FastPep: A Fast Parasitic Extraction Program for Complex Three-Dimensional Geometries

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Abstract

In this paper we describe a computationally efficient approach to generating reduced-order models from PEEC-based three-dimensional electromagnetic analysis programs. It is shown that a recycled multipole-accelerated approach applied to recent model order reduction techniques requires nearly two orders of magnitude fewer floating point operations than direct techniques thus allowing the analysis of larger, more complex three-dimensional geometries.

1 Introduction

A well known approach to modeling coupled inductive and capacitive effects is the Partial Element Equivalent Circuit (PEEC) approach [1]. However, for complex three dimensional structures, the number of densely coupled circuit elements can be in the tens of thousands obviating the PEEC circuit's use in circuit simulators. Recent model order reduction approaches could be applied, but even computing the reduced model may be too computationally expensive. In this paper we describe a computationally efficient approach to generating low order models from these large circuit models. The approach is also ripe for acceleration techniques such as the Fast Multipole Method [2, 3] or the Precorrected-FFT [4] approach allowing the analysis of larger, more complex three-dimensional geometries.

In Section 2 we discuss the PEEC discretization from which we derive the large dense linear system describing the interconnect. In Section 3 we describe using recent model order reduction techniques to generate models for efficient coupled circuit-interconnect simulation, and describe methods of accelerating the computation. Finally in Section 4, we present results of this interconnect modeling tool called FASTPEP.

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2 Formulation

In the area of interconnect analysis, perhaps the best known integral equation approach is the Partial Element Equivalent Circuit (PEEC) method [1]. To model current flow in the PEEC method, the interior of conductors is divided into a grid of volume *filaments*, each of which carries a constant current density along its length, as shown in Figure 1-a. To model charge accumulation, the surface of each conductor is covered with *panels*, each of which holds a constant charge density.

The interconnection of the filaments and panels, plus sources, $\boldsymbol{V}_t$, at the terminal pairs, generates a "circuit" whose solution gives the desired admittance parameters. Each filament is a branch of the circuit, as well as each panel as shown in Figure 1-b for a transmission line structure. For illustration, Figure 1-b coarsely models a two conductor line with terminating load Z_L. The length of the line is broken into three sections where each section consists of a bundle of three filaments. Two panels are connected at the nodes in between the sections. Each section of the line is broken into a bundle of filaments in order to model skin and proximity effects as shown in Figure 1-a.

The constitutive relations for the elements can be written as a single matrix in $\mathbb{C}^{b\times b}$, $b = f + p$, where f is the number of filaments and p the number of panels, by noting that the charge on the panels is related to current via $\boldsymbol{I}_b^p = \frac{d}{dt}\boldsymbol{q}_p$, and by viewing the node voltage at the panels as a branch voltage relative to ground, $\boldsymbol{V}_b^p = \boldsymbol{V}_n - 0$:

$$\boldsymbol{V}_b = \begin{bmatrix} \boldsymbol{V}_b^f \\ \boldsymbol{V}_b^p \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_L & 0 \\ 0 & \boldsymbol{P}/s \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_b^f \\ \boldsymbol{I}_b^p \end{bmatrix} = \boldsymbol{Z}\boldsymbol{I}_b, \quad (1)$$

where $s = j\omega$, $\boldsymbol{Z}_L\boldsymbol{I}_b^f \equiv (\boldsymbol{R} + s\boldsymbol{L})\boldsymbol{I}_b^f = \boldsymbol{V}_b^f$ with $\boldsymbol{L}$ as the dense filament partial inductance matrix, $\boldsymbol{R}$, the diagonal resistance matrix, and $\boldsymbol{V}_n = \boldsymbol{P}\boldsymbol{q}_p$ where $\boldsymbol{P}$ is the dense panel potential coefficient matrix.

To generate a system of equations, the circuit solution technique known as Mesh Analysis can be used whose solution gives the terminal admittance, $\boldsymbol{Y}_t$. The mesh approach has been used in the context of interconnect analysis in [5] and [6].

Kirchoff's voltage law, which implies that the sum of branch voltages around each mesh in the network must be zero, is represented by

$$\boldsymbol{M}\boldsymbol{V}_b = \left[\begin{array}{cc} \boldsymbol{M}_f & \boldsymbol{M}_p \end{array}\right] \left[\begin{array}{c} \boldsymbol{V}_b^f \\ \boldsymbol{V}_b^p \end{array}\right] = \boldsymbol{V}_s \tag{2}$$

where $\boldsymbol{V}_s$ is the mostly zero vector of source voltages, $\boldsymbol{M}_f$ sums filament voltages, and $\boldsymbol{M}_p$ sums panel voltages. The mesh currents, $\boldsymbol{I}_m$, are related to the branch currents, $\boldsymbol{I}_b$ via $\boldsymbol{M}^t\boldsymbol{I}_m = \boldsymbol{I}_b$.

