in Fig. 4 and characterized in terms of efficiency and SWR over a 0.26 W to 4.16 W range. For this proof-of-concept characterization, the input was driven directly by an RF source (without antenna). Efficiency is therefore characterized including mismatch loss and rectifier efficiency, but does not include any characterization of antenna efficiency.

Measured results are shown on an input power per rectifier basis in Fig. 6 (solid lines) in order to facilitate a direct comparison. Including the TLRCN between the rectifier loads and the rf source dramatically improves the impedance match over a wide input power range, thus improving both overall efficiency and dynamic range. Despite the high variation in rectifier load impedance, the input SWR remains under 1.4 over the entire 16 dB input power range, and under 1.1 over the upper 7.7 dB of the measurement. The system has rf-to-dc efficiency above 50% over a 10.1-dB range of input powers, with a peak efficiency of 70%. Furthermore, losses associated with including the TLRCN network between the rf source and rectifiers are offset by the improved impedance match, so that efficiency is not degraded at incident power levels where the unmatched rectifier impedance is closest to 50

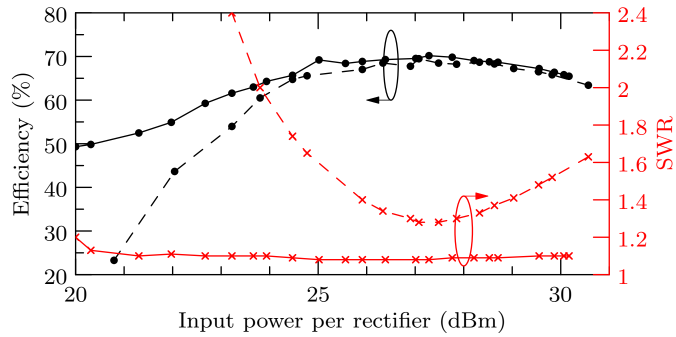

Fig. 6. Efficiency of a single rectifier (dashed), and four rectifiers plus the TLRCN (solid), including losses due to impedance mismatch. The measured SWR is also shown. Including the TLRCN substantially improves the impedance match over the entire input power range.

Ω. This demonstrates the excellent rf-to-dc performance that is achievable with TLRCNs, in terms of efficiency, maintaining a desired load match across a wide power range, and simplicity of implementation in microstrip form.

IV. Conclusion

This paper introduces the use of multi-stage TLRCNs for rf rectification. The approach has several advantages over a matching network designed for a single load impedance, including minimizing the effective input impedance variation across power level and distributing the input power among multiple devices. We introduce two design methods for the selection of TLRCN base length and transmission line characteristic impedance to achieve a desired input resistance. We demonstrate this technique with a design example for resistive load variation over a 18–170 Ω range and operating at 2.45 GHz. When loaded with this range of real impedances at its four load ports, this network achieves an input resistance variation of only 1.07:1. Combined with class-E resonant rectifiers designed to present a theoretically near-resistive range of input impedances, the system has SWR < 1.1 over 7.7-dB power range and < 1.4 over a 16-dB power range. The demonstrated 4-W, 2.45-GHz TLRCN-based rectifier system achieves a peak rf-to-dc energy capture efficiency of 70%, with an efficiency of $> 50\%$ maintained for more than a 10-dB power range.

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