Quantum theory for two-photon-state generation by means of four-wave mixing in optical fiber

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ABSTRACT
We present a quantum theory for generating two-photon states by means of four-wave mixing in optical fiber. We start with an interaction Hamiltonian that can correctly describe all nonlinear interactions among the four waves present in the fiber, namely, the frequency non-degenerate pumps, signal, and idler, including the terms responsible for self-phase modulation (SPM), cross-phase modulation (XPM), and four-photon scattering (FPS). The exact form of this Hamiltonian is obtained through comparison between the classical and quantum equations of motion. The two-photon state is then calculated by means of first-order perturbation theory. It turns out that only the FPS and the pump SPM terms contribute to the formation of the two-photon state. The entangled nature of this state is verified in a coincidence-counting experiment. The results of the theoretical calculation agree well with experimental data.

Keywords: Two-photon state, Quantum entanglement, Four-wave mixing

1. INTRODUCTION
Recent years have seen rapid advances in the field of quantum information and quantum computation. En-...
2. THE INTERACTION HAMILTONIAN

In this section, we derive the Hamiltonian of the FWM process in the interaction picture. In the case of CW pumping, a degenerate-pump interaction Hamiltonian is often employed. However, this approach is not valid when dealing with a pump field that is broadband in the frequency domain. Two photons from the pump field that participate in the FWM process may be of the same frequency or of two distinct frequencies. Instead, a more general form of the interaction Hamiltonian is needed, which can describe the interplay between and the evolution of four distinct optical waves. The so-called non-degenerate-pump interaction Hamiltonian fulfills this task. The four optical waves under consideration are: two pump fields at frequencies \( \omega_{p_1} \) and \( \omega_{p_2} \), signal field at frequency \( \omega_s \), and idler field at frequency \( \omega_i \). In general, there should be 12 terms that phase match, as described in the following expression:

\[
H_I = \epsilon_0 \chi^{(3)} \int_V dV \left[ \alpha_1 E_{p_1}^{(-)} E_{p_1}^{(+)} E_{p_1}^{(+)} + \alpha_2 E_{p_2}^{(-)} E_{p_2}^{(+)} E_{p_2}^{(+)} + \alpha_3 E_s^{(-)} E_s^{(+)} E_s^{(+)} 
+ \alpha_4 E_{p_1}^{(-)} E_{i}^{(+)} E_{i}^{(+)} + \alpha_5 E_s^{(-)} E_s^{(+)} E_s^{(+)} + \alpha_6 E_{p_2}^{(-)} E_{i}^{(+)} E_{i}^{(+)} 
+ \alpha_6 E_{p_1}^{(-)} E_{p_2}^{(-)} E_{i}^{(+)} + \alpha_7 E_{p_1}^{(-)} E_{s}^{(+)} E_{s}^{(+)} + \alpha_8 E_{p_1}^{(-)} E_{i}^{(+)} E_{i}^{(+)} 
+ \alpha_9 E_{p_2}^{(-)} E_{p_2}^{(-)} E_{s}^{(+)} + \alpha_{10} E_{p_2}^{(-)} E_{p_2}^{(-)} E_{s}^{(+)} E_{i}^{(+)} + \alpha_{11} E_{s}^{(-)} E_{s}^{(+)} E_{i}^{(+)} E_{i}^{(+)} \right],
\]

where \( \epsilon_0 \) is the vacuum permittivity, and \( \chi^{(3)} \) is the nonlinear electric susceptibility whose tensorial nature is ignored since all the optical fields are assumed to be linearly co-polarized. The cross-polarized-FWM scattering amplitude is neglected here due to its smallness, and can be included in a straightforward manner if desired. The integral is taken over the entire volume of interaction, namely, the effective volume of the optical fiber. Here we use the convention that the positive-frequency electric-field operators stand for photon annihilation operators, and the negative-frequency electric-field operators stand for photon creation operators. For simplicity, we assume that all the operators in Eq. (1) involve only single frequencies; this assumption will later be relaxed to include multi-frequency fields. The relation between the two sets of operators are: \( E_j^{(+)} = \sqrt{\frac{\hbar \omega_j}{2 \epsilon_0 V_Q}} \hat{a}_j \), where \( j = p_1, p_2, s, i \), and \( V_Q \) is the quantization volume. The above Hamiltonian takes into account all of the possible interactions among the four fields, namely, self-phase modulation (SPM) for each individual field, cross-phase modulation (XPM) between every pair of fields, and four-photon scattering (FPS) among all the four fields.

