DESIGNING FAST 3-D RF EXCITATIONS BY OPTIMIZING THE NUMBER, PLACEMENT AND WEIGHTING OF SPOKES IN K-SPACE VIA A SPARSITY-ENFORCEMENT ALGORITHM

A. C. Zelinski\textsuperscript{1}, K. Setsompop\textsuperscript{1}, V. K. Goyal\textsuperscript{1}, V. Alagappan\textsuperscript{1}, U. Fontius\textsuperscript{2}, F. Schmitt\textsuperscript{3}, L. L. Wald\textsuperscript{4,5}, and E. Adalsteinsson\textsuperscript{4,5}

\textsuperscript{1}Department of Electrical Engineering and Computer Science, MIT, Cambridge, MA, United States, \textsuperscript{2}A. A. Martinos Center for Biomedical Imaging, Massachusetts General Hospital, Harvard Medical School, Charlestown, MA, United States, \textsuperscript{3}Siemens Medical Solutions, Erlangen, Germany, \textsuperscript{4}Harvard-MIT Division of Health Sciences and Technology, MIT, Cambridge, MA, United States

\textbf{INTRODUCTION:} Playing sinc-like RFs in the presence of slice-selective gradient trajectories is useful for exciting a thin slice in $\mathbb{Z}$ and is analogous to playing sinc-like “spokes” along $k$, in excitation k-space. Recently, the use of multiple complex-weighted spokes at different locations in the $(k_x,k_z)$ plane has led to RF pulses that mitigate $B_1$-inhomogeneity in single-coil excitation systems \cite{1} and reduce excitation time (TE) on multi-channel systems \cite{2}. While placing a tall spoke in k-space does indeed significantly increase pulse duration and minimum TE, it is a necessary tradeoff to ensure a sharp slice profile with low sidelobes. Because of this high temporal cost per spoke, an ideal thin-slice design would be one that used very few spokes while achieving a user-specified excitation in the $(x,z)$ plane with high-fidelity. To achieve this, we propose an algorithm that optimizes the number, placement and weighting of spokes, based on sparse approximation theory. First we show the theory encompasses RF design for multi-channel excitation systems, with single-channel systems as a base case. We then show the method generates fast, high-fidelity slice-selective pulses, achieving near-optimal tradeoff of TE & excitation fidelity. Experiments conducted in a phantom on a 3T Siemens Magnetom TRIO with an 8-channel parallel TX array shows the algorithm’s advantages over traditional $(k_x,k_z)$ spoke placement patterns.

\textbf{METHODS AND RESULTS:} Sparse approximation (SA). The goal of SA is to find a vector or matrix of unknowns with a small number of nonzero elements such that a system of equations approximately holds, e.g., $m = \sum_{j=1}^{n} F_{j} p_j + n \cdot$, where $m, n \in \mathbb{C}^{d} , F_j \in \mathbb{C}^{d \times n}, \psi_j \in \mathbb{C}^{n}$, and $N > M$. This problem is ill-posed because there are infinitely many choices of $\psi_j$ vectors that solve it. But consider enforcing sparsity on the $\psi_j$, requiring the $l_1$-norm of each to be small, which is similar to requiring many elements of each $\psi_j$ to equal zero \cite{3}. Suppose we further constrain the $\psi_j$ requiring them to be simultaneously sparse: each of the $\psi_j$ must have nonzeros occurring at a set of indices. With such requirements, the problem is no longer ill-posed. Letting $\Phi = [\psi_1, \ldots, \psi_p]$ a program that finds a simultaneously sparse set of $\psi_j$, and approximately yields $m$ is as follows: $\min_{\Phi_P} (1 - \lambda) \left\| m - \sum_{j=1}^{n} F_j p_j \right\|_2 + \lambda \left\| \Phi \right\|_1$, where the second term, $\|\Phi\|_1$, is the $l_1$-norm of the $l_2$-norms of the rows of $\Phi$, a simultaneously sparse norm that penalizes (rewards) the program when the columns of $\Phi$ have dissimilar (similar) sparsity profiles. The first term keeps the residual error down. As $\lambda$ increases from 0 to 1, sparser solutions are generated while the residual error increases, i.e., $\lambda$ trades off sparsity with residual error. Because the objective function is convex, there exists an optimal solution $\Phi$ that attains the global minimum. $\Phi$ may be computed via a Second-Order Cone program. Refer to \cite{3,4} for more details.

