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LETTER TO THE EDITOR

An ultrabright narrowband source of polarization-entangled photon pairs

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Abstract. The signals and idlers from a pair of matched, doubly resonant optical parametric amplifiers (OPAs) can be combined to yield polarization-entangled vector signal and idler beams. Within any coincidence interval much longer than the OPA cavity lifetime, every signal photon of polarization $\vec{i}$ is accompanied by an idler photon of the conjugate polarization $\vec{i}'$, and vice versa. For periodically poled lithium tantalate OPAs, the pair production rate is estimated to be approximately 1.5×10^6 s^{-1}, within a 30 MHz bandwidth at a 795 nm centre wavelength and a 70 ns duration coincidence interval, using only 0.7 mW of pump power per OPA.

Keywords: Polarization entanglement, optical parametric amplifiers, photon statistics

Superposition and entanglement are the bedrock on which new theoretical paradigms for quantum mechanical information transmission, storage, and processing are being built, see [1] for a recent review. In optical realizations of some of these schemes, e.g., quantum cryptography [2] and quantum teleportation [3], the desired entanglement is in the polarization state of a pair of photons. A type-II phase-matched parametric downconverter can be used to produce such pairs—by selecting two particular emission directions from the signal and idler emission cones—but the resulting low rate of photon-pair generation has hindered proof-of-principle experiments [4]. Recently, Kwiat *et al* [5] have reported a two-crystal type-I downconverter whose generation rate, for photon pairs of full polarization entanglement, was inferred to be 1.5×10^6 s^{-1} over a 5 nm bandwidth at a 702 nm centre wavelength for 150 mW of pump power. Even though this inferred rate is more than 1000 times higher than that of previous type-II downconverter sources [6], it may still be too low for some quantum applications. For example, Lloyd *et al* [7] have proposed a quantum memory, in which the polarization entanglement from a pair of photons can be transferred to—and later read out from—the long-lived hyperfine levels of a pair of ultra-cold rubidium atoms. Each of these laser cooled and trapped atoms is confined (in an ultra-high vacuum chamber with cryogenic walls) within its own high-finesse optical cavity, hence the photon-pair generation rate that matters is the rate for pairs within a cavity linewidth. Taking this linewidth to be 30 MHz, the Kwiat *et al* source has a pair generation rate of only 15 s^{-1}.

In this letter we propose a production scheme for polarization-entangled photon pairs whose *narrowband* generation rate is high enough to make it a practical source for a trapped-atom quantum memory. Like Kwiat *et al*, we rely on two $\chi^{(2)}$ interactions. Unlike Kwiat *et al*, we employ optical parametric amplification in a doubly resonant cavity, rather than spontaneous downconversion, and we can utilize either type-I or type-II phase matching. For a type-I system using periodically poled lithium tantalate (PPLT) OPAs, we estimate the pair production rate to be approximately 1.5×10^6 s^{-1}, within a 30 MHz bandwidth at a 795 nm centre wavelength and a 70 ns duration coincidence interval, using only 0.7 mW of pump power per OPA. Within its 30 MHz bandwidth, this OPA source of entangled pairs is 10^7 times brighter, per mW of pump power, than the downconverter of Kwiat *et al* .

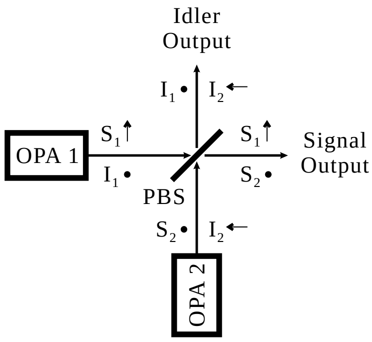

Figure 1. Type-II OPA configuration for generating polarization-entangled photon pairs. For each optical beam, the propagation direction is $\hat{z}$, and $\hat{x}$ and $\hat{y}$ polarizations are denoted by arrows and bullets, respectively. PBS: polarizing beam splitter.

