Magnetic Flux Controlled Josephson Array Oscillators

Donald S. Crankshaw, Enrique Trías, and Terry P. Orlando

Abstract— **One-dimensional parallel arrays of Josephson junctions have the ability to perform as oscillators tunable by magnetic flux. We have designed and tested an array impedance matched to a detector junction load. The amplitude and frequency of the array are controlled by the independent variables of magnetic field and current bias. We report on the array's characteristics and compare them to the results predicted by nonlinear simulations and a linear circuit model. This confirms the accuracy of the circuit model, both for impedance matching and in describing the dynamics of the array, even in a multiple-frequency regime.**

***Index Terms*—array, oscillator, Josephson junction**

I. Introduction

Arrays of parallel Josephson junctions can operate as tunable oscillators [1]. When the array is biased such that it has a non-zero voltage, V_{arr}, each junction oscillates in a periodic manner, with a period $T=\Phi_0/V_{arr}$, where Φ_0 is the flux quantum. In a shunted array, a single frequency wave solution is applicable when the array is biased such that the $V_{arr} > I_cR$ (R is the total resistance of the junction, $R_n \| R_a$, the junction's normal resistance and its shunt, respectively) with frequency $\omega=(2\pi/\Phi_0)V_{arr}$. Then each junction can be modeled as an independent, sinusoidal current source, whose phase differs from its neighbor by $2\pi\Phi_{ext}/\Phi_0$, where Φ_{ext} is the applied external flux per cell of the array. Using this model, we can design a load that is impedance matched to the array for maximum power output [2]. This load can be a Josephson junction that detects the power delivered to it.

What is not apparent is that the same single frequency model is useful when $V_{arr} < I_cR$. Despite the fact that the periodic, AC current of each junction is distinctly non-sinusoidal, if one considers each harmonic individually, this approximation still proves useful. This will be shown by a series of nonlinear simulations.

Manuscript received September 19, 2000. This work was supported in part by the National Security Agency (NSA) and Advanced Research and Development Activity (ARDA) under Army Research Office (ARO) contract number DAAG 55-98-1-0369.

D. S. Crankshaw (telephone: 617-253-4699, e-mail: dscrank@mit.edu), T.P. Orlando (telephone: 617-253-5888, e-mail: orlando@mit.edu), and E. Trías (telephone: 617-253-4213, e-mail: etrias@mit.edu) are with the Electrical Engineering and Computer Science Department, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

We have designed an array where this property can be demonstrated experimentally. This array is not only a useful test structure, but can practically be applied as a tunable oscillator where power and frequency can be altered independently, as will be needed for on-chip, tunable low-power oscillators for applications such as quantum computation experiments.