To write the system in a state space form, the mesh currents and panel voltages are assigned states similar to the nodal voltages and inductor currents in Modified Nodal Analysis. Combining Eqns. (1), and (2) yields a first order system in state space form,

$$\begin{array}{rcl} s\boldsymbol{\mathcal{L}}\,\boldsymbol{x} & = & -\boldsymbol{\mathcal{R}}\,\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{V}_t \\ \boldsymbol{I}_t & = & \boldsymbol{B}^T\boldsymbol{x}. \end{array} \tag{3}$$

where

$$\boldsymbol{x} = \left[\begin{array}{c} \boldsymbol{I}_m \\ \boldsymbol{V}_b^p \end{array}\right], \boldsymbol{V}_t = \left[\begin{array}{c} \boldsymbol{V}_s \\ 0 \end{array}\right], \boldsymbol{\mathcal{L}} = \left[\begin{array}{cc} \boldsymbol{M}_f\boldsymbol{L}\boldsymbol{M}_f^t & 0 \\ 0 & \boldsymbol{P}^{-1} \end{array}\right],$$

$$\boldsymbol{\mathcal{R}} = \left[\begin{array}{cc} \boldsymbol{M}_f\boldsymbol{R}\boldsymbol{M}_f^t & \boldsymbol{M}_p \\ -\boldsymbol{M}_p^t & 0 \end{array}\right], \boldsymbol{B} = \left[\begin{array}{c} \boldsymbol{I} \\ 0 \end{array}\right].$$

Since $\boldsymbol{R}$ is a diagonal matrix, and $\boldsymbol{M}_f$ and $\boldsymbol{M}_p$ are sparse, then $\boldsymbol{\mathcal{R}}$ is sparse. However, to form the first block of $\boldsymbol{\mathcal{L}}$ requires $O(f^2)$ operations and memory since $\boldsymbol{L} \in \mathbb{R}^{f\times f}$ is dense. Similarly, the second block, $\boldsymbol{P}^{-1}$, requires $O(p^2)$ operations and memory to form, and then $O(p^3)$ operations to invert. For complex geometries with tens of thousands of filaments and panels, such growth rates are severely limiting. In the next section we discuss a more efficient technique for generating reduced order models from (3).

3 Coupled Circuit-Interconnect Simulation and Passive Model Order Reduction

To design with the admittance information available from (3), it is necessary to perform coupled simulation with nonlinear devices, such as CMOS drivers and receivers. Nonlinear devices require time domain simulation, and thus the admittance information is necessary from DC conditions up to the highest frequency of interest in the circuit.

One approach to coupling the PEEC-based package models with circuits is to include (3) in a circuit simulator instead of solving for the terminal behavior [1]. This approach has the drawback that the size of the system in (3) can easily be very large if high-accuracy is desired. Another possibility is to construct and solve (3) for various values of ω and then use some rational fitting algorithm to compute a model for the package [7, 8]. However, a more computationally efficient approach is to apply model order reduction techniques to (3) to derive a smaller approximation for direct insertion into a circuit simulator.

3.1 Model Order Reduction

The idea of model order reduction is to reduce (3), which can be on the order of tens of thousands, to a much smaller system which still captures the dominant behavior of the original system. For moment matching techniques, one wishes to derive a rational function whose moments, or terms in the Taylor series expansion, match that of the original admittance function, $\boldsymbol{Y}_t(s)$, up to some order. From (3), the admittance function can be expanded about $s = 0$ as

$$\boldsymbol{Y}_t(s) = \boldsymbol{B}^T\left(\boldsymbol{\mathcal{R}} + s\boldsymbol{\mathcal{L}}\right)^{-1}\boldsymbol{B} = \sum_{k=0}^{\infty} \boldsymbol{m}_k s^k, \tag{4}$$

where the moments can be easily obtained from

$$\boldsymbol{m}_k = -\boldsymbol{B}^T(\boldsymbol{\mathcal{R}}^{-1}\boldsymbol{\mathcal{L}})^k\boldsymbol{\mathcal{R}}^{-1}\boldsymbol{B}.$$