Consider two matched continuous-wave OPAs, each producing entangled signal and idler fields of definite

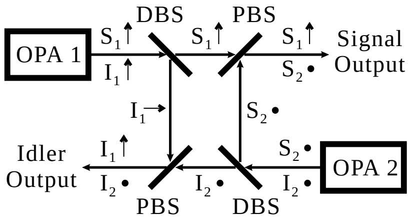

Figure 2. Type-I OPA configuration for generating polarization-entangled photon pairs. For each optical beam, the propagation direction is $\hat{z}$, and $\hat{x}$ and $\hat{y}$ polarizations are denoted by arrows and bullets, respectively. PBS: polarizing beam splitter; DBS: dichroic beam splitter.

polarizations. For type-II phase matching, shown in figure 1, a single polarization beam splitter (PBS) is used to make the signals (idlers) from OPAs 1 and 2 into the $x = \uparrow$ ($y = \bullet$) and $y = \bullet$ ($x = \leftarrow$) components, respectively, of the output vector signal (idler) field. This combining arrangement works whether or not the signal and idler are frequency degenerate. For type-I phase matching, shown in figure 2, frequency degeneracy is not used, so that dichroic beam splitters (DBSs) can be employed to separate the signal and idler beams from each OPA prior to PBS combining into the output vector beams. Let $\{\hat{\vec{E}}_S(t), \hat{\vec{E}}_I(t)\}$ be the photon-units, positive-frequency, vector field operators for the signal and idler output beams, and let $\vec{\boldsymbol{i}} = \boldsymbol{i}_x\hat{x} + \boldsymbol{i}_y\hat{y}$ be an arbitrary complex-valued unit vector ($\vec{\boldsymbol{i}}^* \cdot \vec{\boldsymbol{i}} = 1$) representing an arbitrary polarization state. We will show that within any photon-counting interval that is much longer than the OPA cavity lifetime, every $\hat{\vec{E}}_S(t)$ photon of polarization $\vec{\boldsymbol{i}}$ is accompanied by an $\hat{\vec{E}}_I(t)$ photon of the conjugate polarization $\vec{\boldsymbol{i}}'$, and vice versa. For type-I operation with the two pump beams in phase, the conjugate polarization is $\vec{\boldsymbol{i}}' = \vec{\boldsymbol{i}}^* = \boldsymbol{i}_x^*\hat{x} + \boldsymbol{i}_y^*\hat{y}$; and for type-II operation with the two pump beams in phase it is $\vec{\boldsymbol{i}}' = \boldsymbol{i}_y^*\hat{x} + \boldsymbol{i}_x^*\hat{y}$.

We assume ideal doubly resonant OPAs, with no pump excess noise, no pump depletion, zero detuning, no cavity loss beyond that associated with output coupling, and equal linewidths for the signal and idler. Under these conditions, the internal equations of motion for the phase-matched polarization components of the jth OPA ($j = 1, 2$) are as follows [8], when the two pump beams have equal power and are in phase:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \Gamma\right)\hat{a}_{S_j}(t) = G\Gamma\hat{a}_{I_j}^\dagger(t) + \sqrt{2\Gamma}\hat{A}_{S_j}^{\mathrm{IN}}(t), \tag{1}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \Gamma\right)\hat{a}_{I_j}(t) = G\Gamma\hat{a}_{S_j}^\dagger(t) + \sqrt{2\Gamma}\hat{A}_{I_j}^{\mathrm{IN}}(t). \tag{2}$$

