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Acknowledgements

We thank S. Bale, C. Chaston, B. Sonnerup, J. Drake and A. Bhattacharjee for discussions and T. Nagai for providing a preprint of his Geotail manuscript at an early stage. This work is supported by grants from NASA.

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Quantum-enhanced positioning and clock synchronization

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A wide variety of positioning and ranging procedures are based on repeatedly sending electromagnetic pulses through space and measuring their time of arrival. The accuracy of such procedures is classically limited by the available power and bandwidth. Quantum entanglement and squeezing have been exploited in the context of interferometry[1–5], frequency measurements[6], lithography[7] and algorithms[8]. Here we report that quantum entanglement and squeezing can also be employed to overcome the classical limits in procedures such as positioning systems, clock synchronization and ranging. Our use of frequency-entangled pulses to construct quantum versions of these protocols results in enhanced accuracy compared with their classical analogues. We describe in detail the problem of establishing a position with respect to a fixed array of reference points.

Any position (say, that of Alice) may be obtained simply by sending pulses that originate from that position and measuring the time it takes for each pulse to reach the reference points. The time of flight, the speed of the pulses and the arrangement of the reference points determine Alice's position. The accuracy of such a procedure depends on the number of pulses, their bandwidth and the number of photons per pulse. Here we show that, by measuring the correlations between the times of arrival of M pulses which are frequency-entangled, it is in principle possible to increase the accuracy of such a positioning procedure by a factor $\sqrt{M}$ as compared to positioning using unentangled pulses with the same bandwidth. Moreover, if number-squeezed pulses can be produced[9], it is possible to obtain a further increase in accuracy of $\sqrt{N}$ by employing squeezed pulses of N quanta, rather than employing 'classical' coherent states with a mean number of quanta N. Combining entanglement with squeezing gives an overall enhancement of $\sqrt{MN}$. In addition, the procedure exhibits improved security: because the timing information resides in the entanglement between pulses, it is possible to implement quantum cryptographic schemes that do not allow an eavesdropper to obtain information on the position of Alice (V.G., S.L. and L.M., unpublished results). The primary drawbacks of this scheme are the difficulty of creating the requisite entanglement and the sensitivity to loss. On the other hand, the frequency entanglement allows similar schemes to be highly robust against pulse broadening due to transit through dispersive media[10].

The clock synchronization problem can be treated using the same method. In refs 11 and 12 two techniques for clock synchronization using entangled states were presented. However, the authors of ref. 11 themselves point out that the resources needed by their scheme could be used to perform conventional clock synchronization without entanglement. Similarly, all the enhancement reported in ref. 12 arises from employing high-frequency atoms, which themselves could be used for clock synchronization to the same degree of accuracy without any entanglement. In neither case do these proposals give an obvious enhancement over classical procedures that use the same resources. Here, by contrast, we show that quantum features such as entanglement and squeezing could in principle be used to supply a significant enhancement of the accuracy of clock synchronization as compared with classical protocols using light of the same frequency and power. In fact, the clock synchronization could be accomplished by sending pulses back and forth between the parties whose clocks are to be synchronized, and measuring the times of arrival of the pulses (Einstein's protocol). In this way synchronization may be treated on the same basis as positioning, and the same accuracy enhancements may be achieved through entanglement and squeezing. Here we will address in detail only the enhancement of positioning accuracy.

In order to introduce the formalism, we present the simple case of position measurement with classical coherent pulses. As each dimension can be treated independently, the analysis will be limited to the one-dimensional case. For the sake of simplicity, consider the situation in which Alice wants to measure her position x by sending a pulse to each of M detectors placed in a known position (Fig. 1). This can be easily generalized to different set-ups, such as the case in which the detectors are not all in the same location, the case in which only one detector is employed with M time-separated pulses, the case in which the pulses originate from the reference points and are measured by Alice (as in GPS, the global positioning system), and so on. Alice's estimate of her position is given by $x = (c/M)\Sigma_{i=1}^{M} t_i$, where t_i is the travel time of the ith pulse and c is the speed of light. The variable t_i has an intrinsic indeterminacy dependent on the spectral characteristics and mean number of photons N of the ith pulse. For example, given a gaussian pulse of frequency spread $\Delta\omega$, then, according to the central limit theorem, t_i cannot be measured with an accuracy better than $1/(\Delta\omega\sqrt{N})$ as it is estimated at most from N data points (that is, the times of arrival of the single photons, each having an indeterminacy $1/\Delta\omega$). Thus, if Alice uses M gaussian pulses of equal frequency spread, the accuracy in the measurement of the average time of arrival is:

