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A MULTIPOLE-ACCELERATED BOUNDARY-ELEMENT APPROACH TO TRANSIENT SIMULATIONS OF THREE-DIMENSIONAL INTEGRATED CIRCUIT INTERCONNECT

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Abstract

It has recently been shown that accurate electro-quasistatic, or transient interconnect, analysis for three-dimensional structures can be performed using the same surface discretization used for ordinary capacitance extraction. In this paper we describe how to use multipole-accelerated, iterative algorithms to reduce the computational complexity of surface-discretization based transient interconnect analysis from order N^3 to order N, where N is the number of surface unknowns.

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1. Introduction. In order to analyze the parasitic coupling effects of nearby lines of interconnect on an integrated circuit, referred to as cross-talk, fully three-dimensional simulations are required to accurately model the distributed effects and the detailed geometric configuration of the interconnect structure. It has been suggested[4] that reasonably accurate cross-talk simulations could be performed by computing the time evolution of the electric field both inside and outside the conductors via a finite-difference discretization of Laplace's equation. More recently, a boundary-element approach based on Green's theorem was proposed[5], which performed the same calculation but avoids the volume mesh associated with finite-difference methods. In both formulations, the electro-quasistatic assumptions are made; *i.e.* magnetic induction and volume charges are negligible compared with capacitive effects and surface charges, respectively. The boundary-element approach is effective because only surface quantities are required to model capacitive coupling.

Though boundary-element methods avoids volume meshes, the approach generates dense matrices which require $O(N^3)$ operations to factor directly, or $O(N^2)$ operations to solve iteratively, where where N is the number of surface unknowns. This rapid computational growth with problem size makes the method impractical for very large problems. In this paper, we describe an $O(N)$, preconditioned, multipole-accelerated, iterative algorithm for solving the Green's-theorem based boundary-element formulation, and show that significant reduction in computation time and memory cost can be obtained even for modestly sized interconnect examples.

2. Green's Formulation. It has been shown[5] that, since the potential satisfies Laplace's equation inside and outside a conductor in the electro-quasistatic regime, a Green's theorem-based approach is suitable for the boundary-element technique. The governing equation is

$$4\pi\tau\frac{\partial\psi(x)}{\partial t} = -4\pi\psi(x) + \int_S \psi(x')\frac{\partial}{\partial n'}\frac{1}{\|x-x'\|}da' + \frac{1}{\sigma}\int_{S_{contact}} \frac{J_{external}(x')}{\|x-x'\|}da', \tag{1}$$

where ψ is the electric potential, $\tau = \epsilon/\sigma$ is the dielectric relaxation time, x is a point on a conductor surface, S is the union of all conductor surfaces, $S_{contact}$ are the surfaces in contact with external voltage sources, $J_{external}$ is the current supplied externally by contacts, and $\|x-x'\|$ is the Euclidean distance betweeen x and x'.

To numerically solve (1) for ψ at non-contact conductor surfaces and for J at contact surfaces, the surfaces of all conductors are broken into small tiles or panels. It is then assumed that on each panel l, there is a constant potential ψ_l. In addition, for each contact panel l_c, there is a constant current density J_c. Then a collocation scheme, in which (1) is enforced at the center point in each of N panels, is used to generate a system of N equations[1]. Using the fixed-timestep, backward-Euler method, the linear system to be solved is

$$(\bar{D}_L|P)\begin{pmatrix}\Psi^f\\ J^c\end{pmatrix}_{t=(m+1)h} = -\bar{D}_R\Psi^c_{t=mh} + \frac{4\pi\tau}{h}\Psi_{t=mh}, \tag{2}$$

where, h is the timestep, P is potential matrix of the panels, $\bar{D}$ contains the dipole potential matrix of the panels, and $\bar{D}_L$, $\bar{D}_R$ are the left and the right portions of $\bar{D}$, respectively. Ψ and J are the vectors of panel potentials and currents, and the superscripts c and f represent the contact and free panels, respectively. Given the panel potentials at the m-th timestep, the above back-Euler equation can be solved to yield the potentials at the free panels and the current densities at the contact panels at the $(m+1)$-st timestep, with h being the timestep size.

