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Network Capacity, Power Control, and Effective Bandwidth

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As the preceding chapters have emphasized, the mobile wireless environment provides several unique challenges to reliable communication not commonly found in wireline networks. These include scarce bandwidth, limited transmit power, interference between users, and time-varying channel conditions. A central problem in the design of wireless networks is how to use the limited resources most efficiently in such adverse environments, in order to meet the quality-of-service (QoS) requirements of applications as quantified in terms of bit rate and loss. The problem will become more acute for next-generation, integrated-services networks that aim to support a heterogeneous mix of high bandwidth media types with diverse QoS requirement and bursty traffic characteristics. As the demand for ubiquitous access to the backbone wireline network grows, the capacity of the wireless link will likely be a severe bottleneck.

To meet these challenges, there have been intense efforts in developing more sophisticated physical layer communication techniques, examples of which are described in preceding chapters. A significant thrust of work has been on developing *multiuser* receiver structures of the type described in Chapter 2, which mitigate the interference between users in spread spectrum systems. (See also, for example,

[10, 11, 12, 15, 16, 20, 23].) Recall that, unlike the conventional matched-filter receiver used in the IS-95 CDMA system, these techniques take into account the structure of the interference from other users when decoding a user. Another important line of work is the development of processing techniques in systems with antenna arrays, a class of which is described in Chapter 4. As discussed in Chapter 1 as well, while spread-spectrum techniques provide *frequency diversity* to the wireless system, antenna arrays provide *spatial diversity*, both of which are essentially *degrees of freedom* through which communication can take place.

Despite significant work done in the area, there is still much debate about the network capacity of the various approaches to deal with multiuser interference in spread-spectrum and multiple-antenna systems. One important reason is that the networking level problems of resource allocation and power control are less well understood in the context of multiuser techniques than with more traditional multiaccess schemes, such as TDMA, FDMA, and conventional-receiver CDMA systems. For example, in a TDMA or FDMA system, the network resource is shared among users via disjoint frequency and time slots, and this sharing provides a simple abstraction for resource allocation problems at the networking layer. Such clean separation between the networking and the physical layers does not exist when more sophisticated multiuser techniques are used. This in turn hampers the understanding of the capacity of networks with multiuser receivers and of the associated network-level resource allocation problems such as call admissions control, cell handoffs, and resource allocation for bursty traffic.

In this chapter, we show that under some conditions, a simple abstraction of the amount of resource consumed by a user is indeed possible for several important multiuser receivers. The specific scenario is a set of power-controlled mobile users communicating to a base-station in a single cell. Assuming that each user's QoS can be expressed in terms of a target signal-to-interference ratio (SIR), we show that a notion of *effective bandwidth* can be defined such that the QoS requirements of all the users can be met if and only if the sum of the effective bandwidths of the users is less than the total number of degrees of freedom in the system. These degrees of freedom can be provided by the processing gain in a spread-spectrum system or by the number of antenna elements in a system with an antenna array. These capacity characterizations are simple in that the effective bandwidth of a user depends only on its SIR requirement and nothing else. While these results are proved in an idealized model, they have the potential to provide a first step in bridging between resource allocation problems at the networking layer and multiuser techniques at the physical layer.

The effective bandwidth of a user depends on the multiuser receiver employed. Results for three receivers are obtained. They are the minimum mean-square error (MMSE) receiver [12, 15, 16, 23], the decorrelator [10, 11], and the conventional matched filter receiver. We show that the effective bandwidths are

respectively $e_{\text{mmse}}(\beta) = \frac{\beta}{1+\beta}$, $e_{\text{dec}}(\beta) = 1$, and $e_{\text{mf}}(\beta) = \beta$, where β is the SIR requirement of the user. These effective bandwidth expressions also provide a succinct basis for performance comparison between different receiver structures. The MMSE receiver occupies a special place since it can be shown to lead to the minimum effective bandwidth among all linear receivers.

These effective bandwidth characterizations also illustrate the inherent flexibility in resource-sharing among users with heterogeneous SIR requirements in a CDMA system: the total degrees of freedoms can be divided arbitrarily according to each user's SIR. This case is in contrast to traditional FDMA or TDMA system where the resource allocation is much more rigid and coarse-grained. Such flexibility is supported by appropriate power control, and this philosophy is behind much of recent work in power control for conventional CDMA systems. (See, for example, [1, 3, 6, 25, 26, 27]). Our work can be viewed as an extension of this philosophy to more sophisticated multiuser receivers.

