

Projective measurement scheme for solid-state qubits

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We present an effective measurement scheme for the solid-state qubits, which does *not* introduce extra decoherence to the qubits until the measurement is switched on by a resonant pulse. The resonant pulse then maximally entangles the qubit with the detector. The scheme has the feature of being projective, noiseless, and switchable. This method is illustrated on the superconducting persistent-current qubit, but can be applied to the measurement of a wide variety of solid-state qubits, the direct detection of the electromagnetic signals of which gives poor resolution of the qubit states.

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Quantum computation in solid-state systems is a growing field.^{1–8} Among various physical realizations, solid-state qubits have the advantage of being scalable to a large number of qubits, and that the quantum states can be engineered by various techniques. Successful implementations of qubits have been achieved in several mesoscopic systems.^{9–13}

Effective measurement of quantum bits is a crucial step in quantum computing. An ideal measurement of the qubit is a projective measurement¹⁴ that correlates each state of the quantum bit with a macroscopically resolvable state. In practice, it is often hard to design an experiment that can both projectively measure a solid-state qubit effectively and meanwhile does *not* couple environmental noise to the qubit. Often in solid-state systems, the detector is fabricated onto the same chip as the qubit and couples with the qubit all the time. On one hand, noise should not be introduced to the qubit via coupling with the detector. This requires that the detector is a quantum system well isolated from the environment. On the other hand, to correlate the qubit states to the macroscopically resolvable states of the detector, the detector should behave as a classical system that has strong interaction with the environment, and at the same time interacts with the qubit strongly. These two aspects contradict each other, hence measurements on the solid-state quantum bits are often limited by the trade-off between these two aspects.^{15,16} In the first experiment on the superconducting persistent-current qubit (pc qubit),¹⁰ the detector is an underdamped dc superconducting quantum interference device (SQUID) that is well isolated from the environment and behaves quantum mechanically. The detected quantity of the qubit—the self-induced flux—is small compared with the width of the detector’s wave packet. As a result, the detector has very bad resolution on the qubit states. This is one of the major problems in the study of the flux-based persistent-current qubits.^{3,4,10}

Various attempts have been made to solve the measurement problem^{17–19} and to provide proposals for a scalable quantum computer with superconducting qubits.²⁰ In a recent experiment on the flux qubit, the measurement efficiency has been greatly improved by optimizing the bias current, and coherence oscillation has been observed.²¹ In this paper, we present a scheme that effectively measures the pc qubit by an

on-chip detector in a “single-shot” measurement and does *not* induce extra noise to the qubit until the measurement is switched on. The idea is to make a switchable measurement (but a fixed detector) that only induces decoherence during the measurement. During the regular qubit operation, although the qubit and the detector are coupled, the detector stays in its ground state and only induces an overall random phase to the qubit. The measurement process is then switched on by resonant microwave pulses. First, we maximally entangle the qubit coherently to a supplementary quantum system. Then we measure the supplementary system to obtain the qubit’s information. This approach of exploiting conditional resonant transitions for signal amplification is different from the other approaches.

In the following discussion, we illustrate this method by applying it to the measurement of the superconducting pc qubit. In the previous experiment,¹⁰ the qubit is inductively coupled to the detector—a dc SQUID. The flux of the qubit affects the critical current of the SQUID by an offset $\Delta I_c = 2I_c |\sin(\varphi_{ext} + \delta\varphi_q)/2 - \sin\varphi_{ext}| \sim \pm I_c \delta\varphi_q \approx \pm 10^{-3} I_c$, where φ_{ext} is the flux in the SQUID loop in units of $\Phi_0/2\pi$. This offset is recorded by measuring the switching current distribution of the dc SQUID, the average of which is offset by $\sim \Delta I_c$ as well. Due to the quantum fluctuation and thermal activation, the switching current distribution has a finite width that is much larger than ΔI_c . Hence, the two-qubit states result in two switching histograms whose separation is much narrower than the width of the histogram. As a result, the histogram is not perfectly correlated with the qubit states and the measurement has to be repeated many times (10^4 times) to derive the information about the qubit. This problem can be overcome with our method by using a rf SQUID as the supplementary system. Our method is not only closely related to the ongoing experiments of the pc qubit, but also brings a better idea for effectively measuring the solid-state qubits with a resonant pulse technique.

