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Clock synchronization and dispersion

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Abstract
We present a method to defeat effects of dispersion of timing signals when synchronizing clocks. It is based on the recently proposed 'conveyor belt synchronization' scheme and on the quantum dispersion cancellation effect.

Keywords: Entanglement, clock synchronization, biphoton, dispersion

(Some figures in this article are in colour only in the electronic version)

The usual protocols used to synchronize distant clocks consists of one of the variants of the Einstein clock synchronization scheme [1]. To employ this procedure, the two parties involved (say Alice and Bob) exchange pulses or some other timing signals and measure, according to their own clocks, when such signals arrive at their positions. By comparing the results, they can easily recover the time difference between their clocks and hence get synchronized. One of the main problems in high-accuracy realization of such protocols is the dispersion that the timing signals encounter while travelling between the clocks [2]. The two principal effects of dispersion are the spread of the pulse width and the shift of its mean position.

In this paper we analyse a scheme that allows one to defeat these effects in a large variety of cases by employing the quantum dispersion cancellation [3]. Our analysis, instead of using a variant of Einstein's procedure, employs the recently proposed 'conveyor belt synchronization' protocol [4]. In this scheme, the two parties have access to a common communication channel (conveyor belt) in which the signals that travel from one to the other take the same amount of time as on their way back. These signals possess a degree of freedom (e.g. frequency, phase) that can be continuously varied in time by Alice and Bob. Such a quantity is employed to encode Alice's and Bob's proper times (i.e. the time as measured by their own clocks). By appropriately designing the encoding procedure, it is possible to recover the time difference between their clocks by simply measuring this quantity after a round trip of the signal [4]. The conveyor belt synchronization protocol is particularly suited to address the problem of dispersion, since it does not require a measurement of the time of arrival of any signal, which would be spoiled by dispersion. This fact allows one to devise procedures based on classical resources only, which, if the medium satisfies appropriate conditions, are not affected by dispersion [5]. In this paper we focus on how quantum mechanics can help relax these conditions.

It is well known that the coincidence rate P_c in a Hong–Ou–Mandel [6] interferometer displays a dip (the so-called Mandel dip) the position of which depends on the path length difference of the two arms of the interferometer. The width of the Mandel dip is fixed by the bandwidth $\Delta\omega$ of the two-photon state that must be injected in the interferometer and which is produced at the output of a parametric down-conversion crystal, i.e. the state

$$|\Psi\rangle \equiv \int \mathrm{d}\omega\, \phi(\omega) |\omega_0 + \omega\rangle_I |\omega_0 - \omega\rangle_S, \qquad (1)$$

where $\phi(\omega)$ is the spectral function with centre at $\omega = 0$ and width $\Delta\omega$, ω_0 is the mean frequency of the two photons, and the ket subscripts I and S refer to the idler and signal beam respectively.

In [3] it has been shown that introducing a dispersive medium in one of the two arms of the interferometer does not affect the width of the Mandel dip. This is equivalent to saying that the second-order term in the Taylor expansion of the dispersive wavevector around the mean frequency of the signal field (responsible for the pulse spread) does not affect the result of the measurement. The key ingredient of such an effect is the frequency entanglement between the signal and idler photons of the initial state $|\Psi\rangle$. This quantum dispersion cancellation guarantees that the accuracy of the measurement of the optical path length difference in the interferometer is dispersion independent.