$$\Delta t = \frac{1}{\Delta\omega\sqrt{MN}} \tag{1}$$

Quantum mechanics allows us to do much better. In order to demonstrate the gain in accuracy afforded by quantum mechanics, we provide first a fully quantum analysis of the problem of determining the average time of arrival of a set of M classical pulses, each having a mean number of photons N. Such a quantum treatment for a classical problem may seem excessive, but once the quantum formalism is presented, the speed-up attainable in the fully quantum case can be derived directly. In addition, it is important to verify that no improvement over equation (1) is obtainable using classical pulses. The M coherent pulses are described by a state of the radiation field of the form

$$|\Psi\rangle_{\rm cl} \equiv \bigotimes_{i=1}^{M} \bigotimes_{\omega} \left|\alpha\left(\phi_\omega\sqrt{N}\right)\right\rangle_i \tag{2}$$

where ϕ_ω is the pulses' spectral function, $|\alpha(\lambda_\omega)\rangle_i$ is a coherent state of amplitude λ_ω in the mode at frequency ω directed towards the ith detector, and N is the mean number of photons in each pulse. The pulse spectrum $|\phi_\omega|^2$ has been normalized so that $\int d\omega|\phi_\omega|^2 = 1$. For detectors with perfect time resolution, the joint probability for the

ith detector to detect N_i photons in the ith pulse at times $t_{i,k}$ is given by[13]

$$p(\{t_{i,k}\}) \propto \left\langle : \prod_{i=1}^{M} \prod_{k=1}^{N_i} E_i^{(-)}(t_{i,k}) E_i^{(+)}(t_{i,k}) : \right\rangle \quad (3)$$

where $t_{i,k}$ is the time of arrival of the kth photon in the ith pulse, shifted by the position of the detectors $t_{i,k} \to t_{i,k} + x/c$. The signal field at the position of the ith detector at time t is given by $E_i^{(-)}(t) \equiv \int d\omega\, a_i^{\dagger}(\omega) e^{i\omega t}$ and $E_i^{(+)} \equiv (E_i^{(-)})^{\dagger}$, where $a_i(\omega)$ is the field annihilator of a quantum of frequency ω at the ith detector, which satisfies $[a_i(\omega), a_j^{\dagger}(\omega')] = \delta_{ij}\delta(\omega - \omega')$. The estimation of the ensemble average in equation (3) on the state $|\Psi\rangle_{\rm cl}$, using the property $a(\omega')\bigotimes_{\omega}|\alpha(\lambda_\omega)\rangle = \lambda_{\omega'}\bigotimes_{\omega}|\alpha(\lambda_\omega)\rangle$, gives

$$p(\{t_{i,k}\}) \propto \prod_{i=1}^{M} \prod_{k=1}^{N_i} \left| g(t_{i,k}) \right|^2 \quad (4)$$

where $g(t)$ is the Fourier transform of the spectral function ϕ_ω. Averaging over the times of arrival $t_{i,k}$ and over the number of photons N_i detected in each pulse, we have:

$$\langle t \rangle = \left\langle \frac{1}{M} \sum_{i=1}^{M} \frac{1}{N_i} \sum_{k=1}^{N_i} t_{i,k} \right\rangle = \bar{\tau};\ \Delta t \geq \frac{\Delta\tau}{\sqrt{MN}} \quad (5)$$

with approximate equality for $N \gg 1$. Here $\bar{\tau} \equiv \int dt |g(t)|^2 t$ and $\Delta\tau^2 \equiv \int dt |g(t)|^2 (t - \bar{\tau})^2$ are independent of i and k as all the photons have the same spectrum. Equation (5) is the generalization of (1) for non-gaussian pulses.

Quantum light can exhibit phenomena that are not possible classically, such as entanglement and squeezing, which, as will now be seen, can give significant enhancement for determining the average time of arrival. We first consider entanglement. The framework just established allows the direct comparison between frequency-entangled pulses and unentangled ones. For the sake of clarity, we consider initially single-photon entangled pulses.

We define the 'frequency state' $|\omega\rangle$ for the electromagnetic field as the state in which all modes are in the vacuum state, except for the mode at frequency ω which is populated by one photon. Thus the state $\int d\omega \phi_\omega |\omega\rangle$ represents a single-photon wave packet with spectrum $|\phi_\omega|^2$. We consider the M-photon frequency-entangled state given by

$$|\Psi\rangle_{\rm en} \equiv \int d\omega\, \phi_\omega |\omega\rangle_1 |\omega\rangle_2 \cdots |\omega\rangle_M \quad (6)$$

where the subscripts 1, 2, M indicate the detector each photon is travelling to. Inserting $|\Psi\rangle_{\rm en}$ in equation (3), and specializing to the case $N_i = 1$, it follows that

$$p(t_1, \ldots, t_M) \propto \left| g\left(\sum_{i=1}^{M} t_i \right) \right|^2 \quad (7)$$

