A mixed rigid/elastic formulation for an efficient analysis of electromechanical systems [1]

D. Ramaswamy*, N.R. Aluru** and J. White*

* Research Laboratory of Electronics
Room 36-886, MIT, Cambridge, MA 02139,[deepak,white]@rle-vlsi.mit.edu
** University of Illinois,Urbana-Champaign,aluru@uiuc.edu

ABSTRACT

In this paper we describe both how to extract rigid bodies given an input file of elements, and how to efficiently construct and solve the rigid-elastic system of equations. Results are given to demonstrate that on a typical comb drive problem, our efficiently implemented mixed rigid-elastic simulator is more than 300 times faster than a purely elastic simulator. We also demonstrate that the mixed-regime simulator can be used as part of a coupled-domain simulator to perform 3-D electromechanical analysis of an entire comb drive in under ten minutes.

Keywords: MEMS,Rigid,Electromechanical,Coupled, Simulation.

INTRODUCTION

A MEMS structure often consists of a behaviourally rigid part and an elastic part even though it is built from a single material. Therefore, it is possible to model part of the structure as rigid for a rapid self-consistent electromechanical analysis (Senturia et al [1]). An example of such a structure is shown in Figure 1.

One approach to reducing the high computational cost of coupled electromechanical simulation of three dimensional micro-electro-mechanical devices is to allow designers to use rigid body approximations where appropriate. To gain from such an approach, it is essential that the mixed-regime rigid-elastic system be constucted automatically and simulated efficiently. In this paper we describe both how to extract rigid bodies given an input file of elements, and how to efficiently construct the rigid-elastic system stiffness matrix. In the next section, we give a brief background in finite-element elastostatic analysis and rigid body motion. In section three, we describe how we implemented a mixed rigid-elastic simulator. In section four we give computational results demonstrating that the rigid-elastic simulator is more than 300 times faster than a purely elastic simulator for a comb-drive problem. Finally, we demonstrate that the mixed-regime simulator can be used as part of a coupled-domain simulator to perform 3-D electromechanical analysis of an entire comb drive in under ten minutes.

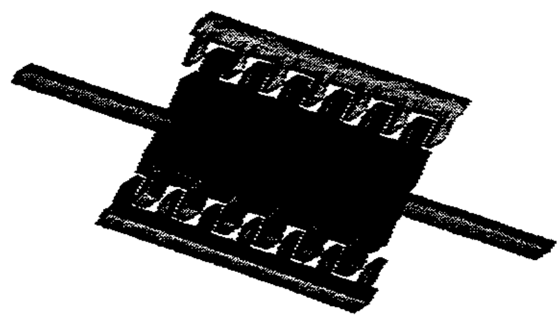


Figure 1: A comb drive accelerometer

BACKGROUND

Finite-Element Elastostatics

In steady-state elastostatics, a structure's deformation due to applied forces can be determined by solving a nonlinear partial differential equation representing force equilibrium. Following the total Lagragian formulation [7], let x be a point on the undeformed structure, and denote the displacement of that point x during deformation as $u(x)$. Then $u(x)$ must satisfy the force equilibrium equation

$$f(u(x)) - p(u(x)) = 0, \quad (1)$$

where $p(u(x))$ is the force on the deformed structure and f represents the nonlinear differential operator which relates the structure's deformation to the resulting stress. If a standard isoparametric Galerkin finite-element method is used to discretize (1), the result is a nonlinear system of equations for the node displacements,

$$F(U) - P(U) = 0, \quad (2)$$

where $U \in \Re^{3n}$ is a vector of node displacements, $F : \Re^{3n} \rightarrow \Re^{3n}$ is the discretized stress-displacement relation, and P is the discretized applied force.

Some variant of Newton's method is typically used to solve (2) for U, in which case a sequence of linear systems must be solved as in

$$K(U^{k+1} - U^k) = P(U^k) - F(U^k) \quad (3)$$

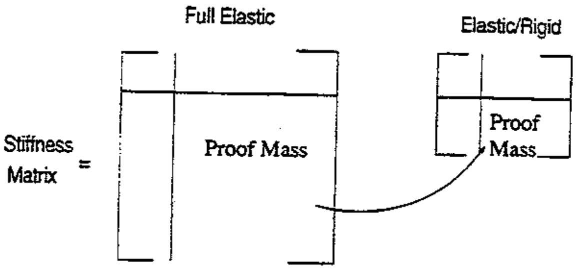


Figure 2: Matrix size reduction

where k is the Newton iteration index, U^k is the k^{th} Newton iterate for the displacements, and $K = \frac{\partial F}{\partial U}$ is referred to as the stiffness matrix.