How can one use such an effect in order to synchronize distant clocks? The idea is to encode the time difference between Alice's and Bob's clocks into a path length difference

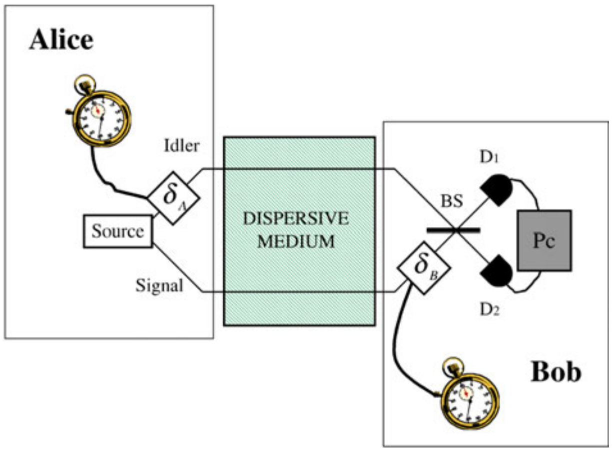

Figure 1. Clock synchronization with quantum dispersion cancellation. Alice produces the state $|\Psi\rangle$ of equation (1). The variable delays δ_A and δ_B are connected with Alice's and Bob's clocks. Bob measures the coincidence rate P_c at the two detectors. It exhibits a dip as a function of the beam splitter BS position. The position of the dip allows one to directly recover the time difference between the two clocks. In this configuration, the second-order effects of the dispersion are cancelled through the frequency entanglement of the two-photon state at the source. This scheme is, however, sensitive to the first-order term of the dispersion, which will shift the coincidence rate dip: thus, it can be used only if the dispersion is constant in time.

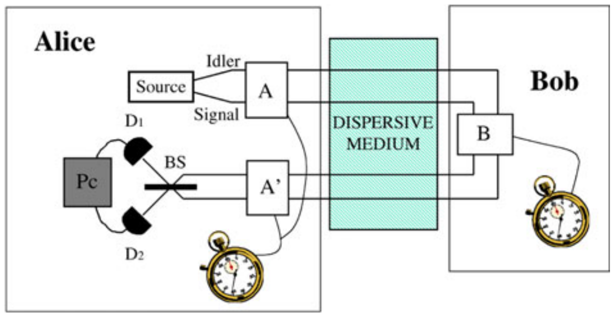

Figure 2. Realization of the 'conveyor belt synchronization' scheme using quantum states. Under appropriate conditions, Alice and Bob may synchronize their clocks without having to bother about dispersion. As in the scheme of figure 1, the time difference between the two clocks is directly connected to the coincidence dip position, but here there is no dependence on the distance between Alice and Bob, as shown by equation (4).

of the two interferometer arms. The simplest way to do so is by introducing time-varying delays connected to the two clocks, as depicted in figure 1. In this first scheme (which does not implement the complete conveyor belt synchronization protocol), Alice operates the two-photon source and introduces the time-dependent delay $\delta_A(t)$ proportional to the time shown on her clock on the idler photon. On the other hand, Bob, separated from Alice by a region in which there is dispersion, introduces the delay $\delta_B(t)$ proportional to the time shown on his clock on the signal photon and measures the coincidence rates P_c at the detectors at the output. If the delays are such that the two path lengths increase at the same rate when the two clocks are synchronized, then one needs to monitor only the coincidence rate at the two detectors in order to find out if the clocks are 'in sync'. The frequency entanglement guarantees that the width of the coincidence rate dip is not changed by the dispersion. However, as pointed out in [3], the dip position will be shifted by the linear term contribution of the dispersion. Moreover, since Alice's time-varying delay is introduced before the dispersive medium and Bob's after, the dip position will also depend on the distance between them. This is unfortunate if one wants to deal with non-constant dispersion. A more symmetric configuration is needed.

The scheme shown in figure 2 solves the limitations of the scheme of figure 1 by using the conveyor belt synchronization protocol. This new scheme is obtained by symmetrizing the configuration of the previously described apparatus. In this case, Alice introduces time-varying delays according to her clock at A and A′ and operates both the source and the detectors. On the other hand, Bob introduces the time-varying delay according to his clock at B. The delays at A and A′ increase the path length of the idler photon and decrease the path length of the signal. The delay at B must be opposite in sign and thus decreases the idler path and increases the signal path. The delay positions are chosen such that the distances $\overline{AB}$ and $\overline{BA'}$ are equal. As in the previous scheme, a minimum in the coincidences at the detectors will indicate that the two clocks are synchronized. In the initial proposal of [4] a similar apparatus was analysed, and the delays were modelled by moving mirrors. Here we consider the more realistic scenario in which electro-optic modulators (i.e. devices in which the refractive index is varied with electric fields) are used to continuously increment or decrement the refractive indices of the delay elements at A, A′, and B.