3. Multipole Acceleration for the Mixed Problem. The linear system to be solved (2) combines both three-dimensional potential integral equations of the first kind and the second kind, with $\frac{1}{r}$ and $\frac{\partial}{\partial n}\frac{1}{r}$ kernels, respectively. It is generally solved with a collocation (as is the case here) or a Galerkin scheme in which the singularities are integrated analytically. The mixed $N \times N$ matrix , $(\bar{D}_L|P)$, thus generated is dense and non-symmetric. Since the elements of P are generated with the $\frac{1}{r}$ kernal and those of $\bar{D}_L$ are generated with the $\frac{\partial}{\partial n}\frac{1}{r}$ kernal, the unknowns can be interpreted as source distributions, with the panels associated with Ψ^f acting as constant dipole sources, and the panels associated with J^c acting as constant charge (monopole) sources. The right-hand side then corresponds to a generalized potential distribution. This linear system can be solved by direct

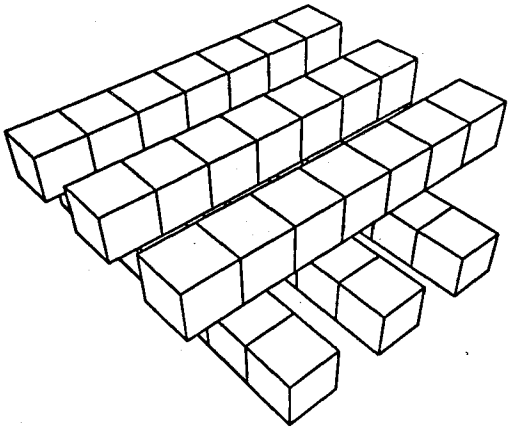

FIG. 1. *3 × 3 Bus Crossing Example (actual discretization is finer.)*

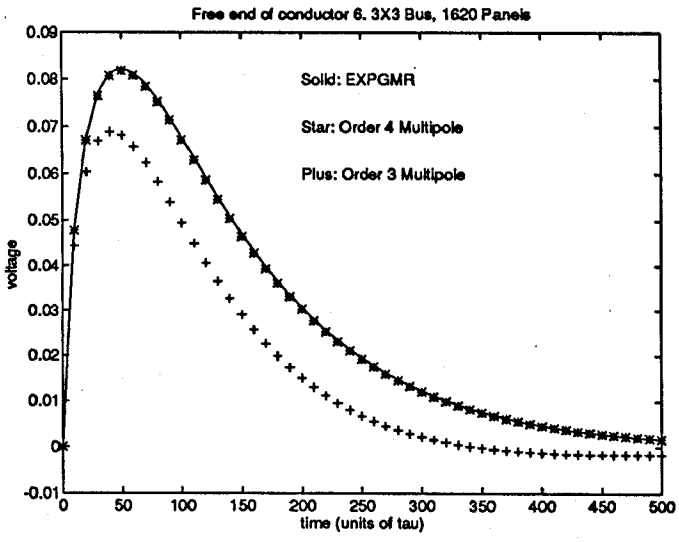

FIG. 2. *Comparison between the explicit and multipole calculations.*

factorization, or with an iterative technique like a Krylov-subspace method[6]. For dense matrices, the cost of direct factorization grows lilke N^3, and even a rapidly converging iterative method will still require computation time and memory which grows like N^2. A fast-multipole algorithm can be combined with an interative method to reduce the time and memory required to solve (2) to nearly order N[3].

4. Application Experiments. We modified an existing multipole-accelerated program[2] to implement our Greens' theorem-based transient interconnect simulation program. To show that the multipole-accelerated, Green's theorem-based method is suitable for practical applications, we perform transient simulations on a 3 × 3 bus-crossing example depicted in Figure 1. The wires are one-unit wide, seven units long, and all spacings are unit distances. The discretization is uniform and finer than shown, and a total of up to 6500 panels have been generated and analyzed from the example. For convenience, we label the conductors 1 through 6 by counting their visible ends clockwise. We investigate the effects of capacitive coupling among them by grounding the near ends of conductors 1,3,4,5,6, and applying a unit-step voltage source to the near end of conductor 2. The far ends of all six wires are left open-circuited, which might correspond to high-impedance terminals. The coupling from the on-switching of the second wire to the open ends of the other five is the cross-talk that must be minimized in packaging and circuit design.