II. Array Model

A. Nonlinear simulation

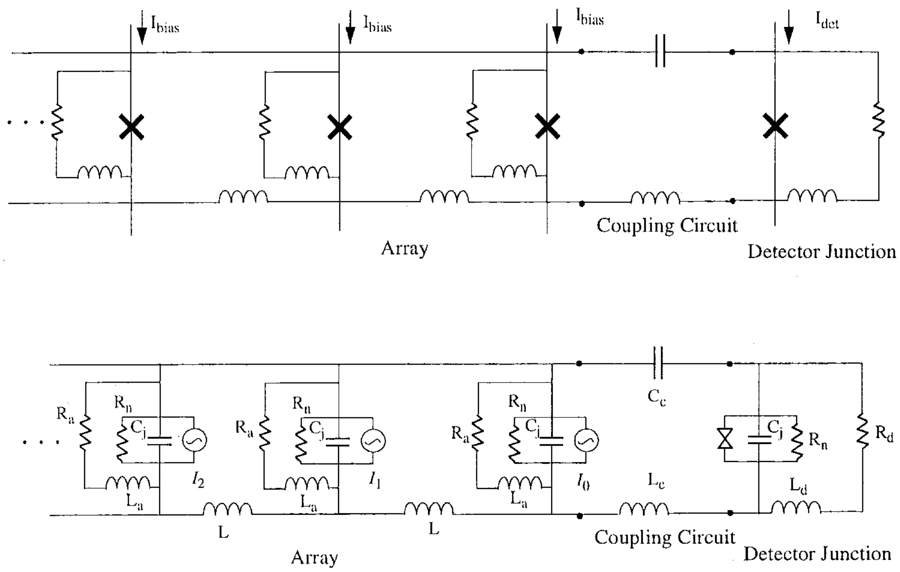
The array that we are using is a one-dimensional, parallel array consisting of 54 junctions. Each cell of the array has a mesh inductance calculated from the cell geometry using a program called FastHenry [3]. Each Josephson junction in the array is modeled using the RCSJ model. The junctions are resistively shunted, and both the resistance and the inductance of the shunt are included in the model. The complete array is modeled in a numerical simulation which solves for flux quantization and Kirchhoff's voltage and current laws. The array is shown in Fig. 1(a). The parameters for each junction are typical of the experimental parameters: I_c=120 μA, R_n=8.6 Ω, and C_j=300 fF. The cell inductance, L=15.2 pH, giving a value of L_J/L_s =0.177. The shunt resistance of each junction is R_a=2.6 Ω, while the inductance of the loop it forms is L_a=1.28 pH. This gives a Stewart-McCumber parameter of β_c=0.5.

B. Linear circuit model, $V_{arr} > I_cR$

When the array is biased at a high enough current I_B, and thus a voltage of $V_{arr}>I_cR$, a traveling wave solution is apparent. At this bias, each of the array's junctions is in a whirling mode [4], producing an approximately sinusoidal current of magnitude I_c. Modeling these as independent sources, which have an amplitude of I_c and a frequency of $\omega=(2\pi/\Phi_0)V_{arr}$, and which differ in phase by $2\pi\Phi_{ext}/\Phi_0$, allows a fairly simple circuit model, as shown in Fig. 1(b). This model can be solved analytically. From this linear circuit network, we can calculate the equivalent impedance of the array. First, each individual junction has an impedance of

$$Z_{J,a} = \frac{1}{\frac{1}{R_n} + j\omega C_j + \frac{1}{R_a + j\omega L_a}} . \tag{1}$$

These parameters are labeled in Fig. 1(b). For an array



(a)

(b)

Fig 1. (a) Schematic of array. (b) Diagram of circuit used to model array. $I_i=\alpha I_c\sin(\omega t+\theta_i)$, where $\theta_i=2\pi if$.

with many junctions, adding one more junction and its cell inductance should not change the impedance of the array. Thus, the impedance of the array can be calculated recursively, so that the equivalent impedance at one end is the recursive solution in parallel with the final junction:

$$Z_r = \frac{j\omega L + \sqrt{(j\omega L)^2 + 4j\omega L Z_{J,a}}}{2}$$
$$Z_{arr} = \left(\frac{1}{Z_{J,a}} + \frac{1}{Z_r}\right)^{-1} \tag{2}$$

Furthermore, the circuit model allows the calculation of how the power delivered to the load varies with frustration, $f=\Phi_{ext}/\Phi_0$. One would normally assume that the power is maximum when all the junctions are in phase, $f=0$. This need not be the case, however. The inductance of the cell induces a current lag between the junctions. This means that the currents from two neighboring junctions sum only if the phase difference compensates for the current lag. In fact, the inductance paired with the resistor and capacitor of the RCSJ model together form a low pass filter. This means that the load, at one end of the array, will not see a power equal to the sum of all the junctions' output, but rather the total power will asymptotically approach a finite level as more junctions are added to the array. The number of junctions where this levels off is dependent on the parameters of the array, but here it is about 10. Nonlinear simulations show that this model is effective. The phase difference between the junctions is approximately as predicted (Fig. 2(a)), as is the current amplitude (Fig. 2(b)).