One might worry that coupling the qubit to the rf SQUID brings to the qubit a new source of noise that couples directly to the rf SQUID. However, in our scheme, until the measurement, the coupling between the qubit and the rf SQUID nearly commutes with the Hamiltonian of the rf SQUID, and the rf SQUID stays in its ground state. The rf SQUID be-

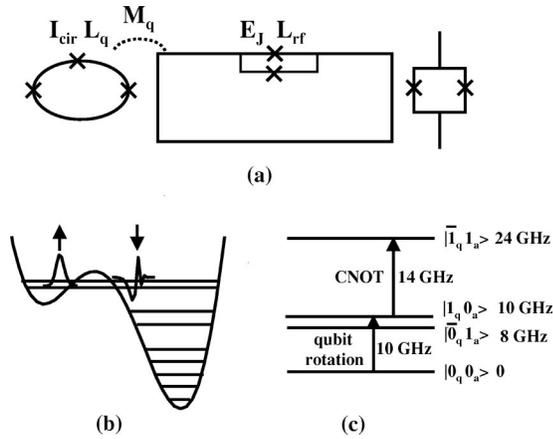


FIG. 1. (a) Circuit of the measurement scheme, from left to right: the qubit, the rf SQUID, and the dc SQUID magnetometer. (b) Eigenstates and potential energy of the rf SQUID when biased at $f_{rf} = 0.4365$ flux quantum. The ETLS are labeled with arrows and the wave functions are shown. (c) The states of the interacting qubit and the rf SQUID.

has as a poor transmitter of the noise, and cannot transfer noise to the qubit as we show below. During the entanglement pulse, the rf SQUID induces decoherence to the qubit in microseconds, which is much longer than the 10 nsec duration of the entanglement pulse.

The superconducting persistent-current qubit^{3,4} is a superconducting loop that has three Josephson junctions in series, see Fig. 1(a). The qubit is controlled by the magnetic flux $f_q \Phi_0$ in the loop, where Φ_0 is the flux quantum. The qubit states of this circuit are nearly localized flux states with opposite circulating currents. The qubit states are analogous to the states with spin 1/2 and are described by the $su(2)$ algebra of the Pauli matrices. The qubit Hamiltonian can be written as $\mathcal{H}_q = (\epsilon_0/2)\sigma_z^q + (t_0/2)\sigma_x^q$, where $\epsilon_0 \propto (f_q - 1/2)$ and t_0 is the coherent tunneling between the two localized flux states over the potential-energy barrier. The operator for the circulating current is $\hat{I}_q = I_{cir}\sigma_z^q$. Typically, the circulating current of the qubit is $I_{cir} \approx 0.7I_c$, where I_c is the critical current of the Josephson junctions and $I_c = 200$ nA. With a loop inductance of $L_q = 10$ pH, the self-induced flux of the qubit is $\delta\varphi_q = 10^{-3}\Phi_0$.

To understand what prevents the effective measurement of the pc qubit in Ref. 10, we analyzed the previous measurement in detail in Refs. 23,24. Considering the dc SQUID as coupled oscillators, the direct coupling between the qubit and the dc SQUID offsets the origin of the SQUID oscillator by $\pm \delta\varphi_0 = \pi M_q I_{cir} / \Phi_0 \approx 0.002$ with typical experimental parameters, where $M_q = 8$ pH is the mutual inductance between the two circuits. The overlapping between the shifted oscillator ground states is $\langle \psi_g^- | \psi_g^+ \rangle = \exp(-\delta\varphi_0^2/2\langle \varphi_m^2 \rangle) \approx 1 - 0.0002$, where $\sqrt{\langle \varphi_m^2 \rangle} \approx 0.1$ is the width of the ground-state wave packet of the inner oscillator of the SQUID. The measurement of the qubit becomes the detection and the resolution of the overlapping and highly nonorthogonal oscillator states $|\psi_g^\pm\rangle$. The overlapping of the oscillator states limits the efficiency of the previous measurement.