Here: $\hat{a}_{S_j}(t)$ and $\hat{a}_{I_j}(t)$ are the intracavity annihilation operators of the signal and idler; $\hat{A}_{S_j}^{\mathrm{IN}}(t)\exp(-\mathrm{i}\omega_S t)$ and $\hat{A}_{I_j}^{\mathrm{IN}}(t)\exp(-\mathrm{i}\omega_I t)$, where ω_S and ω_I are the signal and idler frequencies, are vacuum-state, photon-units, positive-frequency input-field operators; Γ is the cavity-loss rate; and G is the normalized OPA gain, $G^2 = P_P/P_T$, where P_P is the pump power and P_T is the oscillation threshold. The signal and idler outputs from the jth OPA are given by [8],

$$\hat{A}_{S_j}(t) = \sqrt{2\Gamma}\hat{a}_{S_j}(t) - \hat{A}_{S_j}^{\mathrm{IN}}(t), \tag{3}$$

$$\hat{A}_{I_j}(t) = \sqrt{2\Gamma}\hat{a}_{I_j}(t) - \hat{A}_{I_j}^{\mathrm{IN}}(t). \tag{4}$$

Equations (1)–(4) can be used to prove that each OPA produces signal and idler that are in an entangled, zero-mean, Gaussian pure state, and that this state is completely characterized by the following non-zero normally ordered and phase-sensitive correlation functions [9]:

$$\langle\hat{A}_{S_j}^\dagger(t+\tau)\hat{A}_{S_j}(t)\rangle = \langle\hat{A}_{I_j}^\dagger(t+\tau)\hat{A}_{I_j}(t)\rangle$$
$$= \frac{G\Gamma}{2}\left[\frac{\exp[-(1-G)\Gamma|\tau|]}{1-G} - \frac{\exp[-(1+G)\Gamma|\tau|]}{1+G}\right], \tag{5}$$

$$\langle\hat{A}_{S_j}(t+\tau)\hat{A}_{I_j}(t)\rangle$$
$$= \frac{G\Gamma}{2}\left[\frac{\exp[-(1-G)\Gamma|\tau|]}{1-G} + \frac{\exp[-(1+G)\Gamma|\tau|]}{1+G}\right]. \tag{6}$$

It is a simple matter to verify that the scalar field operators,

$$\hat{A}_S(t) \equiv \vec{\boldsymbol{i}}^* \cdot \hat{\vec{E}}_S(t)\exp(\mathrm{i}\omega_S t), \tag{7}$$

$$\hat{A}_I(t) \equiv \vec{\boldsymbol{i}}'^* \cdot \hat{\vec{E}}_I(t)\exp(\mathrm{i}\omega_I t), \tag{8}$$

associated with the $\vec{\boldsymbol{i}}$ polarization of $\hat{\vec{E}}_S(t)$ and the $\vec{\boldsymbol{i}}'$ polarization of $\hat{\vec{E}}_I(t)$, are in the same entangled, zero-mean, Gaussian pure state as $\{\hat{A}_{S_1}(t), \hat{A}_{I_1}(t)\}$ and $\{\hat{A}_{S_2}(t), \hat{A}_{I_2}(t)\}$ are, *regardless* of the choice of $\vec{\boldsymbol{i}}$.

Unity quantum efficiency photon counting over the time interval $0 \leqslant t \leqslant T$ on the $\vec{\boldsymbol{i}}$ polarization of $\hat{\vec{E}}_S(t)$ and the $\vec{\boldsymbol{i}}'$ polarization of $\hat{\vec{E}}_I(t)$ measures the operators,

$$\hat{N}_S \equiv \int_0^T \mathrm{d}t\, \hat{A}_S^\dagger(t)\hat{A}_S(t), \tag{9}$$

and

$$\hat{N}_I \equiv \int_0^T \mathrm{d}t\, \hat{A}_I^\dagger(t)\hat{A}_I(t), \tag{10}$$

respectively. From the normally ordered correlation function equation (5) we then have that the mean photon counts obey

$$\langle\hat{N}_S\rangle = \langle\hat{N}_I\rangle = \frac{G^2\Gamma T}{1-G^2}, \tag{11}$$

whence the photon-count difference—the $\Delta\hat{N} \equiv \hat{N}_S - \hat{N}_I$ measurement—is zero mean.

