Before-class Questions #11:

Question 1: The spontaneous emission rate scales with (omega_0)^x in three dimensions. What is x, and explain how this scaling arises.
Answer 1: x = 3. (omega_0)^2 comes from the density of modes while another factor of omega_0 comes from the quantization of the EM field.

Question 2: What is a typical lifetime for an atom to decay through an optical transition? How does a hyperfine microwave transition lifetime compare?
Please use orders of magnitude, and don't forget to account for the matrix element of a hyperfine transition differing from that of a dipole transition.
Answer 2: 10 ns lifetime for an optical transition. GHz microwave transition is 10^5 smaller in frequency compared to optical, giving us a factor of 10^(-15) from (omega_0)^3 scaling. The magnetic dipole transition matrix element is alpha times smaller than the dipole transition, therefore the transition is alpha^2 weaker. Combining these factors leads us to a hyperfine lifetime of 1000 years.

Question 3: Off-shell terms, transitions where energy is not conserved, arise from the Jaynes-Cummings Hamiltonian. Since they do not conserve energy, they are virtual transitions. Do they have any effect on atoms?
Answer 3: Yes, they lead to level shifts such as the Lamb shift.

Question 4: What is the strong coupling regime of cavity QED, in words and in an inequality?
Answer 4: When the atom couples to the cavity more strongly than the free space modes: omega_(single_photon) >> Gamma.

Question 5: Place an atom in a cavity, both of which have the same resonance frequency. What will the transmission peak(s) look like if there is one photon in the cavity? 100 photons?
Answer 5: The transmission peaks will be separated by the Rabi freqency ~ sqrt(n+1), where n is the photon number. Correspondingly the 100 photons in a cavity will have peaks 10x further separated compared to 1 photon.

Question 6: If the photon field in a cavity is a superposition of a number states, what does the excited state probability of atoms in the cavity look like over time?
Answer 6: The atoms will undergo damped Rabi oscillations and the excited state fraction will go to approximately zero when the different phases of the number states destructively interfere. Once enough oscillation periods have passed that the phases of the involved photon states are commensurate again, we will see a revival of the excited state fraction.