Consider for example the situation in which the refractive indices vary linearly in time as described by

$$\begin{aligned} n_A(t) = n_{A'}(t) &= \pm\eta(t - t_0^a) + 1 \\ n_B(t) &= \mp 2\eta(t - t_0^b) + 1, \end{aligned} \tag{2}$$

where t_0^a and t_0^b are the initial times of Alice and Bob's clocks as measured by an external observer, the term 1 stands for the vacuum refractive index, and η is a fixed constant rate. In equation (2) the upper sign refers to the idler and the lower to the signal beam: Alice is increasing the idler path length while decreasing the signal path length by the same amount, while Bob's variations are double and opposite in sign as compared with Alice's (as required by the conveyor belt protocol). In what follows we will assume that during the transit time of the photons through the delays we can consider such delays as constant. In this regime we can neglect any frequency modulation of the pulse introduced by the time-varying delays. According to equation (2) the annihilator operators of the signal and idler photons undergo the following transformation in their travel from the source to the beam splitter:

$$\begin{aligned} a_S(\omega) &\longrightarrow a'_S(\omega) = a_S(\omega)\mathrm{e}^{+2\mathrm{i}\omega\eta l(t_0^b - t_0^a)/c + 2\mathrm{i}\omega L/c + \mathrm{i}\kappa_S(\omega)} \\ a_I(\omega) &\longrightarrow a'_I(\omega) = a_I(\omega)\mathrm{e}^{-2\mathrm{i}\omega\eta l(t_0^b - t_0^a)/c + 2\mathrm{i}\omega L/c + \mathrm{i}\kappa_I(\omega)}, \end{aligned} \tag{3}$$

where l is the length of the delay elements and L is the distance $\overline{AB}$ (that plays no role in the coincidence rate). In equation (3), κ_S and κ_I take into account the effect of dispersion on the signal and idler photons respectively during their round trip from Alice to Bob and back. In equation (3) a_S and a_I are the annihilation operators at the source position, and a'_S and

a'_I are the annihilation operators before the beam splitter BS of figure 2. Following the same analysis as presented in [4], one can show that the coincidence rate (i.e. the probability of having a click in both the detectors D_1 and D_2 of figure 2) is given by

$$P_c = \int d\omega\, |\phi(\omega)|^2 \times \left[1 - \cos(4\omega\eta l(t_0^b - t_0^a)/c - \Delta\kappa(\omega + \omega_0))\right], \quad (4)$$

where $\phi(\omega)$ is the two-photon state spectral function of equation (1) and

$$\Delta\kappa(\omega) = \kappa_S(\omega) + \kappa_I(2\omega_0 - \omega) - \kappa_I(\omega) - \kappa_S(2\omega_0 - \omega), \quad (5)$$

($2\omega_0$ being the pump frequency for producing the input two-photon state). If $\Delta\kappa(\omega)$ is equal to zero, then any effect of the dispersion is erased and P_c exhibits a dip of width $\Delta\omega^{-1}$ as a function of $t_0^b - t_0^a$. The condition $\Delta\kappa(\omega) = 0$ is the main signature of the frequency entanglement of the state $|\Psi\rangle$ of equation (1). It does *not* require necessarily that the signal and idler undergo the same dispersion (i.e. $\kappa_S = \kappa_I$) as would be necessary to remove dispersion without using entanglement [5].

In conclusion we have presented a scheme which allows one to use the quantum dispersion cancellation effect [3] to help synchronize distant clocks.

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