Simple dimensional analysis shows that the \( \alpha \)'s in Eq. (1) are real, dimensionless quantities describing the interaction strengths between the various optical waves. In what follows, we use a direct comparison between the quantum and classical equations of motion to obtain the ratios of these \( \alpha \)'s.

The quantum equations of motion are obtained when the Heisenberg equation of motion is applied to the FWM process, namely, \( i\hbar \frac{\partial \hat{E}}{\partial t} = [\hat{E}, H_I] \), where \( \hat{E} \) stands for any electric-field operator that appears in the Hamiltonian \( H_I \). In total, there are 4 equations for the positive-frequency operators, and 4 conjugate equations for the negative-frequency operators. We shall only list the first 4 equations for simplicity:

\[
\frac{\partial E_{p_1}^{(+)}(z)}{\partial z} = i \left[ 2\alpha_1 E_{p_1}^{(-)} E_{p_1}^{(+)} E_{p_1}^{(+)} + \alpha_5 E_{p_2}^{(-)} E_s^{(+)} E_i^{(+)} + \alpha_6 E_{p_2}^{(-)} E_{p_2}^{(+)} E_{p_1}^{(+)} 
+ \alpha_7 E_s^{(-)} E_s^{(+)} E_{p_1}^{(+)} + \alpha_8 E_i^{(-)} E_i^{(+)} E_{p_1}^{(+)} \right],
\]

\[
\frac{\partial E_{p_2}^{(+)}(z)}{\partial z} = i \left[ 2\alpha_2 E_{p_2}^{(-)} E_{p_2}^{(+)} E_{p_2}^{(+)} + \alpha_5 E_{p_1}^{(-)} E_s^{(+)} E_i^{(+)} + \alpha_6 E_{p_1}^{(-)} E_{p_1}^{(+)} E_{p_2}^{(+)} 
+ \alpha_9 E_s^{(-)} E_s^{(+)} E_{p_2}^{(+)} + \alpha_{10} E_i^{(-)} E_i^{(+)} E_{p_2}^{(+)} \right],
\]

\[
\frac{\partial E_s^{(+)}(z)}{\partial z} = i \left[ 2\alpha_3 E_s^{(-)} E_s^{(+)} E_s^{(+)} + \alpha_5 E_{p_1}^{(-)} E_{p_1}^{(+)} E_s^{(+)} + \alpha_7 E_{p_1}^{(-)} E_{p_1}^{(+)} E_i^{(+)} 
+ \alpha_9 E_{p_2}^{(-)} E_{p_2}^{(+)} E_s^{(+)} + \alpha_{11} E_i^{(-)} E_i^{(+)} E_s^{(+)} \right],
\]

\[
\frac{\partial E_i^{(+)}(z)}{\partial z} = i \left[ 2\alpha_4 E_i^{(-)} E_i^{(+)} E_i^{(+)} + \alpha_5 E_{p_2}^{(-)} E_{p_2}^{(+)} E_i^{(+)} + \alpha_7 E_{p_2}^{(-)} E_{p_2}^{(+)} E_s^{(+)} 
+ \alpha_9 E_{p_1}^{(-)} E_{p_1}^{(+)} E_i^{(+)} + \alpha_{10} E_s^{(-)} E_s^{(+)} E_i^{(+)} \right],
\]
\[ \frac{\partial E_i^{(+)}(z)}{\partial z} = i\eta \left[ 2\alpha_4 E_i^{(-)} E_1^{(+)} E_i^{(+)} + \alpha_5 E_i^{(-)} E_{p_1}^{(+)} E_{p_2}^{(+)} + \alpha_8 E_{p_1}^{(-)} E_{p_2}^{(+)} E_i^{(+)} + \alpha_{10} E_{p_2}^{(-)} E_{p_2}^{(+)} E_i^{(+)} + \alpha_{11} E_{s}^{(-)} E_{s}^{(+)} E_i^{(+)} \right] , \]  

(2)

where we have used the following identities:

\[ \frac{\partial}{\partial t} = i_v \frac{\partial}{\partial z} , \]

(3)

\[ \left[ E_i^{(+)}(z), E_m^{(-)}(z') \right] = \frac{i\hbar \omega}{2\epsilon_0 V_Q} \delta(z - z') \delta_{lm} , \]

(4)

\[ \int_V dV \rightarrow A_{\text{eff}} \int dz , \]

(5)

\[ \eta = -\frac{\chi^{(3)} A_{\text{eff}} \lambda \omega}{2V_Q v} . \]

(6)

Here \( A_{\text{eff}} \) is the effective transverse area of the propagating pump mode in the optical fiber, \( L \) is the length of the fiber, and \( v = \frac{\omega}{n} \) is the group speed of light in the fiber.

