CW theory for optical-fiber photon-pair generation

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ABSTRACT

We derive a CW theory for optical-fiber photon-pair sources, including the effect of non-zero response time of the fiber’s Kerr nonlinearity. We also include the effects of realistic transmission and detection losses. This theory predicts stronger photon-number correlations than seen experimentally with a pulsed pump, showing the need for development of a pulsed pump theory.

Keywords: Quantum entanglement, Quantum communication, Quantum cryptography, Fiber parametric amplifiers, Optical fiber communication, Photon counting, Raman scattering, Fiber Raman amplifiers, Four-wave mixing, Nonlinear optics.

1. INTRODUCTION AND MOTIVATION

Efficient generation and transmission of quantum-correlated photon pairs, especially in the 1550-nm fiber-optic communication band, is of paramount importance for practical realization of the quantum communication and cryptography protocols. The workhorse source employed in all implementations, thus far, has been based on the process of spontaneous parametric down-conversion in second-order ($\chi^{(2)}$) nonlinear crystals. Such a source, however, is not compatible with optical fibers as large coupling losses occur when the pairs are launched into the fiber. This severely degrades the correlated photon-pair rate coupled into the fiber, since the rate depends quadratically on the coupling efficiency. From a practical standpoint, it would be advantageous if a photon-pair source could be developed that not only produces photons in the communication band but also can be spliced to standard telecommunication fibers with high efficiency.

Over the past few years, various attempts have been made to develop more efficient photon-pair sources, but all have relied on the $\chi^{(2)}$ down-conversion process. Of particular note is Ref. 4, in which the effective $\chi^{(2)}$ of periodically-poled silica fiber was used. In an attempt to further increase the efficiency, we have recently constructed a photon-pair source that is based on the Kerr nonlinearity ($\chi^{(3)}$) of standard telecom fiber. Using a dispersion-shifted fiber (DSF), we observed and carefully characterized the quantum-correlated nature of photon pairs that constitute the parametric fluorescence associated with the four-wave mixing (FWM) process. However, in the experiment it was noted that the fiber emits more uncorrelated photons than expected from a simple FWM theory. In this paper, we present a CW quantum theory that treats the processes of FWM and Raman gain and loss in a unified way. This theory attributes the observed uncorrelated photons to the Raman effect. However, the theory cannot account for all of the observed excess uncorrelated photons, pointing to the need for a more detailed multimode theory to include the effects of the pulsed pump and the filters used in the experiment.

2. THEORY OF SPONTANEOUS EMISSION IN A FIBER PARAMETRIC AMPLIFIER (FPA)

The frequency response of the $\chi^{(3)}$ nonlinearity can be written as

$$ F(\Omega) = \int dt \ f(t) \exp(i\Omega t), $$

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where \( f(t) \) is the response function of the Kerr interaction. We write the response function in the frequency domain as
\[
F(\Omega) = F_\| + F_\perp r(\Omega),
\]
which is composed of a time-domain-delta-function-like electronic response (< 5 fs) that is constant over the bandwidths of interest and a time-delayed delta function (> 30 fs) that varies over frequencies of interest and is caused by the back action of nonlinear nuclear vibrations on electronic vibrations. Published measurements of the real part of the Kerr nonlinearity in common optical fibers, though widely varying, yield \( F(0) \) when nonlinear interaction times in the measurements are of much longer duration than the Raman response time, but are of shorter duration than the electrostriction time (typically of nanosecond duration). Along with measurement of the Raman gain profile, one may, by means of the Kramers-Kronig transformation, obtain \( F(\Omega) \) at the frequencies of interest.9 Here we have assumed symmetry in the Raman-gain profile, i.e., \( F(\Omega) = F(-\Omega)^* \).

The asymmetric case will be treated in a subsequent longer paper. We also note here the relation between the published spectra of the Raman-gain coefficient and the coefficients used in this paper. Typical measurements of the counterpropagating pump-and-signal Raman-gain spectrum yield the polarization-averaged power-gain coefficient
\[
g_r(-\Omega) = \frac{g_\|(-\Omega) + g_\perp(-\Omega)}{2}.
\]
At the Raman-gain peak, \( g_\perp \approx 0 \). For co-propagating, co-polarized, optical waves \( F(-\Omega) = g_\|(-\Omega)/2 \). We estimate the spectrum of \( g_\| \), normalized to its maximum value, from Ref. 10 and take its magnitude from Ref. 11 for both dispersion-shifted fiber (DSF) and standard single-mode fiber (SMF). For \( F(0) \), we use measurements from Ref. 12. We define a nonlinearity coefficient
\[
H(\Omega) = \frac{2\pi F(\Omega)}{\lambda A_{\text{eff}}},
\]
where \( \lambda \) is the pump wavelength and \( A_{\text{eff}} \) is the fiber effective area.