That is, the entanglement in frequency translates into the bunching of the times of arrival of the photons of different pulses: although their individual times of arrival are random, the average $t \equiv (1/M)\Sigma_{i=1}^{M} t_i$ of these times is highly peaked. (The measurement of t follows from the correlations in the times of arrival at the different detectors.) Indeed, from equation (7) it turns out that the probability distribution of t is $|g(Mt)|^2$. This immediately implies that the average time of arrival is determined to an accuracy

$$\Delta t = \frac{\Delta\tau}{M} \quad (8)$$

where $\Delta\tau$ is the same as in equation (5). This result shows a $\sqrt{M}$ improvement over the classical case of equation (5).

To emphasize the importance of entanglement, equation (8) should be compared to the result that would be obtained from an unentangled state analogous to $|\Psi\rangle_{\rm en}$. To this end, we consider the state defined as

$$|\Psi\rangle_{\rm un} \equiv \bigotimes_{i=1}^{M} \int d\omega_i \phi_{\omega_i} |\omega_i\rangle_i \quad (9)$$

which describes M uncorrelated single-photon pulses each with spectral function ϕ_ω. By looking at the spectrum of the state obtained by tracing away all but one of the modes in equation (6), each of the photons in equation (9) can be shown to have the same spectral characteristics as the photons in the entangled state $|\Psi\rangle_{\rm en}$. Now, using equation (3) for the uncorrelated M photon pulses $|\Psi\rangle_{\rm un}$, it follows that

$$p(t_1, \ldots, t_M) \propto \prod_{i=1}^{M} |g(t_i)|^2 \quad (10)$$

which is the same result that was obtained for the classical state $|\Psi\rangle_{\rm cl}$. Thus equation (5) holds, with $N = 1$, also for $|\Psi\rangle_{\rm un}$. From the comparison of equation (5) and (8), we see that employing frequency-entangled pulses gives an increase in accuracy by a factor $\sqrt{M}$ in the measurement of t with respect to the case of unentangled photons.

As $|\Psi\rangle_{\rm en}$ is tailored to give the least indetermination in the quantity t, it is appropriate for the geometry of the case given in Fig. 1, where the sum of the times of arrival is needed. Other entangled states can be tailored for different geometric dispositions of the detectors, as will be shown through some examples.

How can we create the needed entangled states? In the case $M = 2$, the twin beam state at the output of a continuous-wave pumped parametric down-converter will be shown to be appropriate. It is a two-photon frequency-entangled state of the form $\int d\omega\, \phi_\omega |\omega\rangle_s |\omega_0 - \omega\rangle_i$, where ω_0 is the pump frequency and s and i refer to the signal and idler modes respectively. This state is similar to $|\Psi\rangle_{\rm en}$, and can be employed for position measurements when the two reference points are in opposite directions—for example, one to the left and one to the right of Alice. In fact, it can be seen that

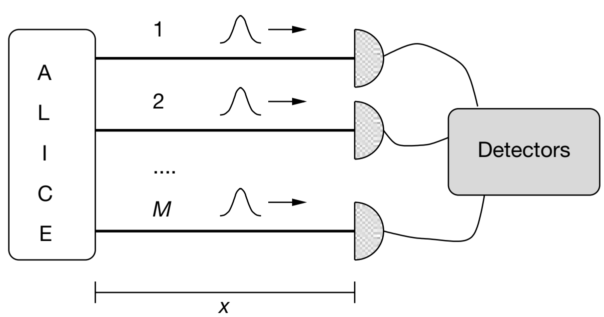

Figure 1 Sketch of the idealized experimental configuration. Alice sends M light pulses to the M detectors. She averages the times of arrival t_i of the pulses to recover her unknown position x.

$p(t_1, t_2) \propto |g(t_1 - t_2)|^2$, and hence such a state is optimized for measurements of differences in the times of arrival, as experimentally reported in ref. 14. In the case of $M = 3$, a suitable state can be obtained starting from a three-photon generation process that creates a state of the form $\int d\omega d\omega' f(\omega, \omega')|\omega\rangle|\omega'\rangle|\omega_0 - \omega - \omega'\rangle$, and then performing a non-demolition (or a post-selection) measurement of the frequency difference of two of the photons. This would create a maximally entangled three-photon state, tailored for the case in which Alice has one detector on one side and two detectors on the other side. However, for $M > 2$, the creation of such frequency-entangled states represents a continuous variable generalization of the Greenberger–Horne–Zeilinger state, and, as such, is a considerable experimental challenge.