Rigid Body representation

The current configuration y of a rigid body under displacement is expressed as

$$y = Rx + c \tag{4}$$

where x is a point on undisplaced body, R is an orthonormal rotational tensor and c a translation vector. R can be represented in terms of the *Euler* angles ϕ, ψ and θ as

$$R = \begin{bmatrix} c\phi c\psi - s\phi s\psi c\theta & -c\phi s\psi - s\phi c\psi c\theta & s\psi s\theta \\ s\phi c\psi + c\psi s\psi c\theta & -s\phi s\psi + c\phi c\psi c\theta & -c\phi s\theta \\ \bar{s}\psi s\theta & c\psi s\theta & c\theta \end{bmatrix} \tag{5}$$

where $c = cos$ and $s = sin$. Note that the entries of the above matrix are bounded and that R is singular for $\theta = \phi = \psi = 0$. Here the angles represent rotations about a set of three orthogonal axes.

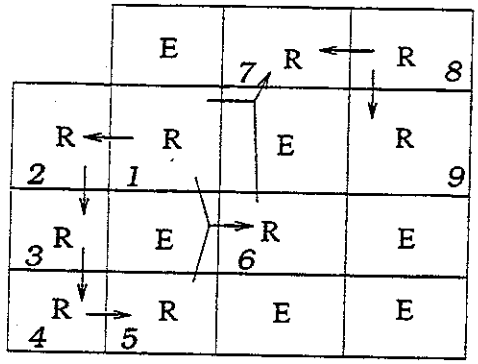
The equations of equilbrium for a rigid body are

$$F_x = F_y = F_z = M_x = M_y = M_z = 0; \tag{6}$$

where F is vector of net forces on the body and M is the vector of net moments of the body about a selected equilibrium point.

Mixed-Regime Algorithms

For a problem like the comb drive in figure (1), most of the unknown displacements are associated with the comb's proof mass. Since the proof mass does not deform under most operating conditions, it is possible to eliminate most of the unknowns by treating the proof mass as a single rigid body. To make this point clear, consider that a large fraction of the stiffness matrix in (3) can be eliminated by treating the proof mass as a rigid body, as shown in Figure (2). In order to exploit this matrix reduction, it is necessary to assemble rigid elements into a single rigid body and then determine the rigid-elastic stiffness matrix efficiently. We describe our approaches below.



R - Rigid Element , E - Elastic Element

Figure 3: Rigid body assembly

Rigid Body Assembly

An implementation issue is the integration of individual rigid bodies into a single rigid body whenever they behave as a single unit. A necessary input to this process is knowing which elements behave as rigid. In this study,the criterion for checking the rigidity is the elasticity modulus of the element which is set to a very large value. Connection to a rigid body in 3-D requires at least three noncollinear common nodes between the rigid body and the element.

If K denotes the number of elements and the $\mathcal{R}$ denotes the set of the rigid elements then the assembly algorithm is written as

$\forall\, i = 1$ *to* K
  if material $(i) = \infty$ *and* $i \notin \mathcal{R}$
    $\mathcal{R} = \mathcal{R} \cup i$
    $\forall\, j \in$ *neighbors* (i)
      if material $(j) = \infty$
        $\mathcal{R} = \mathcal{R} \cup j$
      end
    end
  end
end

The 2d view of this process is shown in Figure 3 where at the end of this process only a single rigid body will exist.

This depth first algorithm is most efficient for structures which have clustered rigid elements. For otherwise a breadth first algorithm will be more efficient.

Finite element-Rigid Bonding

Figure 4 shows a 20 node finite element interface with a rigid element. We note that a rigid element has a need for interface nodes only for communicating the displacement conditions at the interface. The stiffness matrix or Jacobian for the elastic element shown will have components due to the rigid body parameters. Also the

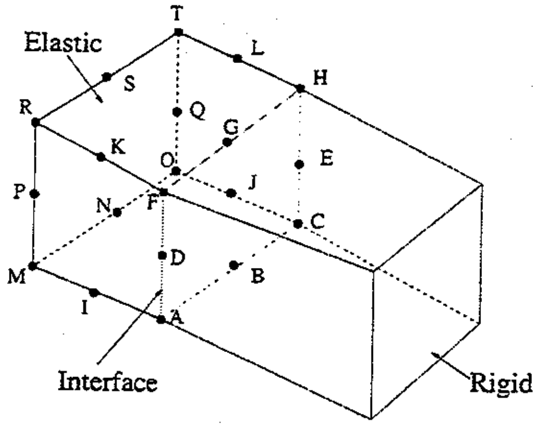


Figure 4: FE-Rigid Interface

forces acting at A through H, on the rigid body will be a function of nodes I through T.