Thus we seek an approximation, $\tilde{\boldsymbol{Y}}_t(s) = \sum_{k=0}^{\infty}\tilde{\boldsymbol{m}}_k s^k$, such that $\tilde{\boldsymbol{m}}_k = \boldsymbol{m}_k, k = 1,\ldots,q$. Since $\boldsymbol{Y}_t(s)$ represents a passive circuit, we require $\tilde{\boldsymbol{Y}}_t(s)$ also be passive, which can be guaranteed via the numerically stable Arnoldi-based PRIMA model order reduction algorithm [9]. In order to apply PRIMA about $s = 0$, both $\boldsymbol{\mathcal{L}}$ and $\boldsymbol{\mathcal{R}}$ must be positive-semidefinite. The condition on $\boldsymbol{\mathcal{L}}$ follows since $\boldsymbol{P}$ and $\boldsymbol{L}$ are positive definite. Also, since every filament is modeled as resistor in series with an inductor, $\boldsymbol{\mathcal{R}}$ is both positive semidefinite and nonsingular.

3.2 Recycled iterative solver

Application of any moment matching scheme about $s = 0$ requires the computation of repeated matrix-vector products with the matrix $(\boldsymbol{\mathcal{R}}^{-1}\boldsymbol{\mathcal{L}})$ in order to obtain a reduced-order model. For instance, an

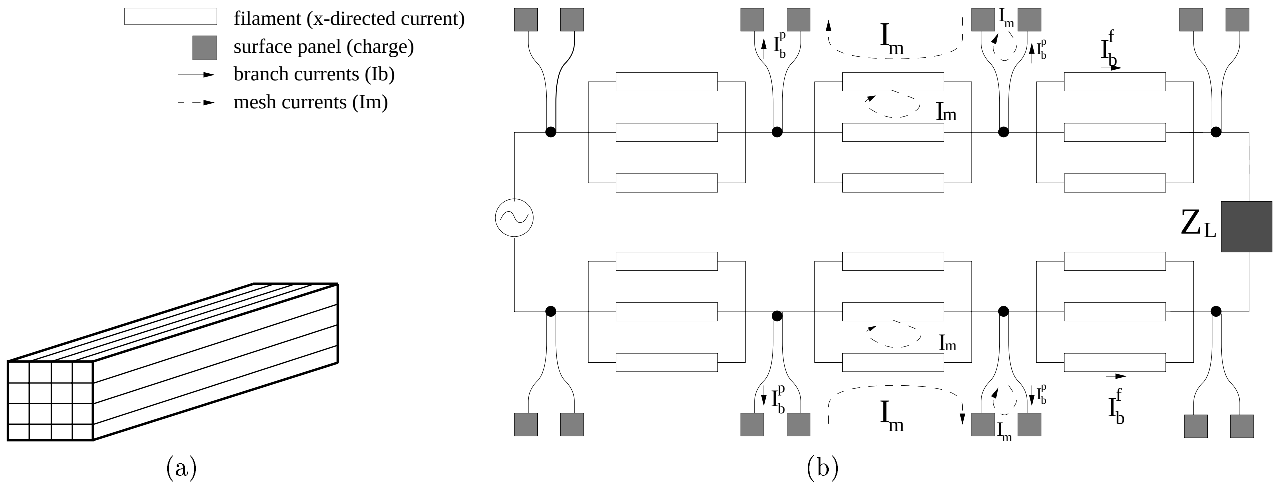

Figure 1: a) A section of conductor and its bundle of volume filaments; b) A circuit describing the mesh quantities for two parallel conductors divided into 3 sections each.

Arnoldi type algorithm requires $q-1$ such products to produce an order q model. However, because the partial inductance matrix $\boldsymbol{L}$ and potential coefficient matrix, $\boldsymbol{P}^{-1}$, which appear in $\boldsymbol{\mathcal{L}}$, are both large and dense, many multiplications by $\boldsymbol{\mathcal{L}}$ can be prohibitively expensive. In particular, if done directly, multiplication by $\boldsymbol{P}^{-1}$, would require an initial dense matrix factorization which is $O(p^3)$ operations. For modern packaging structures, for which p exceeds ten thousand, such a factorization is prohibitive.

The expensive factorization can be avoided by noting that the computation $\boldsymbol{q} = \boldsymbol{P}^{-1}\boldsymbol{v}$ is equivalent to solving for the panel charges, $\boldsymbol{q}$, given a set of voltages, $\boldsymbol{v}$. It is thus possible to use a preconditioned, Krylov-subspace iterative method to solve $\boldsymbol{Pq} = \boldsymbol{v}$ as outlined in Algorithm 1 [10]. Note that the dominant cost of each iteration is the $O(p^2)$ computation of a dense matrix-vector product, $\boldsymbol{Pw}$, to acquire the next vector in the subspace.

