8.421: Pre-Class Questions February 12, 2020

1. Does the spin operator commute with the Zeeman Hamiltonian?

- The Zeeman Hamiltonian is of the form $H = -\vec{\mu} \cdot \vec{B} = -\gamma \hat{S}_z \hat{B}_z$. Spin operators follow the commutator relations $[S_i, S_j] = \epsilon_{ijk} S_k$. Therefore, the general spin operator $\hat{S} = \hat{S_x} + \hat{S_y} + \hat{S_z}$ does not commute with $\hat{S_z}$.

2. What is the equation of motion that describes the time evolution of a quantized magnetic moment in an applied external magnetic field?

- $\frac{\partial}{\partial t} \hat{\mu} = \gamma \hat{\mu} \times \mathbf{B}$

3. Does the equation of motion depend on the value of the spin?

- No, it is valid for any spin. However, it is derived from a spin $1/2$ system.

4. For spin $S = 1/2$ atoms in a time-dependent magnetic field, what is the Rabi frequency?

- $\Omega_R = \gamma B_{\text{eff}} = \sqrt{(\gamma B_0 - \omega)^2 + \gamma^2 B_1^2}$

5. For a classical magnetic moment (which has the same equation of motion as the quantum spin): Can you give an example what can cause T_1 and T_2 relaxation processes?

- damping due to collisions or fluctuations in the strength of the quantizing magnetic field B_0 causes T_1 decay, and/or the frequency of the driving magnetic field B_1 can cause T_2 decay.