Impact of time-ordered measurements of the two states in a niobium superconducting qubit structure

K. Segall, D. Crankshaw, D. Nakada, T. P. Orlando, L. S. Levitov, and S. Lloyd
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

N. Markovic, S. O. Valenzuela, and M. Tinkham
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

K. K. Berggren
MIT Lincoln Laboratory, Lexington, Massachusetts 02421, USA

(Received 28 January 2003; published 27 June 2003)

Measurements of thermal activation are made in a superconducting, niobium persistent-current qubit structure, which has two stable classical states of equal and opposite circulating current. The magnetization signal is read out by ramping the bias current of a dc superconducting quantum interference device. This ramping causes time-ordered measurements of the two states, where measurement of one state occurs before the other. This time ordering results in effective measurement time, which can be used to probe the thermal activation rate between the two states. Fitting the magnetization signal as a function of temperature and ramp time allows one to estimate a quality factor of $3 \times 10^5$ for our devices, a value favorable for the observation of long quantum coherence times at lower temperatures.

DOE: 10.1103/PhysRevB.67.220506

PACS number(s): 74.40.+k, 85.25.Cp, 85.25.Dq

The concept of thermal activation of a particle over an energy barrier plays a critical role in understanding many problems in condensed-matter physics. Starting with Kramers,1 expressions for the thermal activation rate have been derived in both the low and high damping regimes.2 These expressions are often applied to analyses of Josephson-junction circuits, where the particle coordinate represents the phase difference of the superconducting order parameter.3 One such example is the rf superconducting quantum interference device (SQUID), which is a loop of superconductor with a single Josephson junction. Thermal activation of the phase causes flipping between two classically stable states of equal and opposite circulating current in the loop. Thermal activation rates have been measured in an rf SQUID by coupling it to a damped dc-SQUID magnetometer, which measures its magnetization signal.4 In fitting the temperature dependence of the thermal activation rate one can extract important parameters of the rf SQUID, such as its inductance and Josephson energy. These measurements can be valuable as a complement to lower-temperature experiments, where the rf SQUID has shown a macroscopic quantum superposition of states.5

A system similar to the rf SQUID is the persistent-current (PC) qubit, a loop of superconductor with three junctions.6 It has also demonstrated a macroscopic superposition of states.7 The rf-SQUID qubit must have a large loop ($\sim 100-\mu m$ radius) to have enough inductance to have two stable states. The PC qubit does not depend on the loop inductance to define its two stable states; thus it can be made much smaller ($\sim 1-10-\mu m$ radius) and therefore more isolated from the environment. The trade off is that its signal is two or three orders-of-magnitude smaller than that in the rf SQUID. Typically the PC qubit is read out with an underdamped, hysteretic dc-SQUID magnetometer, in order to couple it more strongly to the qubit without introducing extra dissipation. By reading out the qubit in this fashion, the SQUID performs time-ordered measurements of the two states, where one state is measured before the other.

In this report we present measurements of thermal activation in a Nb PC qubit coupled to an underdamped dc SQUID and investigate the impact of the time-ordered measurements of the two states. The two magnetization states of the qubit cause two distinctly different switching points in the SQUID $I-V$ curve, allowing a near single-shot readout. The time to ramp the current between these two switching points forms an intrinsic time scale for the measurement. We show that thermal activation during this period can be seen in the magnetization signal, and derive a model to account for this effect. By varying both the temperature and the SQUID ramp rate we can fit the measured data to the standard thermal activation rates and extract the system parameters. We present the results of this fitting and find the amount of dissipation to be favorable for the observation of quantum effects at lower temperatures.