The outline of this chapter is as follows. In Section 5.1, we introduce our notation for the basic model of a multiple-access spread-spectrum system and the structure of the MMSE receiver. In Section 5.2, we present our key result: that in a large system with each user using random spreading sequences, the limiting interference effects under the MMSE receiver can be calculated as if they were additive; to each interferer can be ascribed a level of *effective interference* that it provides to the user to be decoded. In Sections 5.3 and 5.4, we apply this result to study the performance under power control and obtain a notion of *effective bandwidth*. In Section 5.5, we obtain analogous results for the decorrelating receiver. In Section 5.6, we show that similar ideas carry through for systems with antenna diversity. Section 5.7 contains some concluding remarks.

Proofs of results are not presented here but can be found in [18].

5.1 Basic Spread-Spectrum Model and the MMSE Receiver

In a spread-spectrum system, each of the user's information or coded symbols is spread onto a much larger bandwidth via modulation by its own *signature* or *spreading sequence*. The following is a model for a symbol-synchronous, multiple-access, spread-spectrum system:

$$\mathbf{Y} = \sum_{m=1}^{M} X_m \mathbf{s}_m + \mathbf{W},$$

where X_m is a real scalar and $\mathbf{s}_m$ is a real L-dimensional vector that denote the transmitted symbol and signature spreading sequence of user m, respectively, and $\mathbf{W}$ is zero-mean, variance-σ^2 Gaussian background noise. The length of the signature sequences is L, which one can also think of as the number of degrees of freedom or

diversity. The L-dimensional received vector is $\mathbf{Y}$. We assume the X_m's are independent and identically distributed (i.i.d.) and that $E[X_m] = 0$ and $E[X_m^2] = P_m$, where P_m is the received power of user m.

Rather than looking at *symbol-by-symbol* detection, we are interested in the more general problem of *demodulation*, extracting good estimates of the (coded) symbols of each user as soft decisions to be used by the channel decoder [16]. From this point of view, the relevant performance measure is the SIR of the estimates.

We shall now focus on the demodulation of user 1, assuming that the receiver has already acquired the knowledge of the spreading sequences. The optimal linear demodulator that generates a soft decision $\hat{X}_1$, maximizing the SIR at the output of the demodulator, is the MMSE receiver [12, 15, 16].

As a comparison, note that the conventional CDMA approach simply matches the received vector to $\mathbf{s}_1$, the signature sequence of user 1. This is indeed the optimal receiver when the interference from other users is white. However, in general, the multiple-access interference is not white and has structure as defined by $\mathbf{s}_2, \mathbf{s}_3, \ldots, \mathbf{s}_M$, assumed to be known to the receiver. The MMSE receiver exploits the structure in this interference in maximizing the SIR of user 1.

The formulae for the MMSE demodulator and its performance are well known (cf. Chapter 2):[1]

$$X_{\text{mmse}}(\mathbf{Y}) = \frac{1}{\mathbf{s}_1^{\mathrm{T}}(SDS^{\mathrm{T}} + \sigma^2 I)^{-1}\mathbf{s}_1}\,\mathbf{s}_1^{\mathrm{T}}(SDS^{\mathrm{T}} + \sigma^2 I)^{-1}\mathbf{Y}, \tag{5.1}$$

and the signal to interference ratio β_1 for user 1 is

$$\beta_1 = \mathbf{s}_1^{\mathrm{T}}(SDS^{\mathrm{T}} + \sigma^2 I)^{-1}\mathbf{s}_1 P_1, \tag{5.2}$$

where $S = [\mathbf{s}_2, \ldots, \mathbf{s}_M]$ and $D = \operatorname{diag}(P_2, \ldots, P_M)$.

5.2 Performance under Random Spreading Sequences

Equation (5.2) is a formula for the performance of the MMSE receiver, which one can compute for specific choice of signature sequences. However, it is not easy to obtain qualitative insights directly from this formula. For example, the effect of an individual interferer on the SIR for user 1 cannot be seen directly from this expression. In practice, it is often reasonable to assume that the spreading sequences are *randomly* and independently chosen (see, e.g., [13]). For example, they may be pseudorandom sequences, or the users may choose their sequences from a large set of available sequences as they are admitted into the network. In this case, the performance of the

[1]The superscript ^{T} denotes the transpose operator.

optimal demodulator can be modeled as a random variable since it is a function of the spreading sequences. In this section, we show that, unlike the deterministic case, there is a great deal of analytical information one can obtain about this random performance in a large network. In the development below, we assume that though the sequences are randomly chosen, they are known to the receiver once they are picked. In practice, this means that the change in the spreading sequences is at a much slower time scale than the symbol rate, so that the receiver has the time to acquire the sequences. (There are known adaptive algorithms for which this can even be done blindly; see [8].) However, the *performance* of the MMSE receiver depends on the initial choice of the sequences and, hence, is random.