\textbf{Proposed algorithm.} Our goal is to excite a thin, sharp slice that approximately equals a user-specified target excitation $m(x,y)$ at $z = z_0$ and zero at $z \neq z_0$. To accomplish this, we must decide on a number of spokes to use, their locations in $(k_x,k_z)$, and weights for each. Using spokes in $k$ will let us obtain a thin slice in $\mathbb{Z}$. But achieving the in-slice target $m(x,y)$ is more complicated: ideally, many weighted spokes would be placed in $(k_x,k_z)$ such that $m(x,y)$ was almost exactly achieved, but this would require many spokes and result in a long TE. To keep TE short, we must use a small number of spokes, but with few spokes, achieving $m(x,y)$ becomes difficult. Let us define $m$ to be vector of spatial samples of the $(x,y,z)$ excitation in some region of interest (ROI).

\textbf{Analogy between spokes and Diracs in 2-D Fourier domain.} Placing a spoke in $(k_x,k_z)$ with some arbitrary complex weight $\phi$ is analogous to placing a weighted-Dirac delta, $\delta(k_x,k_z)$, in the 2-D Fourier domain. Since spokes are expensive in terms of pulse duration, each $\delta$ is also expensive. Using this analogy, our goal is now: using a small number of complex-weighted $\delta$'s in 2-D Fourier space, ensure that their 2-D Fourier transform is close to $m(x,y)$ at all points in the region of interest.

\textbf{Base case ($P = 1$) formulation.} Assume a finite grid of discrete points exists in $(k_x,k_z)$, each of which is a Dirac delta that produces a complex exponential in the spatial domain. An arbitrary choice of complex weights at different points on the grid results in the weighted grid by a Fourier transform. Arranging the complex weights into the vector $\phi$, the following holds: $r = D \cdot A \cdot p = F \cdot p$, where $r$ is a vector of spatial samples of the resulting $m(x,y)$ excitation in the ROI, $D$ is a diagonal matrix of samples of the coil sensitivity pattern in the ROI, and $A_{\psi_j} = \exp(j2\pi k_{x}n_{x}[m]k_{z}n_{z}[m])$ \cite{5}. If the $i$-th element of $\phi$, is nonzero, this corresponds to a spike at $(k_{x}n_{x},k_{z}n_{z})$. Thus, a sparse $\phi$, that results in an $r$ close to $m$ is ideal: it implies a short, high-fidelity excitation.

\textbf{Extension to parallel systems ($P > 1$).} For parallel systems, the formulation extends as follows: $r = D_{\text{parallel}} \cdot A_{\text{parallel}} \cdot p = F \cdot p$, with the constraint that the $\phi$, must be simultaneously sparse, which way that the RF pulses along each of the $P$ coils (the $b_{d}(t)$ waveforms) must each play along the same k-space trajectory. This constraint arises because the system’s set of gradients determines a unique k-space trajectory $k(t)$. If the $\phi$, were not simultaneously sparse, it would imply the RFs are concurrently played along different k-space trajectories, which is not possible.

\textbf{Step I: determine spoke locations.} Using as few points on the frequency grid as possible, we want to attain the user-specified $m$ within the thin-slice, i.e., we want to find a simultaneously sparse $\Phi$ matrix such that the residual error term $\left\| m - \phi \right\|_1 = \left\| m - \sum_{j=1}^{n} F_j p_j \right\|_1$ is small. Finding this $\Phi$ is accomplished by fixing $\lambda$ and solving the optimization program above. With the proper choice of $\lambda$, a simultaneously sparse, globally optimal $\Phi$ matrix is found that keeps the residual error down.

\textbf{Step II: keep $T$ spokes and determine $PT$ weights.} Since each row of $\Phi$ that contains nonzeros corresponds to a spoke that must be traversed in k-space, we zero out all but $T$ rows of $\Phi$, keeping those with the largest $l_2$-energy. Thus, $T$ is a control parameter explicitly trading off the number of spokes, and hence TE, with excitation fidelity. Since all but $T$ rows of $\Phi$ equal zero, the affine system of equations is now reduced to $r = \sum_{j=1}^{n} F_{j,T} p_j$, where each $F_{j,T}$ is a truncated matrix whose columns correspond to the $T$ columns of $F_j$ that remain after discarding all but $T$ rows of $\Phi$. Now, $\Phi_{T}$ is recalculated to attain an excitation closer to $m$, i.e., the weights at each of the $T$ spoke locations are retuned for each of the $P$ coils in a least-squares sense via the pseudo-inverse of $[F_{1,T}, \ldots, F_{T,T}]$.

\textbf{Step III: generate an RF pulse set.} At this point, $T$ spokes have been placed in k-space at points on the $(k_x,k_z)$ grid implicitly defined by the $\phi_j$. Further, $P$ weights have been determined for each spoke (one per excitation coil). With this information, a set of RF pulses may be designed using the method in \cite{2}.

\textbf{Results.} The proposed method was compared to a non-optimized spoke placement on a grid within a fixed radius. Experiments were conducted in a phantom in a 3T Siemens Magnetom TRIO equipped with an 8-channel TX array. Each method used 20 spokes and both attempted to excite the dual-space bifurcation target shown above.