A self-consistent quantum theory of light propagation in a non-zero \( \chi^{(3)} \)-response-time medium has been developed13 and the associated Raman-noise limit on the generation of squeezing in such a medium via fully frequency-degenerate four-wave mixing has been found.14 This theory is consistent with the classical mean-field solutions and preserves the continuous-time field commutator. Whereas the theory in Ref. 13 provides integral-form expressions for the propagation of a multimode total field, dispersion was not explicitly included. In the following, we present a theory for parametric amplification in the undepleted-pump approximation that yields analytical expressions while preserving the commutators for the signal and idler fields.

Consider the field operator
\[
\hat{A} = \hat{A}_p + \hat{A}_s \exp(i\Omega t) + \hat{A}_a \exp(-i\Omega t)
\]
for the total field propagating through a FPA having a frequency and polarization degenerate pump. The lower frequency field we will call the Stokes field, \( \hat{A}_s \), and the higher frequency the anti-Stokes field, \( \hat{A}_a \). The fields propagate in a lossless, polarization-preserving, single-transverse-mode fiber under the slowly-varying-envelope approximation. Here the frequency deviation from the pump frequency is \( \Omega = \omega_s - \omega_p = \omega_a - \omega_s \). The quantum equation of motion for the total field can be written as13,14
\[
\frac{\partial \hat{A}(t)}{\partial z} = i \frac{\beta_2}{2} \frac{\partial^2 \hat{A}}{\partial t^2} + i \int d\tau h(t-\tau)\hat{A}^\dagger(\tau)\hat{A}(\tau) \hat{A}(t) + \hat{m}_z(t),
\]
where \( \beta_2 \) is the group-velocity-dispersion coefficient at \( \omega_p \) and the operator \( \hat{m}_z(t) \) is a phase-noise operator that is required to preserve the continuous-time commutators
\[
[\hat{A}(t), \hat{A}^\dagger(t')] = \delta(t-t'),
\]
\[
[\hat{A}(t), \hat{A}(t')] = 0.
\]
operators contributing. Under the undepleted-pump approximation it is also acceptable to neglect the fluctuation operators at all frequencies except the Stokes and anti-Stokes frequencies because only the pump mean-field interacts with the modes of interest to a non-negligible degree, as can be shown by linearization of the quantum fluctuations. Unlike in Ref. 15, we obtain
\[
\frac{dA_p}{dz} = i H(0) \hat{A}_p^\dagger \hat{A}_p,
\]
\[
\frac{dA_a}{dz} = i [H(0) + H(\Omega)] \hat{A}_a^\dagger \hat{A}_a + i H(\Omega) \hat{A}_p^\dagger \hat{A}_a \exp(-i\Delta k z) + \hat{M}_z(\Omega),
\]
\[
\frac{dA_s}{dz} = i [H(0) - H(\Omega)] \hat{A}_s^\dagger \hat{A}_s + i H(-\Omega) \hat{A}_p^\dagger \hat{A}_a \exp(-i\Delta k z) + \hat{M}_z(-\Omega),
\]
where \(\Delta k = \beta_2 \Omega^2\). In Eqs. (10) and (11), respectively, we identify
\[
i[H(0) + \text{Re}\{H(\pm\Omega)\}] \hat{A}_p^\dagger \hat{A}_p \hat{A}_{a(s)}
\]
as the pump cross-phase modulation term,
\[
-\text{Im}\{H(\pm\Omega)\} \hat{A}_p^\dagger \hat{A}_p \hat{A}_{a(s)}
\]
as the Raman loss (gain) term,
\[
i\text{Re}\{H(\pm\Omega)\} \hat{A}_a^\dagger \hat{A}_{a(s)} \exp(-i\Delta k z)
\]
as the electronic and in-phase Raman-mediated four-wave-mixing term, and
\[
-\text{Im}\{H(\pm\Omega)\} \hat{A}_s^\dagger \hat{A}_{a(s)} \exp(-i\Delta k z)
\]
as the quadrature Raman-mediated four-wave-mixing term. All of the above interactions are photon-number preserving, and all but the Raman loss and gain terms conserve energy in the multimode optical field. Thus only the Raman terms require the addition of commutator-preserving quantum-noise operators that couple the field to the molecular vibration modes in the \(\chi^{(3)}\) medium. The solution for the mean fields \(\langle \hat{A}_j \rangle = \bar{A}_j\) can be written as:
\[
\hat{A}_p(z) = \bar{A}_p(0) \exp[iH(0)|\bar{A}_p(0)|^2 z],
\]
\[
\hat{A}_a(z) = \mu_a(z) \bar{A}_a(0) + \nu_a(z) \bar{A}_s(0),
\]
\[
\hat{A}_s(z) = \mu_s(z) \bar{A}_s(0) + \nu_s(z) \bar{A}_a(0),
\]
where
\[
\mu_a(z) = \exp[-i(\Delta k/2 - H(0)|\bar{A}_p|^2) z] \left[ \cosh(gz) + \frac{i\kappa}{2g} \sinh(gz) \right],
\]
\[
\mu_s(z) = \exp[i(\Delta k/2 - H(0)|\bar{A}_p|^2) z] \left[ \cosh(g^* z) + \frac{i\kappa^*}{2g^*} \sinh(g^* z) \right],
\]
\[
\nu_a(z) = \exp[-i(\Delta k/2 - H(0)|\bar{A}_p|^2) z] \frac{iH(\Omega)|\bar{A}_p|^2}{g} \sinh(gz),
\]
and \(\nu_s = -\nu_a^*\) with
\[
g = \left[(\kappa/2)^2 + (H(\Omega)|\bar{A}_p|^2)^2\right]^{1/2},
\]
\[
\kappa = \Delta k + 2H(\Omega)|\bar{A}_p|^2,
\]
and \(|\bar{A}_j|^2\) has units of Watts.
Figure 1. Gain spectrum for 1 km-long FOPA pumped at 1537 nm with 1.5 W power. Fiber’s dispersion zero is at 1537 nm, the dispersion slope is 0.064 ps/(nm²/km), and the nonlinear coefficient $H(0) = 1.8 W^{-1/3} k m^{-1}$. $\text{Im}\{H(\Omega)\}$ calculated from Raman measurements$^{10-13}$ (dots) and $\text{Im}\{H(\Omega)\} = 0$ (solid line).