Assume the qubit state is $|\psi_q\rangle = c_0|0_q\rangle + c_1|1_q\rangle$. With the

inductive coupling, the density matrix of the dc SQUID (in the previously used method of Ref. 10) quickly relaxes to a mixed state of $|\psi_g^\pm\rangle$, $\rho_m = |c_0|^2 |\psi_g^+\rangle\langle\psi_g^+| + |c_1|^2 |\psi_g^-\rangle\langle\psi_g^-|$. Let the desired measurement accuracy be $A_m = x_{err}/L_x$, where x_{err} is the square-root error from the expected value of the measured variable and L_x is the range of the variable x . For a von Neumann measurement, within N measurements, the average time we find $|0\rangle$ is $|c_0|^2 N$ with the deviation $\Delta N/N = 1/(2\sqrt{N})$. $N_v = 1/(2A_m)^2$ repetitions are required to achieve the accuracy A_m . For a measurement with overlapping distributions as in the previous discussions, assume each distribution is a Gaussian for simplicity. The average of the Gaussian functions y_0 and y_1 are slightly different, but much smaller than $\sqrt{\sigma}$, where σ is the deviation of the Gaussian distributions. Given the qubit state, the average of the measured y 's is $y_{ave}^{expt} = |c_0|^2 y_0 + |c_1|^2 y_1$, from which we can infer $|c_0|^2$ of the qubit state. With a finite number of measurements, $y_{ave} = (1/N)\sum_{k=1}^N y_k$ is described by a Gaussian distribution according to the central limit theorem with an average y_{ave}^{expt} and a deviation σ/N (N measurements). The accuracy with N measurements is $\Delta y_{ave}/|y_1 - y_0| = \sqrt{\sigma/N}|y_1 - y_0|^2$. $N_p = (4\sigma/|y_1 - y_0|)^2 N_v$ repetitions are required to achieve the accuracy A_m . In the previous experiment, $2\sqrt{\sigma}/|y_1 - y_0| = 50$, so $N_p/N_v > 10^3$ is required to get satisfactory results.

This shows that the measurement in Ref. 10 is not an efficient measurement. The quantum nature of the dc SQUID (which is intentionally designed to reduce decoherence) prevents efficient detection of the qubit's information. To get a more efficient measurement, we should either encode information in the pc qubit in some other way or measure the qubit states with another approach.

Given the Hamiltonian of a qubit, a projective measurement that correlates the eigenstates to the distinct macroscopic states can always be constructed according to the Neumark's theorem.¹⁴ However, in experiments, it is not obvious that we can build an apparatus that effectively measures the pc qubit without introducing extra noise. In the following, we present a measurement scheme that improves the previous measurement significantly and is both effective and noiseless.

The idea is that instead of directly detecting the flux of the qubit, we first apply a short microwave pulse to entangle the qubit with a supplementary quantum system that behaves as an effective two-level system (ETLS). The flux or charge of the ETLS is designed to be much larger than that of the qubit. Then the ETLS whose state exactly reflects the qubit state is measured. This scheme is a highly effective "single-shot" measurement, and meanwhile it avoids transferring extra noise from the detector to the qubit.

Let the pc qubit interact with this supplementary system via inductive interaction $M_q \hat{I}_q \hat{I}_a$. We assume that the supplementary system is also a current loop with $\hat{I}_a = I_a \sigma_z^a$ and σ_z^a is the Pauli matrix of the ETLS. The Hamiltonian is

$$\mathcal{H}_0 = \frac{\epsilon_0}{2} \sigma_z^q + \frac{t_0}{2} \sigma_x^q + \frac{\hbar \omega_a}{2} \sigma_z^a + \frac{t_0^a}{2} \sigma_x^a + \frac{\hbar \omega_\Delta}{2} \sigma_z^q \sigma_z^a, \quad (1)$$

where ω_a is the energy splitting of the ETLS and ω_Δ is the inductive interaction. We design the tunneling t_0^a to be adjustable. During the qubit operation, $t_0^a=0$; during measurement, t_0^a is a resonant pulse that flips the ETLS. The energy levels are shown in Fig.1(c).

During regular computation, we store the supplementary system in its ground state $|0_a\rangle$. Due to the interaction, the qubit's energy is modified as $\hbar\omega_q = \sqrt{(\epsilon_0 - \hbar\omega_\Delta)^2 + t_0^2}$ and the qubit states are $|0_q\rangle = [-\sin(\theta/2), \cos(\theta/2)]^T$ and $|1_q\rangle = [\cos(\theta/2), \sin(\theta/2)]^T$ with $\sin\theta = t_0/\hbar\omega_q$. By applying an external oscillation with this frequency, single-qubit gates are achieved. In this process, the ETLS stays in its ground state and has trivial dynamics. When the ETLS is at state $|1_a\rangle$, the qubit's energy is $\hbar\bar{\omega}_q = \sqrt{(\epsilon_0 + \hbar\omega_\Delta)^2 + t_0^2}$ and the qubit states are $|\bar{0}_q\rangle = [-\sin(\bar{\theta}/2), \cos(\bar{\theta}/2)]^T$ and $|\bar{1}_q\rangle = [\cos(\bar{\theta}/2), \sin(\bar{\theta}/2)]^T$ with $\sin\bar{\theta} = t_0/\hbar\bar{\omega}_q$.