Representing the rigid body through Euler angles, the equilibrium equation is

$$F(U, \theta, \phi, \psi, xR, yR, zR) = 0 \tag{7}$$

Assuming a convenient ordering of the Jacobian,

$$J_F = \begin{bmatrix} K_{EE} & K_{ER} \\ K_{RE} & K_{RR} \end{bmatrix} \tag{8}$$

The submatrix K_{EE} is the elastic - elastic interaction and a standard finite-element stiffness matrix, excluding the entries due to the rigid-elastic interface nodes.

The submatrix K_{ER} is the elastic - rigid interaction. We have for node variables x_i ($i = A..H$) on the interface

$$\frac{\partial F}{\partial \theta} = \frac{\partial F}{\partial x_A}\frac{\partial x_A}{\partial \theta} + \frac{\partial F}{\partial x_B}\frac{\partial x_B}{\partial \theta} + \cdots + \frac{\partial F}{\partial x_H}\frac{\partial x_H}{\partial \theta} \tag{9}$$

Here the chain rule has been applied. Also we know

$$x_i = x_i(\theta, \phi, \psi, xR, yR, zR) \tag{10}$$

from Equation 4.

The submatrix K_{RE}, the rigid - elastic term, is present because the equivalent nodal forces on the interface with the rigid body are dependent on the variables associated with nodes $I..T$ of the elastic element as noted earlier and these forces contribute in the equilibrium of the rigid body. For example the Moment M of the rigid body is

$$M = L(F_i^I) \tag{11}$$

where L is an operator linear in F_i^I, a interface nodal force component. Then

$$\frac{\partial M}{\partial x} = L(\frac{\partial F_i^I}{\partial x}) \tag{12}$$

but $\frac{\partial F_i^I}{\partial x}$ is directly obtained from K_{EE}.

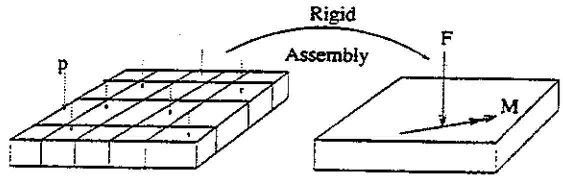


Figure 5: Summing of elemental forces on rigid body

The only forces in the rigid/elastic interface are the equivalent nodal forces and it is important to realize that these elastic force projections are such that the virtual work on the element due to them equals the virtual work due to element internal stresses and are therefore not an exactly equivalent force system to the interface surface pressure exerted by the elastic element on the rigid element.

The submatrix K_{RR} is the purely rigid-rigid interaction term. This term arises from the positional variation of the external pressure and point loads on the rigid body with respect to its parameters. In case of pressure forces such as that due to a fluid, the force is always normal to the surface of the body. As a result thereof the pressure is geometry dependent and hence its contribution to J, specifically K_{RR} must be computed. Figure 5 shows the summing of individual pressure forces and projecting them into the center of rotation of the rigid body which also results in a moment.

It should be noted that J_F is not symmetric.

PRELIMINARY RESULTS

The comb drive accelerometer shown in Figure 1 was tested using a mixed rigid/elastic formulation on a DEC Alpha 433 MHz. The proof mass and fingers were taken as rigid and the tethers were elastic. The material was polysilicon. The proof mass has a dimension of 100 X 100 X 10 , the tethers 60 X 10 X 10 and the fingers 30 X 10 X 10. There is no ground plane. The CPU time required for the mechanical linear system solve for the rigid/elastic case is 55.63 ms whereas it is 21,628.15 ms for the full elastic simulation.

We then combined our mixed regime mechanical simulator with the precorrected FFT accelerated electrostatic solver [4] using the multilevel Newton method [3]. Figure 6 shows the nonlinear variation of the output functional taken to be the maximum absolute displacement, becoming increasingly stiff with increasing voltage, v on one of the supports of the structure. The central structure and the other support were kept at 0 volts. The CPU time for this coupled-domain mixed-regime simulation is plotted as a function of applied voltage in Figure 7. Note that computing the displacement for a given voltage takes less than ten minutes.