As a model for random sequences, let $\mathbf{s}_m = \frac{1}{\sqrt{L}}(V_{1m},\ldots, V_{Lm})^{\mathrm{T}}$, $m = 1, \ldots M$, where the random variables V_{km}'s are i.i.d., zero-mean, and unit-variance. The normalization by $\frac{1}{\sqrt{L}}$ ensures that $E[\|\mathbf{s}_m\|^2] = 1$. In practice, it is common that the entries of the spreading sequences are 1 or -1, but we want to keep the model general so that we can later apply our results to problems with other modes of diversity.

Our results are asymptotic in nature, for a large network. Thus, we consider the limiting regime where the number of users are large, i.e., $M \to \infty$. To support a large number of users, it is reasonable to scale up L as well, keeping the number of users per degree of freedom (equivalently, per unit bandwidth), $\alpha \equiv \frac{M}{L}$, fixed. We also assume that as we scale up the system, the empirical distribution of the powers of the users converges to a fixed distribution, say $F(P)$. The following is the main result of [18], giving the asymptotic information about the SIR for user 1. The proof of this result makes use of the theory of random matrices [14, 17].

Theorem 5.1 Let $\beta_1^{(L)}$ be the (random) SIR of the MMSE receiver for user 1 when the spreading length is L. Then, $\beta_1^{(L)}$ converges to β_1^* in probability as $L \to \infty$, where β_1^* is the unique solution to the equation

$$\beta_1^* = \frac{P_1}{\sigma^2 + \alpha \int_0^\infty I(P, P_1, \beta_1^*) dF(P)} \tag{5.3}$$

and

$$I(P, P_1, \beta_1^*) \equiv \frac{PP_1}{P_1 + P\beta_1^*}.$$

Heuristically, this means that in a large system, the SIR β_1 is deterministic and approximately satisfies

$$\beta_1 \approx \frac{P_1}{\sigma^2 + \frac{1}{L}\sum_{i=2}^{M} I(P_i, P_1, \beta_1)}, \tag{5.4}$$

where, as before, P_i is the received power of user i. This result yields an interesting interpretation of the effect of each of the interfering users on the SIR of user 1: for a large system, the total interference can be decoupled into a sum of the background noise and an interference term from each of the other users. (The factor $\frac{1}{L}$ results from the processing gain of user 1.) The interference term depends only on the received power of the interfering user, the received power of user 1, and the attained SIR. It does not depend on the other interfering users except through the attained SIR β_1.

One must be cautioned not to think that this result implies that the interfering effect of the other users on a particular user is additive across users. It is not: the interference term $I(P_i, P_1, \beta_1)$ from interferer i depends on the attained SIR which in turn is a function of the entire system. However, it can be shown that the equation:

$$x = \frac{P_1}{\sigma^2 + \frac{1}{L}\sum_{i=2}^{M} I(P_i, P_1, x)} \tag{5.5}$$

has a unique fixed point x^*, and moreover, the equation has the following monotonicity property: for any x, $x^* \geq x$ if and only if

$$\frac{P_1}{\sigma^2 + \frac{1}{L}\sum_{i=2}^{M} I(P_i, P_1, x)} \geq x. \tag{5.6}$$

It follows then that to check if the target for user 1's SIR, β_T, can be met for a given system of users, it suffices to check the following condition:

$$\frac{P_1}{\sigma^2 + \frac{1}{L}\sum_{i=2}^{M} I(P_i, P_1, \beta_T)} \geq \beta_T.$$

Based on this interpretation, it is natural to refer to the term $I(P_i, P_1, \beta_T)$ as the *effective interference* of user i on user 1, at a target SIR of β_T.

To gain further insight into this concept of effective interference, it is helpful to compare the above situation with that when the conventional matched filter $\mathbf{s}_1$ is used for the demodulation. For that case, it can be shown that if $\beta_{1,\mathrm{mf}}$ is the (random) SIR of the conventional matched-filter receiver for user 1, then for large processing gain L, $\beta_{1,\mathrm{mf}}$ converges in probability to

$$\beta^*_{1,\mathrm{mf}} = \frac{P_1}{\sigma^2 + \alpha\int_0^\infty P dF(P)}, \tag{5.7}$$

where, as before, F is the limiting distribution of the powers of the users and α is the number of users per degree of freedom. Hence, for large L, the performance of the matched receiver is approximately

$$\beta_{1,\mathrm{mf}} \approx \frac{P_1}{\sigma^2 + \frac{1}{L}\sum_{i=2}^{M} P_i}. \tag{5.8}$$