The classical equations of motion for non-degenerate FWM are derived in textbooks (see, e.g., Agrawal\textsuperscript{10}) and are given as follows:

\[ \frac{\partial A_{p_1}}{\partial z} = \iota \gamma \left[ (|A_{p_1}|^2 + 2|A_{p_1}|^2 + 2|A_i|^2) A_{p_1} + 2A_{p_2} A_s A_t e^{i\Delta k z} \right] , \]

\[ \frac{\partial A_{p_2}}{\partial z} = \iota \gamma \left[ (|A_{p_2}|^2 + 2|A_{p_2}|^2 + 2|A_i|^2) A_{p_2} + 2A_{p_1} A_s A_t e^{i\Delta k z} \right] , \]

\[ \frac{\partial A_s}{\partial z} = \iota \gamma \left[ (|A_s|^2 + 2|A_{p_1}|^2 + 2|A_{p_2}|^2 + 2|A_i|^2) A_s + 2A_{p_1} A_{p_2} A_s e^{-i\Delta k z} \right] , \]

\[ \frac{\partial A_i}{\partial z} = \iota \gamma \left[ (|A_i|^2 + 2|A_{p_1}|^2 + 2|A_{p_2}|^2 + 2|A_s|^2) A_i + 2A_{p_1} A_{p_2} A_s e^{-i\Delta k z} \right] \]

(7)

where the \( A \)'s are classical electric-field amplitudes of the interacting waves, \( \Delta k = k_s + k_i - k_{p_1} - k_{p_2} \) is the wave-vector mismatch that vanishes for perfect phase matching, \( \gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}} \) is the nonlinear parameter of interaction, and \( n_2 \) is the nonlinear-index coefficient related to \( \chi^{(3)} \) via the relation \( n_2 = \frac{3}{8\pi} \text{Re}(\chi^{(3)}_{xxx}) \). Here, we also use the fact that \( |\lambda_{p_1} - \lambda_i| \approx |\lambda_i - \lambda_s| \ll \lambda_i \) in the FWM process; therefore, we can ignore the wavelength differences between the fields and denote them as a single wavelength \( \lambda \).

Exact term-by-term comparison between Eqs. (2) and Eqs. (7) is not possible in the current context, as there is a dimensional mismatch between them (notice the operators \( E \) are of dimension \( V/m \), and amplitudes \( A \) are of dimension \( \sqrt{W} \)). However, it is sufficient to know just the ratios among the seemingly independent \( \alpha \)'s, up to an overall constant. Since there is a one-to-one correspondence between these two sets of equations, the ratios among the coefficients in Eqs. (7) will determine the ratios among the coefficients in Eqs. (2). The result is \( \alpha_{i \geq 5} = 4\alpha_{i \leq 4} \equiv 4\beta, \beta \) being the unknown overall constant. We then have the following form for our Hamiltonian:

\[ H_I = \beta \epsilon_0 \chi^{(3)} \int_V dV \left[ E_{p_1}^{(-)} E_{p_1}^{(-)} E_{p_1}^{(+)} E_{p_1}^{(+)} + E_{p_2}^{(-)} E_{p_2}^{(-)} E_{p_2}^{(+)} E_{p_2}^{(+)} + E_s^{(-)} E_s^{(-)} E_s^{(+)} E_s^{(+)} + E_i^{(-)} E_i^{(-)} E_i^{(+)} E_i^{(+)} \right] \]

\[ + 4 \left( E_{s}^{(-)} E_i^{(-)} E_{p_1}^{(+)} E_{p_2}^{(+)}, E_{p_1}^{(-)} E_{p_2}^{(-)} E_s^{(+)}, E_{p_2}^{(-)} E_{p_1}^{(-)} E_s^{(+)}, E_{s}^{(-)} E_i^{(-)} E_{p_1}^{(+)}, E_{s}^{(-)} E_i^{(-)} E_{p_2}^{(+)}, E_{p_2}^{(-)} E_{p_1}^{(-)} E_s^{(+)}, E_{s}^{(-)} E_i^{(-)} E_{p_1}^{(+)}, E_{s}^{(-)} E_i^{(-)} E_{p_2}^{(+)}, \right) , \]