A zero-mean photon-count difference is an obvious prerequisite to the photon-pair property we are trying to establish. The definitive result, however, comes from the count-difference variance. Using the normally-ordered and phase-sensitive correlation functions from equations (5) and (6) in equation (29) of [9] we can show that this variance satisfies,

$$\frac{\langle\Delta\hat{N}^2\rangle}{\langle\hat{N}_S\rangle + \langle\hat{N}_I\rangle}$$
$$= \frac{1-\exp(-2\Gamma T)}{2\Gamma T} \rightarrow \begin{cases} 1, & \text{for} \quad \Gamma T \ll 1, \\ 0, & \text{for} \quad \Gamma T \gg 1. \end{cases} \tag{12}$$

For photon-counting intervals much shorter than the OPA cavity lifetime ($T \ll 1/\Gamma$), equation (12) tells us that the difference count has a Poissonian (shot-noise limited) variance. For photon-counting intervals much longer than this lifetime, equation (12) shows that the difference-count variance is very nonclassical: namely, it is strongly sub-Poissonian. The latter property is exactly what we wanted: within any time interval of duration $T \gg 1/\Gamma$, every signal photon in polarization $\vec{i}$ is accompanied by an idler photon of the conjugate polarization $\vec{i}'$ and vice versa.

For a given Γ, let us choose T such that $\Gamma T = 10$, ensuring that we capture the entangled pairs within the counting interval. For the quantum memory application, we want there to be at most one entangled photon pair per counting interval. So let us pump OPAs 1 and 2 at 1% of threshold ($G^2 = 0.01$), thus making $G^2\Gamma T = 0.1$ the average number of signal (or idler) photons in the counting interval. In this low-gain limit, the signal and idler fields will have identical, double-Lorentzian spectra of bandwidth $\Delta\nu \approx 0.64\Gamma/\pi$ Hz. Hence our $\Delta\nu = 30$ MHz example requires $\Gamma \approx 1.5 \times 10^8$ s^{-1} and $T \approx 70$ ns.

In the low-gain ($G^2 = 0.01$) multimode ($\Gamma T = 10$) limit, the $\hat{N}_S$ and $\hat{N}_I$ measurement statistics will be approximately Poissonian, hence the probability P_n of counting n signal (or idler) photons within the duration-T counting interval satisfies $P_0 \approx 9.05 \times 10^{-1}$, $P_1 \approx 9.05 \times 10^{-2}$, and $\sum_{n\geqslant 2} P_n \approx 4.68 \times 10^{-3} \approx P_1/20$. If we neglect the small probabilities that two or more signal (or idler) photons are present within a counting interval, we have that $\langle \Delta\hat{N}^2 \rangle \approx 2P_{01}$ where

$$P_{01} \equiv \Pr(\hat{N}_S = 1, \hat{N}_I = 0) = \Pr(\hat{N}_S = 0, \hat{N}_I = 1), \quad (13)$$

is the probability that we detect a signal (idler) photon but not its idler (signal) companion. The conditional probability of successful pair detection, given we detect a signal photon, is thus

$$P_{\mathrm{I|S}} \equiv 1 - P_{01}/P_1 \approx 1 - 1/2\Gamma T = 0.95. \quad (14)$$

So, if we divide the time axis into 70 ns duration intervals, approximately 10% of them will contain a signal photon of polarization $\vec{i}$, and 95% of these signal-bearing intervals will also have an idler photon of polarization $\vec{i}'$. Furthermore, only about 0.5% of the intervals will contain two or more polarization-$\vec{i}$ signal photons. We have thus obtained a source of 30 MHz bandwidth polarization-entangled photon pairs whose production rate is $G^2\Gamma \approx 1.5 \times 10^6$ s^{-1}, for a $T = 70$ ns coincidence interval. It only remains for us to assess the pump power required to realize this device.