Now we turn to the use of number-squeezed states to enhance positioning. Quantum effects in the propagation of multi-photon states are well known (see, for example, ref. 15). The Nth excitation of a quantum system (that is, the state $|N\rangle$ of exactly N quanta) has a de Broglie frequency N times the fundamental frequency of the state. Its shorter wavelength makes such a state appealing for positioning protocols. In this case, the needed 'frequency state' is given by $|N_\omega\rangle$, defined as the state where all modes are in the vacuum except for the mode at frequency ω, which is in the Fock state $|N\rangle$. The probability of measurement of N quanta in a single pulse at times $t_1, \ldots, t_N$ is given by equation (3) with $M = 1$ detectors. We see that, for a state of the form $\int d\omega\, \phi_\omega |N_\omega\rangle$, the time of arrival probability is given by:

$$p(t_1, \ldots, t_N) \propto \left| g\left(\sum_{k=1}^{N} t_k \right) \right|^2 \qquad (11)$$

Such a result must be compared to what one would obtain employing a classical pulse $|\Psi\rangle_{cl}$ of mean number of photons N, that is, the state equation (2) with $M = 1$. Its probability (equation (4)) shows that employing the N-photon Fock state gives an accuracy increase of $\sqrt{N}$ compared to the coherent state with mean number of photons N. The similarity of this result (equation (11)) to that obtained in equation (7) stems from the fact that the Fock state $|N_\omega\rangle$ can be interpreted as composed of N one-photon pulses of identical frequency. Hence, all the results and considerations obtained previously apply here. An experiment that involves such a state for $N = 2$ is reported in ref. 16.

Entangled pulses of number-squeezed states combine both these enhancements. By replacing $|\omega\rangle$ with the number-squeezed states $|N_\omega\rangle$ in the M-fold entangled state of equation (6), one immediately obtains an improvement of $\sqrt{MN}$ over the accuracy obtainable by using M classical pulses of N photons each.

The enhanced accuracy achieved comes at the cost of an enhanced sensitivity to loss. If one or more of the photons fails to arrive, the time of arrival of the remaining photons do not convey any timing information. The simplest way to solve this problem is to ignore all trials where one or more photons is lost. A more sophisticated method is to use partially entangled states: these states provide a lower level of accuracy than fully entangled states, but are more tolerant to loss. As shown in Fig. 2, even the simple protocol of ignoring trials with loss still surpasses the unentangled-state accuracy limit even for significant loss levels. The use of intrinsically loss-tolerant, partially entangled states does even better (V.G., S.L. and L.M., unpublished results).

It is useful to consider the following intuitive picture of quantum measurements of timing. A quantum system such as a pulse of photons or a measuring apparatus with spread in energy ΔE can evolve from one state to an orthogonal state in time Δt no less than $h/(4\Delta E)$ (ref. 17). Accordingly, to make more accurate timing measurements, we require states with sharp time dependence, and hence high spreads in energy. Classically, combining M systems each with spread in energy ΔE results in a joint system with spread in energy $\sqrt{M}\Delta E$. Quantum-mechanically, however, M systems can be put in entangled states in which the spread in energy is proportional to $M\Delta E$. Similarly, N photons can be joined in a squeezed state with spread in energy $N\Delta E$. The Margolus–Levitin theorem[18] limits the time Δt it takes for a quantum system to evolve from one state to an orthogonal one by $\Delta t \geq h/(4E)$, where E is the average energy of a system (taking the ground state energy to be 0). This result implies that the $\sqrt{MN}$ enhancement presented here is the best possible. □

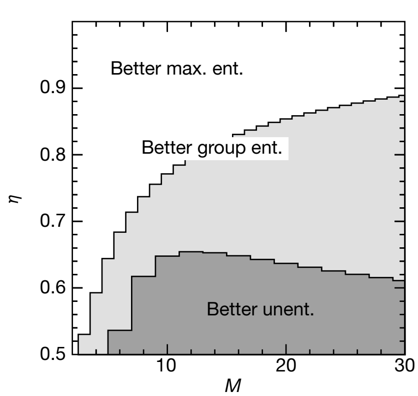

Figure 2 Sensitivity to loss. The quantum efficiency η needed to have an accuracy increase over the unentangled state $|\Psi\rangle_{un}$ is plotted versus the number M of pulses (here the number of photons per pulse is $N = 1$). The upper unshaded region is where the maximally entangled state $|\Psi\rangle_{en}$ does better than the unentangled state $|\Psi\rangle_{un}$. This unshaded region and the light grey region are where a partially entangled state, which exploits a configuration where one partially entangles subgroups of two maximally entangled photons (group entanglement), does better than $|\Psi\rangle_{un}$. The lower dark region is where the unentangled state $|\Psi\rangle_{un}$ does better.

Received 5 March; accepted 11 June 2001.

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Acknowledgements

This work was funded by the ARDA, NRO, and by ARO under a MURI program.

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