(8)
3. THE TWO-PHOTON STATE

In this section, we study the generation of the two-photon state. We start off by specifying the various fields that participate in the FWM process. The pump field is, as we assumed earlier, linearly polarized and propagating in the z direction (parallel with the fiber axis) with a central frequency $\Omega_p$ and an envelope of arbitrary shape $\tilde{E}_p$. Mathematically, it can be written as

$$E_p^{(+)} = e^{-i\Omega_p t} \tilde{E}_p(z, t) = e^{-i\Omega_p t} \int d\nu_p \tilde{E}_p(\nu_p) e^{i k_p z - i\nu_p t}, \quad (9)$$

wherein the bandwidth of the pump field is much smaller than $\Omega_p$, satisfying the quasi-monochromatic approximation. The signal and idler fields are quantized electromagnetic fields, co-polarized and co-propagating with the pump, as given by the following multimode expansion:

$$E_s^{(-)} = \sum_{\omega_s} e^{i \omega_s} a_{k_s}^\dagger e^{-i[k_s(\omega_s)z - \omega_st]} , \quad (10)$$

$$E_i^{(-)} = \sum_{\omega_i} e^{i \omega_i} a_{k_i}^\dagger e^{-i[k_i(\omega_i)z - \omega_it]} , \quad (11)$$

where $a_{k_s}^\dagger$ is the annihilation operator for the signal mode with frequency $\omega_s$, $k_s(\omega_s) = n(\omega_s) \omega_s/c$ is its wave-vector magnitude, and $e_{\omega_s} \equiv \sqrt{\frac{n\omega_s}{2\epsilon_0 V_Q}}$. The idler field is defined in an analogous fashion. The central frequencies of the signal and idler fields are individually denoted by $\Omega_s$ and $\Omega_i$, respectively, which are symmetrically displaced from the central frequency of the pump field $\Omega_p$, satisfying the energy conservation relation: $\Omega_s + \Omega_i = 2\Omega_p$.

To simplify our calculation, the pump is further assumed to have a Gaussian spectral envelope and its self-phase modulation (SPM) is represented in a straightforward manner, i.e.,

$$E_p^{(+)} = e^{-i\Omega_p t} e^{-i\gamma P_p z} E_p^0 \int d\nu_p e^{-\frac{\nu_p^2}{2\sigma_p^2}} e^{i k_p z - i\nu_p t}, \quad (12)$$

where $P_p = 2\sqrt{\pi} A_{eff} \epsilon_0 c n \sigma_p^2 E_p^0$, is the peak power of the pump pulse that is treated as a constant under the undepleted pump approximation and $\sigma_p$ is the optical bandwidth of the pump. The Gaussian filter assumption is made in accordance with the experimental fact that all the narrow-band filters employed in our experiments have Gaussian-shaped spectral transmission functions. The appearance of the pump SPM phase factor in Eq. (12) can be attributed to the classical, analytical solution for the pump field by keeping only the most dominant term in the first equation of Eqs. (7), i.e., the pump SPM term. This classical nonlinear phase factor retains its importance in our quantum theory in that it modifies the pump phase along its propagation, which in turn affects the phase of the pump-generated two-photon state that co-propagates.

The two-photon state at the output of the fiber is calculated by means of the first-order perturbation theory, i.e.,

$$|\Psi\rangle = |0\rangle + \frac{1}{i\hbar} \int_{-\infty}^{\infty} H_f(t) dt \ |0\rangle. \quad (13)$$

Retaining of higher-order terms in the perturbation series involves generation of multi-photon states, which will be ignored in our calculation owing to their smallness. We can see that only the FWM term in the interaction Hamiltonian contributes to the formation of the two-photon state. This is because all terms vanish when acting on the vacuum state $|0\rangle$ with the exception of $E_s^{(-)} E_i^{(-)} E_{p1}^{(+)} E_{p2}^{(+)} + H.c.$, which we denote as

$$H_{FWM} \equiv \alpha \epsilon_0 \chi^{(3)} \int_V dV (E_s^{(-)} E_i^{(-)} E_{p1}^{(+)} E_{p2}^{(+)} + H.c.), \quad (14)$$
where $\alpha = 4\beta$, and H.c. stands for Hermitian conjugate. The state vector is then given by

$$|\Psi\rangle = |0\rangle + \frac{1}{i\hbar} \int_{-\infty}^{\infty} H_{\text{FWM}} \, dt \, |0\rangle,$$

which is a superposition of the vacuum and the two-photon state. The following form of the state vector can be obtained after some algebra:

$$|\Psi\rangle = |0\rangle + \sum_{k_s,k_i} F(k_s,k_i) a_{k_s}^\dagger a_{k_i}^\dagger |0\rangle,$$

where

$$F(k_s,k_i) = g \int_{-L}^{0} \frac{1}{1-ik''(\Omega_p)\sigma_p^2 z} \exp \left\{ -\frac{ik''(\Omega_p)z}{4} (\nu_s - \nu_i + \Delta)^2 - 2i\gamma P_p z - \frac{(\nu_s + \nu_i)^2}{4\sigma_p^2} \right\},$$

$$g = \frac{\alpha \pi^2 \chi^{(3)} P_p}{i \epsilon_0 V Q n^3 \lambda_p \sigma_p}.$$

Here, $k''(\Omega_p) = \frac{d^2 k}{d\omega^2}|_{\omega=\Omega_p}$ is the second-order dispersion at the pump central frequency, which can be obtained from $k''(\Omega_p) = -\frac{\lambda_p^2}{2\pi c} D_{\text{slope}} (\lambda_p - \lambda_0)$, where $\lambda_0$ is the zero-dispersion wavelength of the DSF. $D_{\text{slope}} = 0.06 \text{ ps/(nm}^2 \cdot \text{km)}$ is the experimental value of the dispersion slope in the vicinity of $\lambda_0$. $\Delta \equiv \Omega_s - \Omega_i$ is the central frequency difference between the signal and idler fields and $\nu_s$ and $\nu_i$ are related to $\omega_s$ and $\omega_i$, respectively, through the following relation: $\nu_s = \omega_s - \Omega_s, \nu_i = \omega_i - \Omega_i$.

4. SINGLE AND COINCIDENCE COUNTING RATES

We shall proceed by briefly describing the experiments performed in our lab. Interested readers can refer to Refs.4 and 5 for experimental details.

Figure (1) outlines the main experimental setup as far as the scope of this paper is concerned. FPS occurs in a Sagnac-loop of DSF when the phase matching condition is satisfied. Most of the pump photons are reflected back due to the mirror-like property of the Sagnac loop. The generated two-photon state at the output port of
the loop is further separated by means of an effective wavelength-division (de)multiplexer (WDM). The spatially separated signal and idler photons are then detected after passing through filters (B_s, B_i) placed in their own paths. Single-photon counting as well as coincidence-photon counting can be performed during the experiment. The accidental counts arising from the spontaneous Raman scattering and the dark counts from the detectors are independently measured\textsuperscript{7} and their contribution is subtracted. Overall quantum efficiency of detection in both the signal and idler channels are also separately measured. The single-count rates are divided by the respective quantum efficiencies and the coincidence-count rate by their product to arrive at rates at the output of the fiber for comparison with the prediction of our theory.

Single-photon counting rate as well as coincidence-photon counting rate are calculated using the following formulas.\textsuperscript{13}

\[
S_c = \int_0^\infty dT \langle \Psi|E_s^{(-)}E_s^{(+)}|\Psi\rangle, \quad (19)
\]

\[
C_c = \int_0^\infty dT_1 \int_0^\infty dT_2 \langle \Psi|E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}|\Psi\rangle, \quad (20)
\]

where \(S_c\) denotes single-photon counting rate for the signal channel (the idler-channel counting rate can be defined similarly), \(C_c\) denotes coincidence-photon counting rate, and the electric-field operators have been defined in the photon-number unit as follows.\textsuperscript{14}

\[
E_s^{(+)} = \sum_{k_s} \sqrt{\frac{c A_{\text{eff}}}{4VQ}} a_{k_s} e^{-i \omega_s t_s e^{-\frac{\omega_s - \omega_j^0}{2\sigma_0^2}}}, \quad (21)
\]

\[
E_i^{(+)} = \sum_{k_i} \sqrt{\frac{c A_{\text{eff}}}{4VQ}} a_{k_i} e^{-i \omega_i t_i e^{-\frac{\omega_i - \omega_j^0}{2\sigma_0^2}}}, \quad (22)
\]

\[
E_2^{(+)} = \sum_{k_2} \sqrt{\frac{c A_{\text{eff}}}{4VQ}} a_{k_2} e^{-i \omega_i t_2 e^{-\frac{\omega_i - \omega_j^0}{2\sigma_0^2}}}. \quad (23)
\]

It should be noted that in the above definitions Gaussian filters have already been included as \(f(\omega_j) = e^{-\frac{(\omega_j - \omega_j^0)^2}{2\sigma_0^2}}\) for \(j = s, i\), with \(\sigma_0\) as the filter bandwidth.