For nondegenerate operation with the signal wavelength centred at 795 nm—corresponding to the D_1 line of ^{87}Rb—we can use type-I phase matched PPLT as the nonlinear crystal for the OPAs. Assuming a 10 mm long PPLT crystal, which has an effective nonlinear coefficient of 9 pm/V for a first-order grating [10], and a 5% output coupling for both the signal and the idler fields, the expected threshold is less than 70 mW in a single-pass pump configuration. The required pump power for two of these PPLT-based OPAs at $G^2 = 0.01$ is therefore only 1.4 mW, which yields a pair-production rate of 10^6 s^{-1}mW^{-1} when normalized to the pump power. For 5% output coupling, the cavity length should be adjusted to have a free spectral range of 5.9 GHz so as to yield the required spectral bandwidth $\Delta\nu = 30$ MHz. Note that, in addition to its much higher narrowband pair-production rate than the Kwiat *et al* downconverter, our resonant-OPA approach to generating polarization-entangled photons has the additional advantage of producing these pairs in well-defined spatial modes, which should make achieving near-unity collection efficiency much easier.

Some additional features of our OPA approach to generating polarization entanglement deserve mention. First, it is very easy to change the cavity linewidth $\Delta\nu$, which determines the pair production rate and the duration, T, of the coincidence interval. This can be accomplished by varying the cavity length and/or the output coupling, because the cavity linewidth is inversely proportional to the cavity's optical length and linearly proportional to its output coupling. Furthermore, because the pump power requirement is modest, the nonlinear crystal length can be shortened to allow the implementation of a very compact device. For example, a cavity with a 20 GHz free spectral range can be obtained with a 3 mm long PPLT crystal. Such a short cavity, used in conjunction with a 10% output coupler, yields a spectral width of 200 MHz, a pair-production rate of 10^7 s^{-1}, and a $\Gamma T = 10$ coincidence interval of 10 ns. In this case, the pump power is increased to a still modest 20 mW. Note that the pair-production rate per unit pump power is smaller by a factor of 2 for the short-cavity system because of its lower nonlinear conversion efficiency. For longer wavelength operation, type-I phase matched periodically poled lithium niobate (PPLN) should produce even lower pump power requirements, because of its higher nonlinearity. Potassium titanyl phosphate (KTP) is also a suitable crystal; its type-II phase matching will permit frequency-degenerate operation if that is desirable.

Two final comments are now in order. We have restricted our presentation to the dual-OPA configuration with equal-power, in-phase pump beams. Several interesting things happen if we relax this assumption, both of which have been treated—for the downconverter case—by Kwiat *et al* [5]. If we change the relative phase between the two pump beams, while maintaining their power equality, we still achieve full polarization entanglement, i.e., the vector-signal and vector-idler beams produced in our arrangements in figures 1 and 2 are still randomly polarized individually, with each $\vec{i}$-polarized signal photon still accompanied by an $\vec{i}'$-polarized idler photon and vice versa. What does change, when these equal-power pumps have a non-zero phase difference, is the idler-beam conjugate polarization, $\vec{i}'$, that is associated with the signal-beam polarization $\vec{i}$. For example, in a type-I system with a π-rad phase shift between the pumps, we have that $\vec{i}' = i_x^*\hat{x} - i_y^*\hat{y}$. If, in addition to allowing a non-zero phase difference between the pumps, we let them also have different powers, then we can produce states of arbitrary non-maximal polarization entanglement, as Kwiat *et al* predicted in [5] and White *et al* experimentally demonstrated in [11]. Even when we introduce a non-zero relative phase and/or unequal powers for the pump beams, however, the key distinction between our dual-OPA configuration and the dual-downconverter approach of Kwiat *et al* remains: ours is an ultrabright *narrowband* source of polarization-entangled photons, whereas theirs is intrinsically broadband.

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