In the standard approach, for every product $\boldsymbol{\mathcal{L}x}$, the iterative algorithm would be called to solve $\boldsymbol{P}^{-1}\boldsymbol{v}$, generating a new subspace $span\{\boldsymbol{w}, \boldsymbol{Pw}, \boldsymbol{P}^2\boldsymbol{w}, \ldots\}$, and a new set of search direction, $\boldsymbol{w}_k$. If the number of $\boldsymbol{\mathcal{L}x}$ products is large, the advantage of an iterative method would be degraded by the large number of total $\boldsymbol{Pw}$ products necessary. One is thus lead to consider reusing the search directions from the previous solves [11, 12]. While the recycled vectors are not optimal for the next $\boldsymbol{v}$, the cost of computing the solution along those directions is negligible compared to a single $\boldsymbol{Pw}$ product.

The $O(p^2)$ operations of the iterative algorithm can be reduced further by using a multipole-accelerated iterative algorithm [3] whose cost and memory has been shown to grow only as $O(p)$. Similarly, the computation of the product $\boldsymbol{M}_f \boldsymbol{L} \boldsymbol{M}_f^t$ can be performed in $O(f)$ operations also via the multipole-algorithm [5].

Algorithm **1 (Iterative Scheme for $\boldsymbol{Pq} = \boldsymbol{v}$)**

guess $\boldsymbol{q}^0$
Initialize the search direction
  $\boldsymbol{w}^0 = \boldsymbol{v} - \boldsymbol{Pq}^0$
for $k = 1, \ldots$ {
  Select $\boldsymbol{w}^k \in span\{\boldsymbol{w}^0, \boldsymbol{Pw}^0, \ldots, \boldsymbol{P}^{k-1}\boldsymbol{w}^0\}$
    such that the new solution
      $\boldsymbol{q}^k = \boldsymbol{q}^{k-1} + \boldsymbol{w}^k$
    minimizes $||\boldsymbol{r}^k|| = ||\boldsymbol{v} - \boldsymbol{Pq}^k||$
  if $||\boldsymbol{r}^k|| < tolerance$, return solution $\boldsymbol{q}^k$
}

4 Results

In this section we present results from the mesh formulated parasitic extraction program, FASTPEP. To verify that the formulation is correct, consider modeling a long two conductor transmission line with this 3D tool. The characteristic impedance of the line was computed as $Z_0 = 97.43\Omega$. By using a matched termination, $Z_L = Z_0$, the input impedance is constant, $Z(f) = Z_0$, for all frequencies f. To verify that the FASTPEP formulation is correct, the two conductors were taken to be $1cm$ long and discretized into 558 filaments, and 768 panels. Such a discretization leads to a 2033 state system. As seen from Figure 2, the full system matches closely to the exact solution. Notice

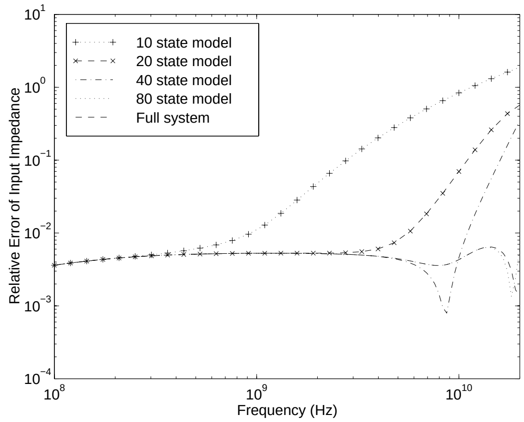

Figure 2: Relative error for models for matched transmission line

also that for a reduced order model with 1% error, a 20th order model is valid up to 6 GHz, a 40th order model up to 12 GHz, and an 80th order model past 20GHz (to about 26GHz).