In our numerical experiments, we used the GMRES iterative method[6]. Potential evaluations for the Krylov-subspace iterations were done either explicitly or by using the fast-multipole algorithm, and the final results are shown for comparison. For a 1620-panel discretization, in which each unit face is divided into 9 equal square panels, we show the computation results for the potential at the free end of conductor 6 (Figure 2). Fifty uniform timesteps were used in the calculation. Results from using third- and fourth- order multipole expansions in the calculations are compared against explicit calculations (EXPGMR). Fourth-order multipole calculations are seen to be in excellent agreement with the explicit method, while third-order multipole calculations give only reasonable results. Note: the position for which the potential is plotted represents a worst-case situation; the third-order results are closer to the explicit results at other locations.

We performed the same calculation on progressively finer discretizations of this 3 × 3 bus-crossing example, and compared the growth in time and memory requirements of the multipole-accelerated versus the explicit approach. Figure 3 shows the CPU run times versus number of panels, and Figure 4 shows the memory allocated for each problem size; the program is run on an IBM Model 590. The data for the explicit approach is extrapolated beyond 1620 panels. For the multipole-accelerated approach, both CPU time and memory gowth as N, where as for the explicit approach, both memory and time grow as N^2. Up to 6500 panels have been analyzed with the multipole algorithm, in about 15 minutes and with about 200 MB of memory; the explicit calculation would have taken nearly five times as long and required almost an order of magnitude more main memory to perform. Thus, fairly realistic and interesting interconnect simulations can be performed with the multipole approach on a scientific workstation within reasonable time.

5. Conclusion. A multipole-accelerated boundary-element approach to transient interconnect simulations has been implemented and shown to have significant advantages in both computation time and memory

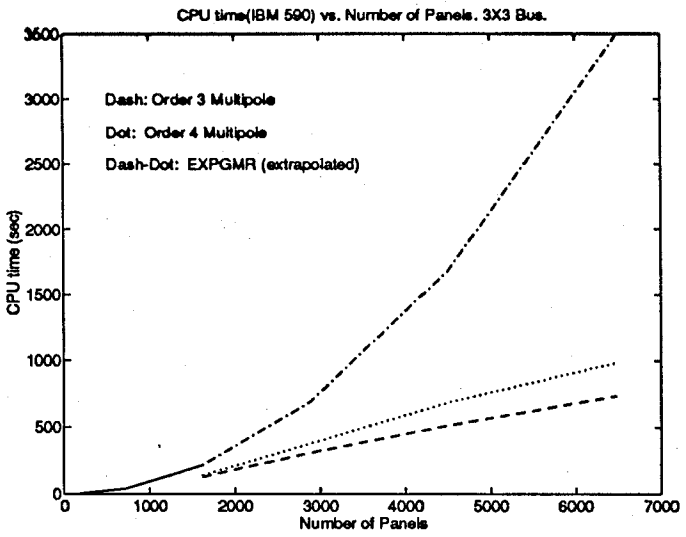

FIG. 3. *Computation time versus problem size.*

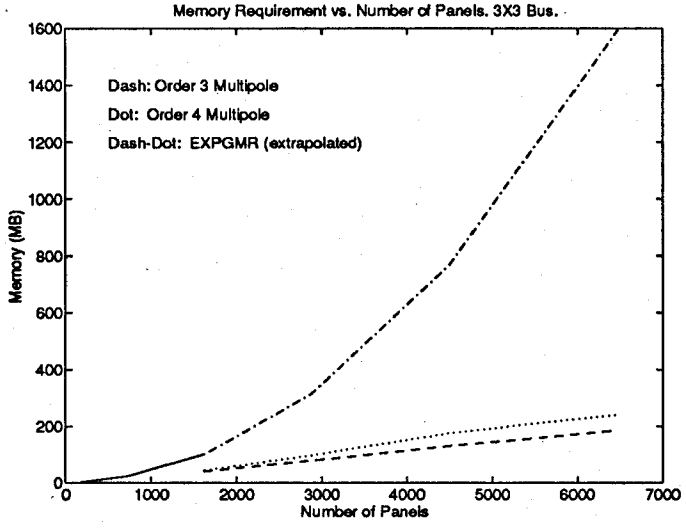

FIG. 4. *Memory allocation versus problem size.*

requirements over the explicit method for a practical interconnect example. For iterative techniques, it is demonstrated that multipole-acceleration reduces both time and memory usage from $O(N^2)$ to $O(N)$. This is to be compared with the direct factorization method used in [5], which required $O(N^3)$ time and $O(N^2)$ memory.

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