Comparing this expression with (5.4), we see that the interference due to user i is simply P_i in place of $I(P_i, P_1, \beta_1)$. Since the matched-filter receiver is independent of the signature sequences of the other users, it is not surprising that the interference is linear in the received powers of the interferers. In the case of the MMSE receiver, the filter does depend on the signature sequences of the interferers, thus resulting in the interference being a nonlinear function of the received power of the interferer. Also, observe that $I(P_i, P_1, \beta_1) < P_i$, which is expected since the MMSE receiver maximizes the SIR among all linear receivers. But more importantly, we see that while for the conventional receiver the interference grows without bound as the received power of the interferer increases, for the MMSE receiver, the effective interference from user i is bounded and approaches $\frac{P_1}{\beta_1}$ as P_i goes to infinity. Thus, while the SIR of the matched-filter receiver goes to zero for large interferers' powers, the SIR of the MMSE receiver does not. This is the well-known *near-far resistance* property of the MMSE receiver discussed in Chapter 2 [see also [12]). The intuition is that as the power of an interferer grows to infinity, the MMSE receiver will null out its signal. While the near-far resistance property has been reported by previous authors, Theorem 5.1 goes beyond that as it not only quantifies the worst-case performance (i.e., large interferer's power) but also the performance for all finite values of the interference. This quantification is useful for example in situations when power control is exercised, as we turn to in the next section.

In general, we have no explicit solution for the SIR β_1^* in (5.3). However, for the special case when the received powers of all users are the same, the equation is quadratic in β_1^* and a simple solution is obtained:

$$\beta_1^* = \frac{(1-\alpha)P}{2\sigma^2} - \frac{1}{2} + \sqrt{\frac{(1-\alpha)^2 P^2}{4\sigma^4} + \frac{(1+\alpha)P}{2\sigma^2} + \frac{1}{4}}.$$

We see that the β_1^* is positive for all values of α and approaches 0 as α, the number of users per degree of freedom, goes to infinity.

Two performance measures commonly used in the literature for multiuser receivers (and discussed in Chapter 2) are their *efficiency* and their *asymptotic efficiency* [21]. In the context of linear receivers, the efficiency for user 1 is defined to be the ratio of the achieved SIR to the SIR when there is no interferer and only

background noise. For the MMSE receiver with random spreading sequences and equal received power for all users, the efficiency is given by:

$$\frac{\beta_1^* \sigma^2}{P},$$

where β_1^* is given by the above expression. Recall from Chapter 2 that the asymptotic efficiency η_1 is the limiting efficiency as the background noise level goes to zero. If $\alpha \leq 1$, this asymptote is given by

$$\eta_1 := \lim_{\sigma \to 0} \frac{\beta_1^* \sigma^2}{P} = 1 - \alpha.$$

For $\alpha > 1$, the limiting SIR is positive but bounded:

$$\lim_{\sigma \to 0} \beta_1^* = \frac{1}{\alpha - 1}, \tag{5.9}$$

and so the asymptotic efficiency is zero.

5.3 Capacity and Performance under Power Control

We observed in Section 5.2 that in the conventional receiver case, the interference of a user is proportional to its power, and hence a strong interferer can completely overcome a weaker signal. This is the so-called near-far problem, and a well-known consequence is that the conventional receiver can only avoid this problem via tight power control. We also observed that the MMSE receiver does not suffer arbitrarily poorly from the near-far problem, and indeed, this is one of the key motivations for the original work on multiuser detection [20]. Nevertheless, a MMSE receiver still suffers interference from other users, and it follows that capacity can be increased and power consumption reduced if power control is employed.

In this section, we consider the case in which all users require an SIR of exactly β^*, given a processing gain of L degrees of freedom per symbol. For a given number of users, we compute the minimum power consumption required to achieve β^* for all users and then look at the maximum number of users per degree of freedom supportable for a given power constraint under power control. Of particular interest is the maximum number without power constraint, which we define to be the *capacity* of the system (in terms of number of users per degree of freedom.) This coincides with the definition of capacity taken in [5]; "capacity" is then the point at which saturation occurs as we put in so many users that we drive

the required power level to infinity. We show that this capacity is different but finite for both the conventional and the MMSE receivers; thus, both are interference-limited systems. As before, our results are asymptotic as the the processing gain L goes to infinity.