Figure 1 shows the power gain vs. signal-pump detuning for a FOPA made with DSF when the nonlinearity is assumed to be instantaneous (solid line) and when the complex nonlinear response at 1.3 THz is included as explained above (dotted line). We note that the power gain of the mean field is modified only slightly by the non-zero time response of the nonlinearity in the DSF.

Each differential element of the fiber couples in noise from an independent reservoir of phonon oscillators to the Stokes and anti-Stokes modes with a coupling strength that preserves the mode commutators. Each phonon mode is assumed to be in a thermal state with mean phonon-occupation-number $n_{th} = 1/\{\exp(h\Omega/kT) - 1\}$ and commutator $[\hat{M}_j(\pm \Omega), \hat{M}_j'(\pm \Omega')] = \delta(\Omega - \Omega')\delta(z - z')$. Here $h$ is Planck’s constant over $2\pi$, $k$ is Boltzmann’s constant, and $T$ is the temperature. Our goal in this paper is to calculate the correlation between the photons that are emitted at the Stokes and anti-Stokes wavelengths. For a coherent-state undepleted pump, one can show by linearizing around the pump’s mean field that the quantum fluctuations of the pump make negligible contributions to the photon correlations that develop through Eqs. (10) and (11). Accordingly, we treat the pump field as if it were a c-number field in Eqs. (9)–(11). Under these conditions, the operator equations (10)–(11) can be solved exactly to give

$$\hat{A}_a(z) = \mu_a(z)\hat{A}_a(0) + \nu_a(z)\hat{A}_s(0) + \sqrt{1 - |\mu_a(z)|^2 - |\nu_a(z)|^2} \hat{\eta}_a,$$  \hspace{1cm} (20)

$$\hat{A}_s(z) = \mu_s(z)\hat{A}_s(0) + \nu_s(z)\hat{A}_a(0) + \sqrt{|\mu_s(z)|^2 - |\nu_s(z)|^2 + 1} \hat{\eta}_s,$$ \hspace{1cm} (21)

where $\hat{\eta}_a$ and $\hat{\eta}_s$ are the thermal-field operators representing the sum of the contributions by each differential element $\hat{M}_j$ propagated through the remaining length of fiber.

In order to properly model the experimental conditions in Ref. 7, we must account for the imperfect collection efficiency of the detection system. The propagation, filtering, and detection efficiencies can be lumped and modeled as a single beamsplitter placed at the output of the fiber, having a transmissivity $\eta_j$ for $j = s, a$ at the Stokes and anti-Stokes wavelengths. A representation of this is shown in Fig. 2. Labelling the input modes as $\hat{b}_j = \hat{A}_j(z)$ and the output modes as $\hat{c}_j$, we can write the mode transformations from the input through the FPA and the beamsplitter as:

$$\hat{c}_a = \sqrt{\eta_a} \mu_a \hat{b}_a + \sqrt{\eta_a} \nu_a \hat{b}_s + \sqrt{\eta_a} w_a \hat{\eta}_a + \sqrt{1 - \eta_a} \hat{\eta}_a,$$  \hspace{1cm} (22)

$$\hat{c}_s = \sqrt{\eta_s} \mu_s \hat{b}_s + \sqrt{\eta_s} \nu_s \hat{b}_a + \sqrt{\eta_s} w_s \hat{\eta}_s + \sqrt{1 - \eta_s} \hat{\eta}_s,$$ \hspace{1cm} (23)

where $\hat{\eta}_j$ are mutually independent thermal- and vacuum-state operators for the Stokes and anti-Stokes modes ($j = s, a$), and

$$w_a = \sqrt{1 - |\mu_a|^2 + |\nu_a|^2},$$ \hspace{1cm} (24)
Figure 2. A schematic of the optical-fiber photon-pair source with loss: BS, beamsplitter with transmissivity $\eta_j$. 