A. Linear circuit model, $V_{arr}<I_cR$

When the voltage is significantly smaller, each junction's Josephson current is non-sinusoidal. Despite this, the model may still be used if each harmonic is examined individually. The magnitude of each harmonic for a resistively shunted junction is given by

$$\alpha = 2\nu\left[\left(1+\nu^2\right)^{1/2} + \nu\right]^{-n} \tag{3}$$

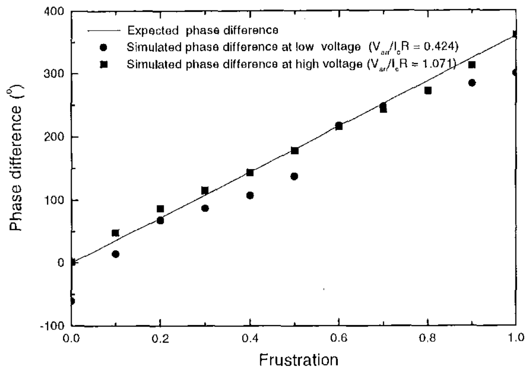
where n is the order of the harmonic, and $\nu = V_{arr}/I_cR$ [5]. As can be seen in Fig. 2(a) and (b), the phase difference and magnitude follow the same trend. There are, however, some obvious outliers near f=0.8.

B. Impedance matching the load

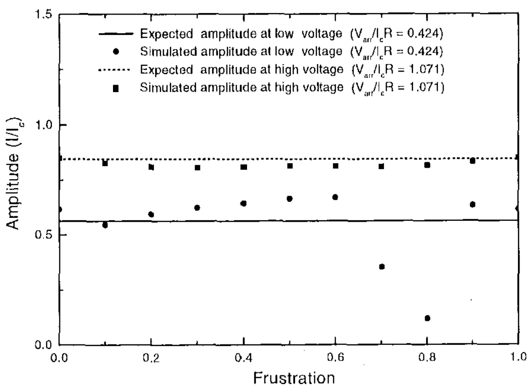
Once the array is modeled by a simple impedance, the detector junction can be matched to it. The detector junction is a single junction, identical to the junctions in the array, and shunted by a resistance. The junction parameters are determined by our choice of critical current and critical current density rather than our desire to match the impedances. Thus it is that the shunt resistance and inductance are adjusted to match the array impedance. The detector's impedance is given by

$$Z_{J,d} = \frac{1}{\frac{1}{R_n} + j\omega C + \frac{1}{R_d + j\omega L_d}}\ . \tag{4}$$

It is separated from the array by a capacitor to block DC current, which forms a loop with an inductance. It has an impedance of



(a)



(b)

Fig 2. A comparison of simulations of the array biased at high and low voltage: (a) the phase difference between adjacent junctions, and (b) the amplitude of the oscillation.

$$Z_c = j\omega L_c - \frac{j}{\omega C_c}. \tag{5}$$

Note that this impedance is purely imaginary. Thus while the detector junction's parameters are adjusted to match the real part of the array's impedance, the coupling circuit's capacitance and inductance can be adjusted until the imaginary part of the impedance is matched, thus achieving the matching requirement: $Z_{arr}=(Z_c+Z_{J,d})^*$. The parameters of the junction itself are identical to the junctions in the array, but its shunt resistance and inductance are R_{sh}=2.6 Ω and L_{sh}=1.36 pH, respectively, while the coupling capacitor and inductance are C_c=200 fF and L_c=50 pH.

In the simulations, the accuracy of this matching is apparent in the fact that the voltage delivered to the load is a clean sinusoidal voltage, while that supplied by the array has many harmonics at low V_{arr}.

III. Experiments

Experiments were performed on Nb arrays manufactured by HYPRES. The devices had a current density of 2.5 kA/cm^2. The measurements were performed with the samples heated to 7.0 K to give a critical current of 120 μA, for which the circuit was designed. The junctions were 3μm x 3μm. An external coil supplied the magnetic field. Figure 3 shows the bias point of the array. The array is maintained at a constant voltage using feedback loop is which adjusts the current bias as frustration is varied.