Let us focus first on the conventional receiver. With the matched filter receiver, (5.7) tells us that, asymptotically, users receive the same level of interference and hence must be received at the same power level to get the same SIR β^*. It is easy to compute that with $L\alpha$ users, and a processing gain of L, the common received power required for the conventional receiver is given asymptotically as $L \to \infty$ by

$$P_{\text{mf}}(\beta^*) = \frac{\beta^* \sigma^2}{1 - \alpha\beta^*} . \tag{5.10}$$

For a given constraint P on the received power, the maximum number of users supportable is then

$$\alpha_{\max} = \frac{1}{\beta^*} - \frac{\sigma^2}{P} \text{ users/degree of freedom.}$$

The capacity of the conventional receiver when $P = \infty$ is then

$$C_{\text{mf}}(\beta^*) = \frac{1}{\beta^*} \text{ users/degree of freedom.} \tag{5.11}$$

Phrased differently, as $\alpha \to \frac{1}{\beta^*}$, the system saturates and the required power level goes to infinity.

Now, let us turn to the MMSE receiver. To satisfy given target SIR requirements for each user, [9, 19] showed that there is an optimal solution for which the received power of *every user* is minimized; moreover, they gave an iterative algorithm to compute it. However, here we can give an explicit solution and characterize the resulting system capacity.

To begin, we fix the number of users per degree of freedom at α. As in the conventional receiver case, it turns out that the system saturates if α is too high, so we first obtain a necessary and sufficient condition for feasibility. It can be shown, from the monotonicity property of (5.6), that in the limit of a large number of degrees of freedom, the system is feasible if and only if the SIR can be met with equal received powers for all users. Setting the received powers of all users to be equal in (5.6) tells us that a given target SIR requirement β^* can be met if and only if

$$\alpha < \frac{1 + \beta^*}{\beta^*} .$$

If this condition is satisfied, it can further be shown that the minimum power solution is given by having the received powers of all users be

$$P_{\text{mmse}}(\beta^*) = \frac{\beta^*\sigma^2}{1 - \alpha \dfrac{\beta^*}{1+\beta^*}}. \tag{5.12}$$

Hence, the capacity of the system under MMSE receiver is

$$C_{\text{mmse}}(\beta^*) = \frac{1+\beta^*}{\beta^*} \text{ users/degree of freedom.} \tag{5.13}$$

Moreover, for a given received power constraint P, the maximum number of users that can be supported is attained by assigning each user the same received power, and that number is given by

$$\alpha_{\max} = \frac{1+\beta^*}{\beta^*}\left(1 - \frac{\beta^*\sigma^2}{P}\right) \text{ users/degree of freedom.}$$

Contrasting (5.10) and (5.11) with (5.12) and (5.13), we note that if α is feasible for both types of receiver, then the power consumption of the MMSE receiver system is less than that of the matched-filter system, and the MMSE system has potentially much greater capacity. Indeed, if $\alpha < 1$, then we can accommodate arbitrarily large β^* without saturating the MMSE receiver, whereas the the conventional receiver saturates as β^* approaches $\frac{1}{\alpha}$. For fixed β^*, we also note that the MMSE receiver system saturates at a higher value of α, yielding a capacity of precisely 1 more user per degree of freedom than the system with a conventional receiver. On the other hand, the relative gain of the MMSE receiver system is not so large for small values of β^*.

5.4 Multiple Classes, Maximum Power Constraints, and Effective Bandwidths

It is straightforward to generalize these results to the case in which there are J classes, with all class j users requiring an SIR of β_j. We denote the number of users of class j by $\alpha_j L$ and again consider the limiting regime $L \to \infty$.

The conventional matched-filter receiver results generalize very easily to

$$P_{\text{mf}}(j) = \frac{\beta_j\sigma^2}{1 - \sum_{j=1}^{J}\alpha_j\beta_j},$$

where $P_{\text{mf}}(j)$ denotes the common, received power level of all users of class j (see [7]). Thus, the capacity constraint on feasible values of $(\alpha_1, \ldots, \alpha_J)$ is the linear con-

straint $\sum_{j=1}^{J}\alpha_j\beta_j < 1$. Furthermore, if class j users have a maximum power constraint that $P_{\text{mf}}(j) \le \overline{P}_j$ for each j, then the tighter capacity constraint

$$\sum_{j=1}^{J} \alpha_j\beta_j \le \min_{1\le i\le J}\left[1 - \frac{\beta_i\sigma^2}{\overline{P}_i}\right]$$

emerges [5]. It is convenient to refer to β_j as the *bandwidth* of class j users, in degrees of freedom per class j user. Let us denote this bandwidth by

$$e_{\text{mf}}(\beta_j) \equiv \beta_j \text{ degrees of freedom per class } j \text{ user.}$$