$$w_s = \sqrt{-1 + |\mu_s|^2 - |\nu_s|^2}. \quad (25)$$

The correlation coefficient for simultaneous photon-counting measurements ($\hat{N}_j = \hat{c}_j^\dagger\hat{c}_j$ is the photon-number operator) on these two modes is:

$$C = \frac{\langle \Delta \hat{N}_s \Delta \hat{N}_a \rangle - \langle \hat{N}_s \rangle \langle \hat{N}_a \rangle}{\sqrt{\langle \Delta \hat{N}_s^2 \rangle \langle \Delta \hat{N}_a^2 \rangle}} = \frac{\eta_s \eta_a \mu_s^* \nu_a^* \nu_s^* \mu_a^*}{\sqrt{\langle \Delta \hat{N}_s^2 \rangle \langle \Delta \hat{N}_a^2 \rangle}}. \quad (26)$$

We define the probability of measuring a photon in channel $j$ as $P(N_j = 1) = P_j$ and the probability of detecting a coincidence to be $P(N_s = 1, N_a = 1) = P_c$. When the probability of two or more photons being detected by the photodetector $j$ is negligible [i.e., $P(N_j > 1) \approx 0$], we obtain the following expression for the coincidence probability:

$$P_c = C \sqrt{P_s(1 - P_s)} \sqrt{P_a(1 - P_a)} + P_s P_a. \quad (27)$$

3. RESULTS AND COMPARISON WITH EXPERIMENT

We compare the above theoretical predictions of the coincidence probability $P_c$ with previously obtained experimental results.\textsuperscript{7} In that experiment, pump pulses composed of $\sim 3$ ps full-width at half-maximum (FWHM) and optical filtering in the Stokes and anti-Stokes bands (FWHM bandwidth of 0.47 nm) were used. The peak efficiency for detecting the Stokes photons was $\eta_s = 0.08$ and that for the anti-Stokes photons was $\eta_a = 0.07$. In addition, the quantum efficiency for detection of photons with polarization orthogonal to the pump polarization was approximately 50%. This means that we should include the Raman spontaneous emission into the polarization direction perpendicular to that of the pump as well. At a frequency shift of 9 nm, where the signal and idler photons were measured, and including the lower efficiency for Raman scattering in the perpendicular polarization, we estimate that the average number of Raman photons in the perpendicular polarization is approximately 19% of the number of Raman photons in the parallel polarization. Furthermore, because the power-gain nonlinear coefficient for FWM with signal and idler polarizations perpendicular to the pump polarization is $1/9$ th that of FWM with parallel polarizations, and because the perpendicular FWM does not phase-match well due to the smaller nonlinear coefficient, we neglect the perpendicular parametric fluorescence in our calculation.

We show the results in Fig. 3. Note that without inclusion of the Raman spontaneous emission (triangles), there is very poor fit with the experimental data (circles).\textsuperscript{7} However, when the Raman spontaneous emission is included (dash-dots), the theory is much closer to the experimental results. By artificially degrading the theoretical correlation coefficient $C$ by a factor of 0.65 (i.e., $C_{\text{fit}} = 0.65C$), we obtain a relatively good fit when including the Raman spontaneous emission (dashes). On the other hand, when neglecting the Raman spontaneous emission, one must multiply $C$ by a factor of 0.2 to obtain a good fit (dots). For comparison purposes, we also show theoretical results with $C = 0$ (solid line) and the measured coincidence probability when the signal and idler photons originate from different pump pulses (crosses).\textsuperscript{7} This means that the Raman spontaneous emission is important in the experiment of Ref. 7 and needs to be included in the model. It also means that a CW theory is inadequate for explaining the experimental results. The effect of the pulsed pump...
and optical filtering must also be included to correctly account for the further loss of quantum correlation in the experiment. This is because the pulsed pump gives rise to spreading of the signal-idler correlations in the frequency domain, which is not included in the CW model.

4. CONCLUSION

In conclusion, we have developed a theory for the generation and detection of photon-pairs in a Kerr medium. We have included the effects of Raman scattering and of realistic transmission and detection losses. This theory predicts stronger photon-number correlations than seen experimentally with a pulsed pump, showing the need for development of a pulsed-pump theory.

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REFERENCES