When rf power of frequency ω_s is delivered to a Josephson junction, current steps are expected to develop at voltages corresponding to multiples of the frequency, ω_s. If the junction were voltage biased, these steps should have heights described by Bessel functions of the same order as the step. In a current-biased junction, however, such as the junction that we are using as a detector in this circuit, the steps are less easily described analytically. Finding the power corresponding to the current-voltage (I-V) curve in Fig. 4, for example, must be done by using the nonlinear simulations, as shown by [6] and [7]. Therefore, the measured power was determined from the Shapiro step height by using simulations to determine the step height and critical current suppression corresponding to different amplitudes of independent sources, and then mapping those values to our measured I-V curves. We have considered the height of the first step and the suppression critical current independently, allowing two estimates of the power. Only the calculation from the step height is shown. The power calculated from the critical current suppression, while not identical, follows the same trend.

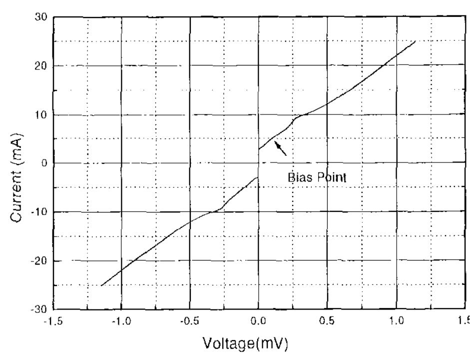


Fig 3. Measured current versus voltage of the array at 7.0 K. The bias voltage which gives a 50 GHz output is indicated, V_{arr} = 0.1035 mV=0.424 I_cR

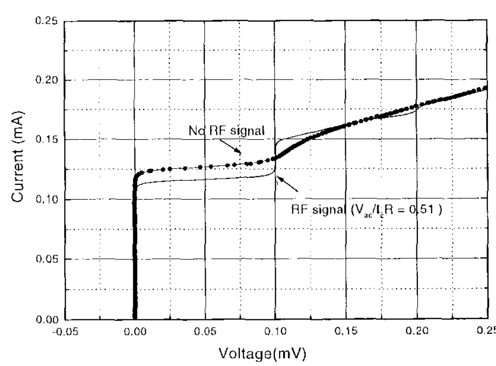


Fig 4. Measured current vs. voltage curves of detector junction at 7.0 K, showing the curve both with and without an RF source of 50 GHz (V_{arr} = 0.1035 mV)

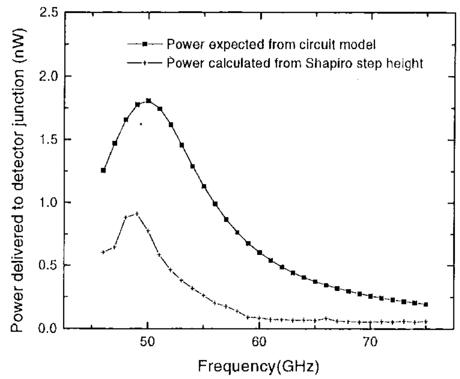


Fig 5. Power delivered to the detector junction varying with the applied frequency. The frequency is determined by the bias point of the array's voltage. $\nu=V_{arr}/\Phi_0$.

Figure 5 shows the power delivered to the load versus the frequency of the array, indicating the frequency response of the coupling to the detector junction. The circuit was designed to be impedance matched at a frequency of 50 GHz, and this graph indicates that the designed impedance matching was successful.