We now show that the MMSE receiver results generalize in a similar manner. It is clear in this case also that the minimal power solution consists of the same received power for each class: let all users in class j be received at power P_j. Then, the power control equations become

$$\frac{P_i}{\sigma^2 + \sum_{j=1}^{J}\alpha_j I(P_j, P_i, \beta_i)} = \beta_i, \qquad i = 1, 2, \ldots, J, \tag{5.14}$$

where, as in Theorem 5.1, $I(P_i, P_j, \beta_j) \triangleq \frac{P_iP_j}{P_j + P_i\beta_j}$. But (5.14) implies that $\frac{\beta_j}{P_j}$ is a constant, which allows us to simplify (5.14) down to

$$P_{\text{mmse}}(i) = \frac{\beta_i\sigma^2}{1 - \sum_{j=1}^{J}\alpha_j \dfrac{\beta_j}{1+\beta_j}} \qquad i = 1, 2, \ldots, J. \tag{5.15}$$

The capacity constraint for the MMSE receiver with J classes is therefore given by

$$\sum_{j=1}^{J} \alpha_j \frac{\beta_j}{1+\beta_j} < 1, \tag{5.16}$$

which is linear in $\alpha_1, \ldots, \alpha_J$.

As above, maximum power constraints provide tighter capacity constraints, and in this context we note that (5.15) implies that

$$\sum_{j=1}^{J} \alpha_j \frac{\beta_j}{1+\beta_j} = 1 - \frac{\beta_i\sigma^2}{P_{\text{mmse}}(i)}, \qquad i = 1, 2, \ldots, J.$$

Thus, if $P_{\text{mmse}}(i) \le \overline{P}_i$ is a maximum power constraint on class i, then the linear constraint

$$\sum_{j=1}^{J} \alpha_j \frac{\beta_j}{1+\beta_j} \le \min_{1\le i\le J}\left[1 - \frac{\beta_i\sigma^2}{\overline{P}_i}\right], \qquad i = 1, 2, \ldots, J$$

defines the restricted capacity region of the system. It is natural to define the effective bandwidth of class j users as $e_{\text{mmse}}(\beta_j)$ degrees of freedom per user, where

$$e_{\text{mmse}}(\beta_j) \equiv \frac{\beta_j}{1+\beta_j}.$$

Linearity in the matched-filter case is a straightforward consequence of the fact that powers add. However, our MMSE effective bandwidth results are rather surprising, as the receiver itself depends on the signature sequences and the received powers of the users. Another interesting observation is that no matter how high β is, the MMSE effective bandwidth of a user is upper bounded by unity. We will gain further insight into why this is so in the next section.

5.5 The Decorrelator

To this point, we have contrasted the performance of the MMSE receiver with that of the conventional matched-filter receiver. It is also illuminating to compare the performance of the MMSE receiver with that of the decorrelator. The decorrelator was in fact the first linear "multiuser detector" described by Lupas and Verdu [10]. As discussed in Chapter 3, this receiver is known to have optimal *near-far resistance* [11], as measured by the worst-case performance over all choices of interferers' powers and in the limit of vanishing background noise power. Here, we focus on the SIR performance for finite noise power and random sequences and obtain simple answers. It can be shown that in a large system with α users per degree of freedom, the (random) SIR under the decorrelating receiver for user 1 converges in probability to β_1^*, given by

$$\beta_1^* = \begin{cases} \dfrac{P_1(1-\alpha)}{\sigma^2} & \alpha < 1 \\ 0 & \alpha \geq 1 \end{cases}. \tag{5.17}$$

We observe that as the number of users α per degree of freedom approaches 1, the SIR goes to zero. Geometrically, as the dimensionality of the orthogonal complement to the span of the interference decreases to zero, the length of the projection of the desired signal onto this orthogonal complement tends to zero, and so in the limit the projected signal is lost in the background noise. This behavior is the high price paid for ignoring the background noise. In contrast, the MMSE receiver can support more users than the number of degrees of freedom because it takes both the interference and the background noise into account.

By comparing (5.17) and (5.3), one can see that the effective interference for an interferer on user 1 under the decorrelator is $\frac{P_1}{\beta_1}$, which does not depend on the

power of the interferers. Equation (5.17) further implies that the capacity constraint on the system is $\alpha < 1$.