The detailed calculation, although not complicated, is lengthy, and therefore will be presented elsewhere.\textsuperscript{11} The end results are given below. The single-counts formula reads:

\[
S_c = A_1 \cdot P_p^2 \cdot \frac{\sigma_0}{\sigma_p} \cdot I_{sc}, \quad (24)
\]

\[
A_1 = \frac{\alpha^2 \pi^3 [\chi^{(3)}]^2 A_{\text{eff}}}{8\sqrt{2} \lambda_0^3 V Q c^2 \lambda_p^2 n^3}, \quad (25)
\]

\[
I_{sc} = \int_{-L}^0 dz_1 \int_{-L}^0 dz_2 \exp \left[ -2i \gamma P_p (z_1 - z_2) - \frac{\sigma_0^2}{1 + b^2} + i \frac{r}{2} \arctan(b) + i \frac{r'}{2} \right] \frac{\sqrt{(1 - ik'' \sigma_0^2 z_1)(1 + ik'' \sigma_0^2 z_2)}}{\sqrt{1 + b^2}}, \quad (26)
\]

where \(b = -\frac{k''(z_1 - z_2)^2(\sigma_0^2 + \sigma_p^2)}{2}, \quad c = \frac{\lambda_0^2}{2(2\sigma_0^2 + \sigma_p^2)^2}, \quad r = -\frac{k''(z_1 - z_2)^2 \Delta^2}{4}\). The coincidence-count formula reads:

\[
C_c = A_2 \cdot P_p^2 \cdot \frac{\sigma_0}{\sigma_p \sqrt{\sigma_0^2 + \sigma_p^2}} \cdot I_{cc}, \quad (27)
\]

\[
A_2 = \frac{\alpha^2 \pi^3 [\chi^{(3)}]^2 A_{\text{eff}}^2}{64 \lambda_0^6 V^{8/3} c^2 \lambda_p^2 n^2}, \quad (28)
\]

\[
I_{cc} = \int_{-L}^0 dz_1 \int_{-L}^0 dz_2 \exp \left[ -2i \gamma P_p (z_1 - z_2) - \frac{\sigma_0^2}{1 + b^2} + i \frac{r}{2} \arctan(b') + i \frac{r'}{2} \right] \frac{\sqrt{(1 - ik'' \sigma_0^2 z_1)(1 + ik'' \sigma_0^2 z_2)}}{\sqrt{1 + b'^2}}, \quad (29)
\]

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where \( b' = -\frac{k''(z_1-z_2)\sigma_0^2}{2} \), \( c' = \frac{\Delta^2}{2\sigma_0^2} \), \( r' = -\frac{k''(z_1-z_2)\Delta^2}{4} \). The explicit dependence on parameters like pump power \( P_p \), pump bandwidth \( \sigma_p \), and filter bandwidth \( \sigma_0 \) has been explicitly taken out in Eq. (24) and Eq. (27). The more intricate dependencies on these parameters are described by the double integrals \( I_{sc} \) and \( I_{cc} \), which take into account phase matching, SPM of the pump field, and the Gaussian shapes of the pump and filter spectra.

We have carried out numerical simulations to compare the theory with the experimental data. In Fig.2 we have fitted our theory to two sets of experimental data, where the ratio of \( \sigma_p/\sigma_0 \) is varied. The fitting parameters are \( \alpha = 0.23 \) and \( V_Q = 1.6 \times 10^{-16} \text{m}^3 \). \( k'' \) has also been found to be \( -0.116 \text{ps}^2/\text{km} \), corresponding to the wavelength difference \( \lambda_p - \lambda_0 = 1.52 \text{nm} \), which agrees well with the measured experimental value. Thus, reasonable agreement between the experiment and the theory is obtained. Since the theory successfully matches the experimental data, it follows that our theory can fully describe the nonclassical nature of the two-photon state produced in the experiment of Fig. 1.4

![Graphs showing experimental data and theoretical predictions.](image)

**Figure 2.** Experiment versus theory: squares correspond to the experimental data and curves correspond to predictions of the theory.

## 5. CONCLUSION

In summary, we have provided a detailed discussion on the interaction Hamiltonian of the FWM process in optical fibers and the ensued two-photon quantum state. Single and coincidence photon-counting formulas are also presented. Numerical simulations show good agreement between the theoretical and experimental results. Extension of the current theory will include multi-photon state generation from one pulse,15 and two-photon polarization entanglement.5
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