The devices tested were made at MIT Lincoln Laboratory, with a planarized niobium trilayer process;8 a circuit schematic is shown in Fig. 1(a). Two such devices were tested, with both showing very similar behavior. For simplicity we discuss the data from only one of them.9 The PC qubit is a loop of niobium, $16\times16\ \mu m$, interrupted by three Josephson junctions. The junctions are Nb-AlO$_3$-Nb, oxidized to yield a critical current density of $730\ A/cm^2$. The ratio of the Josephson energy to the charging energy, $E_J/E_C$, is about 2000. The self-inductance of the loop is about 30 pH. The PC qubit is surrounded by a two-junction dc-SQUID magnetometer, which reads out the state of the PC qubit. The SQUID loop is $20\times20\ \mu m$. The SQUID junctions are about $1.25\times1.25\ \mu m$, with a critical current of about 11 $\mu A$. The self-inductance of the SQUID loop is about 60 pH, with a mutual inductance to the qubit of about 25 pH. Both junctions of the SQUID are shunted with 1-pF capacitors to lower the resonance frequency of the SQUID.
FIG. 1. (a) Schematic of the PC qubit surrounded by a dc SQUID. The X’s represent junctions. (b) Schematic curve of the bias current \( I_q \) vs the SQUID voltage \( V_S \) for the SQUID. At the switching point the SQUID voltage switches to the gap voltage \( V_g \). The 0 and 1 qubit states cause two different switching currents. (c) Timing of the current and voltage in the SQUID as the measurement proceeds. If the qubit is in state 0, \( V_S \) switches to \( V_g \) at time \( t_0 \); if the qubit is in state 1, \( V_S \) switches at time \( t_1 \). The time difference \( t_1-t_0 \) forms a time scale for the measurement.

The SQUID is highly underdamped, so the method of readout is to measure its switching current, which is sensitive to the total flux in its loop. A bias current \( I_q \) was ramped from zero to above the critical current of the SQUID, and the value of current at which the junction switched to the gap voltage was recorded for each measurement [see Figs. 1(b) and 1(c)]. The repeat frequency of the bias current ramp was varied between 10 and 150 Hz. Typically several hundred measurements were recorded, since the switching is a stochastic process. The experiments were performed in a pumped \(^3\)He refrigerator, at temperatures ranging from 330 mK to 1.2 K. A magnetic field was applied perpendicular to the sample in order to flux bias the qubit near to one-half a flux quantum can be approximated as a two-state system, where the states have equal and opposite circulating half a flux quantum can be approximated as a two-state system, where the states have equal and opposite circulating magnetic field is increased, the system probability is gradually modulated until the qubit is found completely in the 1 state, corresponding to the larger switching current. Focusing on the point in flux where the two states are equally likely, one can see that it is formed from a bimodal switching distribution, with the two peaks corresponding to the two different qubit states. The fitting from the model developed below is also shown.

The qubit is found in state 0 with a probability of \( P_0 \) and a qubit circulating current of \( I_q= (-I_p) \); it is found in state 1 with a probability of \( P_1 \) and a circulating current \( I_q= (+I_p) \). Since there are only two states, \( P_0+P_1=1 \). The average circulating current in the qubit is

\[
\bar{I}_q = (-I_p)P_0 + (+I_p)P_1 = 2I_p(1-P_0) - I_p. \tag{1}
\]

In the steady state, the probability \( P_0=\gamma_{10}/(\gamma_{10}+\gamma_{01}) \), where \( \gamma_{10} \) and \( \gamma_{01} \) are the transition rates from 0 to 1 and from 1 to 0, respectively. For thermal activation in an underdamped system, the transition rate \( \gamma_{10} \) is given by

\[
\gamma_{10} = \frac{7.2\Delta U_{10}\omega_0}{2\pi kT}e^{-\Delta U_{10}/kT}, \tag{2}
\]

where \( \omega_0 \) is the attempt frequency, \( \omega_0=\sqrt{8E_cE_f}/h \), \( Q \) is the quality factor (equal to the inverse of the damping coefficient), \( k \) is Boltzmann’s constant, \( T \) is the operating temperature, and \( \Delta U_{10} \) is the size of the energy barrier to go from 1 to 0. A similar expression exists for \( \gamma_{01} \), with \( \Delta U_{10} \) replaced by \( \Delta U_{01} \), the size of the energy barrier to go from 0 to 1. The energy barrier \( \Delta U_{10} \) depends almost linearly on the flux in the qubit (\( \Phi_q \)) and for the parameters listed above is given approximately by

\[
\Delta U_{10} = 3.5E_f(f_q-0.5) + \Delta U^b. \tag{3}
\]

Here the qubit frustration \( f_q \) is equal to \( \Phi_q/\Phi_0 \), and \( \Delta U^b \) is the energy barrier at a frustration of 0.5. The energy barrier \( \Delta U^b \) depends on \( \alpha \), the ratio of the area of the smaller junction to that of the two larger junctions in the three-junction loop. In our devices \( \alpha \) is about 0.6. The same expression holds for \( \Delta U_{01} \), except with a minus sign in front of the first term. The value of \( E_f \) is constant over the temperature range that was studied.
$P_1$ is the instantaneous probability that the system is in $1$. However, to observe the larger switching current corresponding to $1$ requires the following: (i) the qubit must be in $1$ at time $t_0$ in the ramp [see Fig. 1(c)] and (ii) it must remain in $1$ until time $t_1$, at which point the SQUID switches. If (i) is satisfied but (ii) is not, namely, the qubit is in $1$ at time $t_0$ but flips from $1$ to $0$ at time $t$ ($t_0 < t < t_1$), then the SQUID will switch at this time $t$, at a current value between the two switching currents. Note that the same is not true for $0$: if the system is in $0$ at time $t_0$, the SQUID will switch immediately and the state will be measured.