To measure the qubit's state, local operation on the supplementary system is applied to entangle the two systems. With the presence of the qubit, we have $E_{\bar{0}_q 1_a} - E_{0_q 0_a} = \hbar\omega_a - (\hbar\bar{\omega}_q - \hbar\omega_q)/2$ and $E_{\bar{1}_q 1_a} - E_{1_q 0_a} = \hbar\omega_a + (\hbar\bar{\omega}_q - \hbar\omega_q)/2$. By applying an external pulse of $\frac{1}{2}\hbar\Omega_X\sigma_x^a$ in resonance with $E_{\bar{1}_q 1_a} - E_{1_q 0_a}$, the ETLS is flipped to the state $|1_a\rangle$. Let $(\bar{\omega}_q - \omega_q) \gg \Omega_X$, the off-resonant transition between the states $|\bar{0}_q 1_a\rangle$ and $|0_q 0_a\rangle$ is negligible and the dynamics only depends on the resonance properties. After a π pulse that operates as $\exp[i(\pi/2)\sigma_x^a]$, the ETLS is maximally entangled with the qubit, $(c_0|0_q\rangle + c_1|1_q\rangle)|0_a\rangle \rightarrow c_0|0_q 0_a\rangle + ic_1|\bar{1}_q 1_a\rangle$, which gives the density matrix of the ETLS as

$$\rho_a = |c_0|^2 |0_a\rangle\langle 0_a| + |c_1|^2 |1_a\rangle\langle 1_a| + (ic_0^*c_1 \langle 0_q|\bar{1}_q\rangle |1_a\rangle + \text{c.c.}), \quad (2)$$

where $\langle 0_q|\bar{1}_q\rangle = \sin(\bar{\theta} - \theta)/2$. The probabilities $|c_{0,1}|^2$ of the ETLS are then measured by a detector. Note that the supplementary system does not have to be a qubit and be well isolated from the environment itself. But it is required that the noise is not transferred back to the qubit during the regular qubit operations.

In this design, we choose a rf SQUID to be the supplementary system that inductively couples with the qubit. The circuit is shown in Fig. 1(a). The detector is a damped dc SQUID magnetometer. In the following, we adopt the parameters as in Refs. 22,11 for the rf SQUID, where coherent manipulation of the rf SQUID has been achieved experimentally. Typical parameters are $L_{rf} = 154$ pH, $I_c = 4$ μ A, $C_J = 40$ fF, and $E_J/E_C \approx 4000$. The inductance of the rf SQUID is much larger than that of the qubit, which has two consequences: (1) the flux difference between the states localized in the two potential wells of the rf SQUID is of an order of half a flux quantum, and can be resolved by a dc SQUID magnetometer in a "single-shot" detection; (2) the coupling between the rf SQUID and the environmental noise is strong, hence it is harder to keep the coherence of the rf SQUID and to use the rf SQUID as a qubit directly. At $\beta_L = 2\pi L_{rf} I_c / \Phi_0 \approx 1.9$, the rf SQUID has a double-well potential with several eigenstates localized in each well. In prac-

tice, the junction is always made of a SQUID where $E_J(\Phi_{ex})$ is controllable by the external flux Φ_{ex} and hence is t_0^a . The potential energy of the rf SQUID is drawn in Fig. 1(b) with the energies of its eigenstates. By adjusting the parameters, the two states localized in different wells, indicated by the up and the down arrows in Fig. 1(b), are chosen as the effective two-level system. The currents of these two states differ by $\Delta I \approx I_c$ and results in a flux difference of $\Delta\Phi_{rf} = \Delta I L_{rf} \approx 0.3\Phi_0$. By adjusting the flux in the qubit loop, we have $\epsilon_0 = 13$ GHz, $t_0 = 1$ GHz, $\omega_a = 11$ GHz, and $t_0^a = 0$. By adjusting the mutual inductance to be $M_q/L_q = 1/4$, we have $\omega_\Delta = 3$ GHz. The states are drawn in Fig. 1(c) with their energies labeled beside each level.