Figure 6 shows the variation of power with frustration. In this experiment, while the array was current biased, it was the voltage, V_{arr}, which was held constant through a feedback loop. The power is periodic in frustration and can be tuned from its maximum value of about 1 nW to near zero, while the frequency (voltage) of the array is maintained at a constant value. Similar dependencies are found at other frequencies. The expected power is shown from both the linear circuit model (+) and the nonlinear simulations (o). The solid line is a fit through the nonlinear simulation points. Results differ from the predicted values in three significant ways. First, the measured power is less than the predicted power by approximately a factor of 2 (in Fig. 6, the predicted power is shown reduced by this factor in order to more clearly show the match in the shape). Second, the offset of the maximum differs from that predicted (0.7 rather than 0.8). Third, the peak in the power is broader than expected. Notice that the shape of the nonlinear simulation prediction is closer to the experimental results than the linear circuit model, even though its prediction of the amplitude is slightly larger.

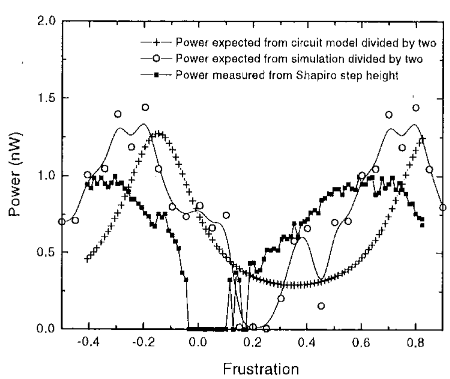


Fig. 6. The power produced by the array, experimental measurements compared to nonlinear simulation and linear circuit model predictions. The array is biased at $V_{arr} = 0.1035$ mV.

IV. Conclusions

The model has proven useful in predicting the general trend and, more importantly, in allowing an impedance match between the array and the junction. The experiments further confirm that amplitude and frequency can be modified independently using frustration and bias current, resulting in a tunable rf source, with a clean output signal at 50 GHz with a frequency bandwidth of about 6 GHz. These independent tunability requirements are needed for applications such as on-chip oscillator controlling circuits for quantum computation. For such applications, the low output power (1 nW) is sufficient.

Acknowledgment

The authors would like to thank Lin Tian, Juan Mazo, and Daniel Nakada for helpful discussions. We would especially like to thank Andrew Kuzmieko for his aid in the experimental work and data processing. The HYPRES foundry, located in Elmsford, NY, manufactured the samples.

References

[1] A.K. Jain, K.K. Likharev, J.E. Lukens, and J.E. Sauvageau, "Mutual phase-locking in Josephson arrays," *Phys. Rep.*, vol. 109, pp. 309-426, 1984.

[2] E. Trías, T.P. Orlando, A.E. Duwel, and S. Watanabe, "Circuit models for arrays of Josephson oscillators with loads," *IEEE Trans. Appl. Supercond.*, vol. 9, pp. 4316-4319, 1999.

[3] M. Kamon, M.J. Tsuk, J.K. White, "FastHenry – a multipole accelerated 3-D inductance extraction program," *IEEE Trans. on Microwave Theory and Techniques*, vol. 42, pp. 1750-1758, 1994.

[4] S. Watanabe, S.H. Strogatz, H.S.J. van der Zant, and T.P. Orlando, "Whirling modes and parametric instabilities in the discrete Sine-Gordon equation: experimental tests in Josephson rings," *Phys. Rev. Lett.*, vol. 74, pp. 379-382, 1995.

[5] J. Lukens, "Josephson arrays as high frequency sources," in *Superconducting Devices*, S.T. Ruggiero and D.A. Rudman, Eds., Boston, MA: Academic Press, Inc., 1990, pp. 135-167.

[6] A.M. Kadin, *Introduction to Superconducting Circuits*. New York, NY: John Wiley&Sons, Inc., 1999.

[7] Y. Taur, P.L. Richards, and F. Auracher, "Application of the Shunted Junction Model to Point-Contact Josephson Junctions," in *Low Temperature Physics--LT-13*, vol. 3, K.D. Timmerhaus, W.J. O'Sullivan, and E.F. Hammel, Eds., New York: Plenum Press, 1974, pp. 276-280.