We derive a form for the average circulating current with these conditions of a finite measurement time. To avoid confusion we distinguish between the “flip” of the qubit state and the “switching” of the SQUID from zero voltage to finite voltage; in the time interval between $t_0$ and $t_1$ in the current ramp, a qubit flip from $1$ to $0$ causes the SQUID to switch to finite voltage because it becomes unstable. The probability that a $1$ to $0$ flip in the qubit occurs in an interval $dt$ about time $t$ is given by

$$p(t) dt = P_1 \exp[-\gamma_1(t-t_0)] \gamma_1 dt.$$  

(4)

Here $\gamma_1 dt$ is the instantaneous probability of a $1$ to $0$ transition during $dt$, and the first two factors on the right-hand side are the probability that the qubit is in $1$ at $t_0$ and survives in $1$ until time $t$. The average circulating current can be calculated from three possibilities: (i) the SQUID switches at $t_0$, with a probability of $P_0$ and a qubit circulating current of $(-I_p)$; (ii) it switches at a time $t$ between $t_0$ and $t_1$ due to a qubit flip, with a probability $p(t) dt$ and a qubit circulating current of $I_q(t)$; and (iii) it switches at time $t_1$, with a probability of $P_1 e^{-x}$, where $x = \gamma_1 \tau$, and a circulating current of $(+I_p)$. Thus,

$$\bar{I}_q = (-I_p)P_0 + \int_{t_0}^{t_1} I_q(t)p(t)dt + (+I_p)P_1 e^{-x}.$$  

(5)

Switching events from the time interval $t_0$ to $t_1$ correspond to apparent values of the qubit circulating current between $(-I_p)$ and $(+I_p)$. In the calculation of $I_q(t)$ in Eq. (5) we assume a linear relationship,

$$I_q(t)=I_p\left[\frac{2(t-t_0)}{\tau} - 1\right].$$  

(6)

and thus Eq. (5) becomes

$$\bar{I}_q = 2I_p(1-P_0) \left[1 - e^{-x}\right] - I_p.$$  

(7)

Note that this expression reduces to Eq. (1) in the limit that $\tau$ goes to zero.

In Fig. 3 we plot $P_0$ and the average circulating current versus flux in the qubit for the two expressions (1) and (7), for $E_j=4000$ and $E_c=2\mu eV$, $T=0.6\text{ K}$, $\tau=100\mu s$, $Q=10^6$, and $\alpha=0.58$. The effects of the finite measurement time [Eq. (7)] are that the zero crossing of the curve is shifted in flux and its shape is slightly changed. The amount of displacement in flux depends on the amount of thermal activation during the measurement; the more thermal activation, the more the curve will move. We define the flux to be where the average circulating current equals zero as $f_z$, defined by

$$\overline{\bar{I}_q}(f_z) = 0.$$  

(8)

One can increase the amount of thermal activation during measurement by either raising the temperature or increasing the measurement time. Thus the value of $f_z$ should depend on both temperature ($T$) and measurement time ($\tau$). In Fig. 3 we can see that if the amount of thermal activation is significant, then $f_z$ occurs significantly displaced from 0.5. In this region of flux, the value of $P_0$ is close to zero. Setting $P_0 = 0$ in Eq. (7) results in a solution where $\gamma_{10}\tau=1$, essentially indicating that the average current is zero when the times for thermal activation and measurement are equal. Solving for $f_z$ in Eq. (8) using $P_0=0$ results in

$$f_z = 0.5 + \frac{kT}{4E_j} \ln \left[\frac{\Delta U_{10} \gamma_{10} \tau}{1.44QkT} - \frac{\Delta U}{4E_j}\right].$$  

(9)

Equation (9) is transcendental, since the energy barrier $\Delta U_{10}$ depends linearly on $f_z$, but this dependence is weak since it is in the logarithm. Ignoring this weak dependence, Eq. (9) predicts a movement of $f_z$ that is linear in temperature and logarithmic in measurement time. The circulating current in the arms of the SQUID couples a flux into the qubit that is not accounted for here, but this flux simply adds a constant offset to $f_z$ and does not significantly affect its temperature and rate dependence.