The rf SQUID is stored in the state $|0_a\rangle$ as in Fig. 1 of Ref. 11 by suddenly switching the flux in the loop. The qubit energy is $\omega_q = 10$ GHz, and the single-qubit operation is implemented with a microwave pulse at the resonant frequency. During the qubit operations, the rf SQUID has trivial dynamics. In the beginning of a measurement, a microwave pulse with frequency $E_{\bar{1}_q 1_a} - E_{1_q 0_a} = 14$ GHz is applied to the rf SQUID for a π rotation. This pulse flips the SQUID state when the qubit is in state $|1_q\rangle$. When the qubit is in state $|0_q\rangle$, $E_{\bar{0}_q 1_a} - E_{0_q 0_a} = 8$ GHz and the applied pulse is not resonant with the rf SQUID. By controlling the external flux Φ_{ex} , a π pulse of 10 nsec ($\omega_X = 50$ MHz) flips the rf SQUID. The off-resonant transition has a probability $(\omega_X/\Delta\omega)^2$ which is lower than 10^{-4} and is irrelevant in this process. The operation is hence an effective controlled-NOT gate on the rf SQUID. After the entanglement pulse, the rf SQUID is measured by a magnetometer, such as the dc SQUID shown in Fig. 1(a). Optimized designs besides the simple dc SQUID configuration can be made for better detection.²⁵⁻²⁷

In designing the rf SQUID, attention should be focused on several issues to successfully implement this measurement scheme. First, the two-level system should be well separated from the other states of the rf SQUID, so that no off-resonant leakage to the other levels happens during the entanglement pulse. In our design, the two states are at least 40 GHz away from all the other states, and the off-resonant transitions can be neglected. Second, a trivial but crucial point, the parameters have to be realistic for sample fabrication. We base our scheme on existing experiments.^{11,22} Although we chose as an example that the qubit couples inductively to a rf SQUID in this paper, other supplementary systems can also be used with different interaction mechanisms and different detection technologies.

With the self-generated flux of an order of one flux quantum, the rf SQUID is subjected to strong perturbation from the environment, such as randomly trapped flux, impurity spins, and nuclear spins. Fortunately, however, the noise does not affect the qubit during the regular qubit operations. The fluxlike noise adds to Eq. (1) a term $\sigma_z^a f(t)$ which shakes the energy levels of the rf SQUID up and down randomly. As the ETLS stays in its ground state $|0_a\rangle$ during the qubit operations, this term only contributes an overall phase to the total wave function of the interacting system and does not decohere the qubit. This is true even when the qubit

Hamiltonian has a nonzero σ_x^a term. For environmental degrees that assume σ_x^a coupling with the rf SQUID, the environmental modes with the frequency around 10 GHz can flip the rf SQUID, in principle. But at the low temperature of 20 mK, no excitations of these transversal modes exist to excite the rf SQUID from the ground state to the excited state.

During the entanglement pulse, the rf SQUID makes a transition from $|0_a\rangle$ to $|1_a\rangle$, and the fluxlike noise affects the dynamics of the qubit-ETLS system. It is important for the gate time to be shorter than the decoherence time of the rf SQUID to maximally entangle the qubit and the ETLS. In the rotating frame of the Hamiltonian \mathcal{H}_0 in Eq. (1), the Hamiltonian during the entanglement is

$$\mathcal{H}_{rot} = \frac{1 + \sigma_z^q}{4} \Omega_x \sigma_x^q + f(t) \sigma_z^a, \quad (3)$$

where the first term is the resonant pulse and the second term is the flux noise to the rf SQUID which dephases the rf SQUID. The noise is treated as a classical fluctuation field $f(t)$. The decoherence rate is determined by the spectral den-

sity $\langle f(\Omega_X) f(-\Omega_X) \rangle$. With noise coupling as in Ref. 23, we estimate the decoherence time to be between 0.1 and 10 μ sec, which is significantly longer than the resonant pulse of 10 nsec.

In conclusion, we have presented an effective measurement scheme that increases the measurement efficiency without introducing extra decoherence. We illustrated this method by applying it to the superconducting persistent-current qubit where a supplementary two-level system—a rf SQUID—was coupled to the persistent-current qubit. This method improves the previous measurement by avoiding the difficulty of measuring a flux of 10^{-3} flux quantum with a quantum detector whose quantum broadening is of 0.1 flux quantum. Our scheme creates for the solid-state qubits a measurement scheme that is projective, noiseless, and switchable. More delicate designs based on this idea can be developed to optimize the measurement.

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