We also observe that if all users require an SIR of β and employ power control, then it is sufficient for each user to be received with power at least $\beta\sigma^2/(1-\alpha)$. Thus, for a given received power constraint $\overline{P}$, the maximum number of users with SIR requirement β supportable is $1 - \beta\sigma^2/\overline{P}$ (per degree of freedom). Similarly, for multiple classes of users with SIR requirement β_j and power constraint $\overline{P}_j$ for the jth class, the system can support α_j users (per degree of freedom) from each class if

$$\sum_{j=1}^{J} \alpha_j \leq \min_{1 \leq j \leq J} \left[1 - \frac{\beta_j \sigma^2}{\overline{P}_j} \right].$$

Thus, the capacity region under the decorrelator is given by

$$\sum_{j=1}^{J} \alpha_j \leq 1 \tag{5.18}$$

when there are no power constraints or, equivalently, when the background noise power σ^2 goes to zero. So, each user occupies an effective bandwidth of 1 degree of freedom, independent of the value of β.

From (5.17), it can be immediately inferred that the efficiency of a decorrelator in a large system with random spreading sequences is $1 - \alpha$ if α, the number of users per degree of freedom, is less than 1, and is zero otherwise. Since this efficiency does not depend on the background noise power σ^2, it is also the asymptotic efficiency.

It is well known [12] that the MMSE receiver has the same asymptotic efficiency as the decorrelator, and hence the decorrelator is optimal in this sense among all linear receivers. However, comparing (5.16) and (5.18), one can see that the capacity region under the MMSE receiver is strictly larger than that under the decorrelator, even as the background noise goes to zero. In particular, the MMSE receiver can in general accommodate more users than the number of available degrees of freedom, while the decorrelator cannot. This apparent paradox can be resolved by noting that when $\alpha > 1$, the SIR attained by the decorrelator is zero (5.17) while the attained SIR by the MMSE receiver is strictly positive but bounded as the noise power σ^2 goes to zero. Since the asymptotic efficiency measures only the *rate* at which the SIR goes to infinity as σ^2 goes to zero, they are the same (zero) for both receivers. On the other hand, the capacity region quantifies the number of users with *fixed* SIR requirements a receiver can accommodate; hence the difference between the decorrelator and the MMSE receiver is reflected. In practice, users have target SIR requirements and hence the capacity region characterization seems to be a more natural performance measure than the asymptotic efficiency. In this context, the decorrelator remains suboptimal even as the noise power σ^2 approaches zero.

5.6 Antenna Diversity

In spread-spectrum systems, diversity gain is obtained by spreading over a wider bandwidth. However, there are other ways to obtain diversity benefits in a wireless system. A technique, particularly effective for combating multipath fading, is the use of an *adaptive antenna array* at the receiver. Multipath fading can be very detrimental because the received signal power can drop dramatically due to destructive interference between different paths of the transmitted signal. By placing the antenna elements greater than half the carrier wavelength apart, one can ensure that the received signal fades more or less independently at the different antenna elements. By appropriately weighing, delaying and combining the received signals at the different antenna elements, one can obtain a much more reliable estimate of the transmitted signal than with a single antenna. Such antenna arrays are said to be *adaptive* since the combining depends on the strengths of the received signals at the various antenna elements. This signal strength in turn depends on the location of the users. Moreover, the combining weights will be different for different users, allowing the array to focus on specific users while mitigating the interference from other users. This process is called *beamforming*. From our previous results, it turns out that the capacity of such an antenna array system can again be characterized by effective bandwidths.

The following is a model for a synchronous, multiple-access antenna-array system:

$$\mathbf{Y} = \sum_{m=1}^{M} X_m \mathbf{h}_m + \mathbf{W}.$$

Here, X_m is the transmitted symbol of the mth user, and $\mathbf{Y}$ is an L-dimensional vector of received symbols at the L antenna elements of the array. The vector $\mathbf{h}_m$ represents the fading of the mth user at each of the antenna array elements. The entries are complex to incorporate both phase and magnitude information. The vector $\mathbf{W}$ is zero-mean, variance-σ^2 Gaussian background noise.

The fading is time-varying, as the mobile users move, but usually at a much longer time scale than the symbol rate of the system. Assuming then that the channel fading of the users can be measured and tracked perfectly at the receiver, we would like to combine the vector of received symbols appropriately to maximize the SIR of the estimates of the transmitted symbols of the users. The optimal linear receiver is clearly the MMSE. Assuming that the fading of each user is independent and identically distributed from antenna element to antenna element, we are essentially in the same setup as for spread-spectrum systems. Thus, for a system with a large number of antenna elements and large number of users, we can treat each of the interfering users as contributing an additive *effective interference*. Under perfect power control, the system capacity is characterized by sharing the L degree of freedom among the users according to their *effective bandwidths* given by the pre-

vious expressions for the different receivers. The only difference here is that the L degrees of freedom are obtained by spatial rather than frequency diversity.