Next, to demonstrate the efficiency of the recycled iterative scheme, consider refining the discretization of the transmission line of the previous example and extracting a 50th order model. Figure 3 shows the number of floating point operations (flops) required for direct factorization with back substitution, a non-recycled Krylov-subspace method, a recycled Krylov method, and multipole-accelerated recycled Krylov method, for various levels of discretization. Our implementation of FastPep uses direct matrix-vector products and thus the multipole-accelerated times are projected based on flop counts from multipole-accelerated capacitance and inductance codes [3, 5]. The error tolerance of the iterative algorithm was chosen such that the difference between models produced by the iterative scheme versus direct factorization differed by less than 1% up to 100 GHz. As can be seen from the figure for an original 15409 state system, the recycled scheme performs an order of magnitude faster than direct factorization, and similarly, the multipole algorithm would provide another order of magnitude speed up. Note that the CPU time comparison would be similar to the flop count comparison for the direct factorization and direct recycled iterative scheme, however the overhead in arranging the multipole computation would shift its curve slightly upward.

To demonstrate FastPep's utility as a 3D solver, consider analyzing a printed circuit board connector from Teradyne, Inc, as shown in Figure 4. The connector has four outer conductors and one middle ground conductor which widens in the center to pro-

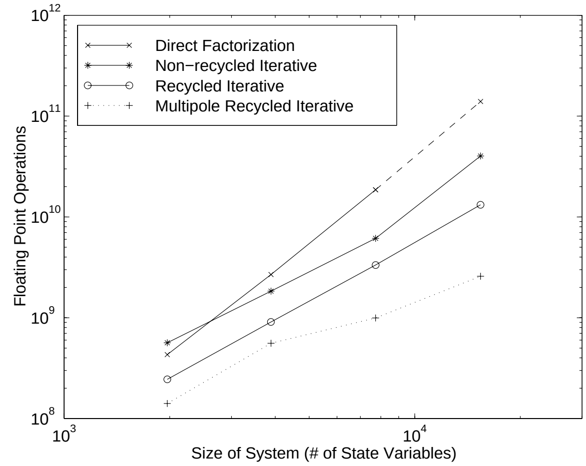

Figure 3: Flop count for different methods of computing $\boldsymbol{P}^{-1}\boldsymbol{x}$.

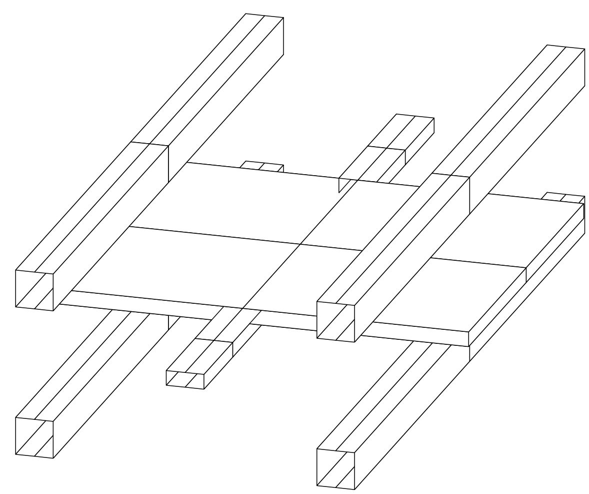

Figure 4: A 3D connector

vide shielding. Here the number of states in the original discretized system is 3018 which was generated from 845 filaments and 1182 panels. The results of generating models with 80 and 250 states are compared to the exact response in Figure 5. Note the strong improvement from 80 to 250 states.

5 Conclusions

In this paper we showed that combining a recycled Krylov-subspace iterative algorithm and multipole-acceleration with the PEEC method with Mesh Analysis can be used to efficiently generate low order models of three-dimensional interconnect structures.

As can be seen from Figure 5, a 250th order model

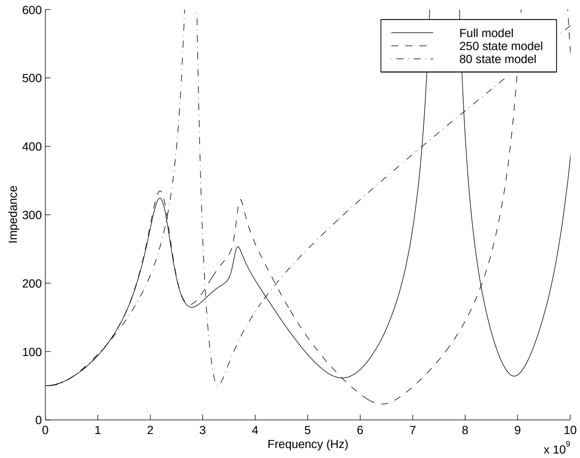

Figure 5: Various reduced order models for the connector

was needed to capture the response up to 3 GHz. Future work involves fast methods for generating models about expansion points other than $s = 0$ for more compact models.

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