In Fig. 2 we show the transition curves for two different base temperatures, 0.33 and 0.62 K. A best fit for each curve from Eq. (7) is also shown. The same fitting parameters (see below) are used in both cases, with only the temperature allowed to vary. The 0.62-K curve has moved in flux relative to the 0.33-K curve, as expected. The theory predicts both the curve’s shape and its relative position in flux. Figure 4 shows how the center point of the transition ($f_z$) varies with the natural log of the ramp rate and the temperature. The data are fit using Eqs. (7) and (8). At values of larger temperature or slower ramp rate (slower ramp rate is equivalent to larger $\tau$), $f_z$ varies in a linear fashion as predicted by Eq. (9). In this region $\gamma_{10}\tau=1$. As either the temperature is lowered or the rate is increased, there is a crossover to a region at which $f_z$ no longer varies. This is the “fast” measurement region, where on average no thermal activation of the qubit occurs during measurement.
The value of $Q$ is found to be $3 \times 10^5$ to within a factor of 3, independent of temperature. The large error results from the uncertainty in $E_C$ combined with the weak (logarithmic) dependence of $f_z$ on $Q$. The $Q$ factor appears to be limited by the coaxial-like impedance of the SQUID current and voltage leads at the plasma frequency, whose current fluctuations couple flux into the qubit. The temperature-dependent subgap current would imply a much larger $Q$ factor.\textsuperscript{10} Other\textsuperscript{13} single junction measurements, also limited by the high-frequency impedance of the leads, typically yield $Q$ factors of order 30. The flux biasing of our devices effectively transforms the impedance seen by the qubit by a factor of $(3L_J/M)^2$, where $L_J = \Phi_0/(2\pi I_s)$ is the Josephson inductance of each of the three junctions in the qubit. Using our values this would then imply a $Q$ of $5 \times 10^4$; our measured value is somewhat larger because current fluctuations in the leads of the dc SQUID divide evenly between the two arms and do not couple flux very efficiently to the qubit. This value of $Q$ corresponds to a relaxation time of roughly $Q\Gamma_0 \sim 1$ $\mu$s. Similar relaxation times have been measured\textsuperscript{14} in aluminum superconducting qubits, and indicate possible long coherence times in the quantum regime.

In short, we have measured the effects of time-ordered measurements and thermal activation in two Nb PC qubit/dc SQUID systems. A model that includes thermal activation during measurement describes the temperature and rate dependence of the signal. Using the model to fit the system parameters we find junction sizes consistent with our fabrication and favorable dissipation values for observing long quantum coherence times in these qubits.

We thank B. Singh, J. Lee, J. Sage, and T. Weir for experimental help and L. Tian for useful discussions. This work was supported in part by the AFOSR Grant No. F49620-01-1-0457 under the Department of Defense University Research Initiative on Nanotechnology (DURINT) and by ARDA. The work at Lincoln Laboratory was sponsored by the Department of Defense under the Department of the Air Force, Contract No. F19628-00-C-0002.

\begin{thebibliography}{9}
\bibitem{1} H. A. Kramers, Physica (Amsterdam) \textbf{7}, 284 (1940).
\bibitem{9} The second device has the same geometry but a current density about a factor-of-2 lower. The fitted parameter values were $E_J = 2400$ $\mu$eV, $\alpha = 0.589$, and $Q = 1.0 \times 10^6$.
\bibitem{10} To be rigorously correct, one should subtract the zero-point energy from the energy barrier; however, that has only a slight impact on the fitting of our parameters and we have omitted it for simplicity.
\bibitem{11} With no self-inductance in the SQUID, this is exactly true. Our SQUID has some self-inductance, but because we operate the SQUID near its true critical current these effects are small. The clear separation of peaks in Fig. 2 is an indication that the linear approximation is appropriate.
\bibitem{12} The theoretical subgap resistance at 1 K is greater than 100 $M\Omega$ near zero voltage at 1 K and increases below that; this impedance suggests a $Q$ factor greater than $10^6$. We have performed measurements of the subgap impedance down to 1.6 K and found good agreement with theory.
\end{thebibliography}