These results should be compared with that of Winters et al. [22], which showed that for a flat Rayleigh fading channel, a combiner that attempts to null out all of the interferers costs one degree of freedom per interferer. This combiner is, of course, the suboptimal decorrelator, which we have shown earlier to be very wasteful of degrees of freedom if interferers are weak. While Winters' result holds for the Rayleigh model and any number of antennas, our results hold for *any* fading distribution but are asymptotic in the number of antennas.

5.7 Concluding Remarks

It is illuminating to compare the effective interference and effective bandwidths of the users in the three cases: the conventional matched filter, the MMSE filter, and the decorrelating filter. This comparison is shown in Figures 5.1 and 5.2. The

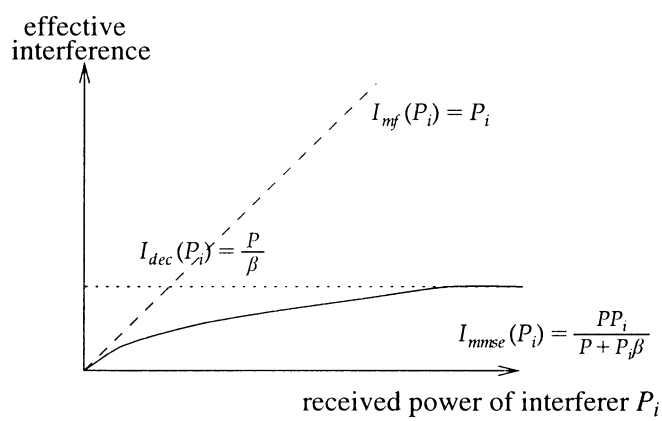

Figure 5.1 Effective interference for the three receivers as a function of interferer's received power P_i. Here, P is the received power of the user to be demodulated, and β is the achieved SIR.

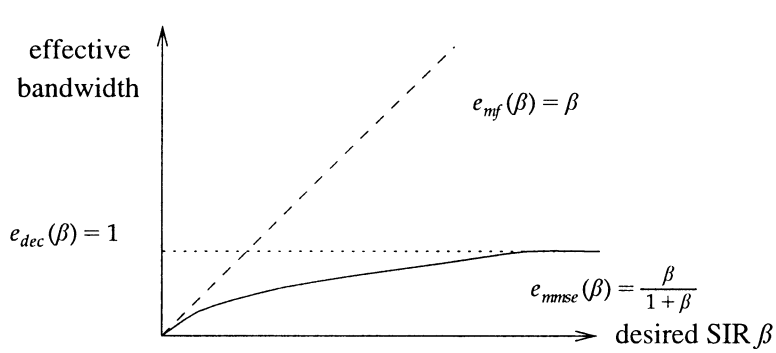

Figure 5.2 Effective bandwidths for three receivers as a function of SIR.

effective interference under MMSE is nonlinear and depends on the received power P of the user to be demodulated as well as on the achieved SIR β. The effective interference under the conventional matched filter is simply P_i, the received power of the interferer. Under the decorrelator, the effective interference is $\frac{P}{\beta}$, independent of the actual power of the interferer. The intuition here is that the decorrelator completely nulls out the interferer, no matter how strong or weak it is.

Assuming perfect power control, we can define effective bandwidths that characterize the amount of network resources a user consumes for a given target SIR. The effective bandwidths under the conventional, MMSE, and decorrelating receivers are β, $\frac{\beta}{1+\beta}$, and 1, respectively. We note that the conventional receiver is more efficient than the decorrelator when β is small and far less efficient when β is large. Intuitively, at high SIR requirements, a user has to transmit at high power, thus causing a lot of interference to other users under the conventional receiver. Not surprisingly, since it is by definition optimal, the MMSE filter is the most efficient in all cases. When β is small, the MMSE filter operates more like the conventional receiver, allowing many users per degree of freedom; but when β is large, each user is decorrelated from the rest, much as in the decorrelator receiver, and therefore the interferers can still occupy no more than one degree of freedom per interferer.

The effective bandwidth concept for the MMSE receiver is valid only in the perfectly power-controlled case. By contrast, the concept of effective interference applies with or without perfect power control and may prove more useful in the multicell context.

While these results provide much insight into the performance of these filters, we must emphasize that they pertain only to a single cell, without fading, and in the time-synchronous case. It remains to be seen how these filters perform in more realistic scenarios.

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