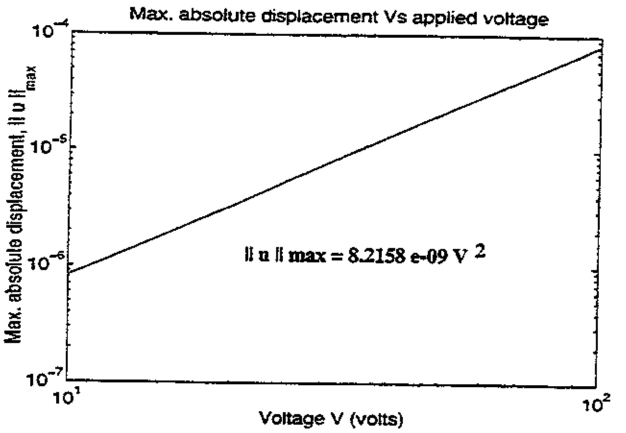


Figure 6: Maximum displacement Vs applied voltage

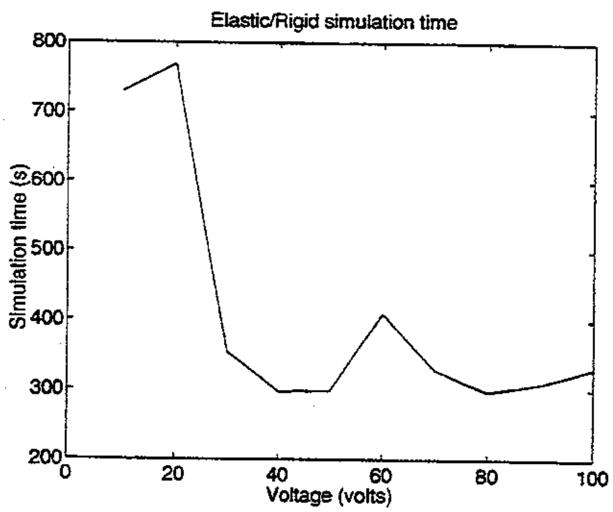


Figure 7: Rigid/Elastic Coupled Simulation

CONCLUSIONS AND ACKNOWLEDGEMENTS

A mixed rigid/elastic formulation leads to a considerable saving in the mechanical solver computation time requiring at the same time much less memory than a full elastic analysis. With an automation of the rigid element identification process, this leads to an efficient coupled electromechanical analysis.

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REFERENCES

[1] Stephen D. Senturia, Narayan Aluru and Jacob White,"Simulating the Behavior of MEMS devices: Computational Methods and Needs",IEEE Computational Science and Engineering, Vol. 4, No. 1, pp. 30-43,jan-mar, 1997.

[2] N.R. Aluru and J. White, "Direct-Newton Finite Element/Boundary-Element Technique for Micro-Electro-Mechanical Analysis" ,Transducers Research Foundation,Tech. Digest, Solid-State Sensor and Actuator Workshop,Cleveland Heights, Ohio, pp 54-57,1996.

[3] N.R. Aluru and J. White,"A Coupled Numerical Technique for Self-Consistent Analysis of Micro-Electro-Mechanical Systems", Microelectromechanical Systems (MEMS), ASME Dynamic Systems andControl (DSC) series,New York,Vol. 59,pp. 275-280,1996.

[4] J.R. Phillips and J. K. White,"Precorrected-FFT Methods for Electromagnetic Analysis of Complex 3-D Interconnect and Packages",Proc. PIERS '95, Progress in Electromagnetic Research Symp., 1995.

[5] S. J. Sherman, W. K. Tsang, T. A. Core, D. E. Quinn, "A low cost monolithic accelerometer",1992 Symposium on VLSI Circuits Digest of technical papers, pp. 34-35.

[6] H. U. Schwarzenbach, J. G. Korvink, M. Roos, G. Sartoris and E. Anderheggen , " A microelectromechanical CAD extension for SESES",J. MEMS,3 , pp. 162-171 , 1994.

[7] K. J. Bathe,"Finite Element Procedures",Prentice-Hall Inc., Englewood Cliffs, NJ, 1996.

[8] Morton E. Gurtin,"An introduction to continuum mechanics", Academic Press,New York,1981.

[9] P. Chadwick,"Continuum mechanics",Halsted Press,New York, 1976.