

Q1: The rotating wave approximation involves counter intuitive terms, such as emission along with a lowering of the atomic state. Using the language of perturbation theory, describe how it is possible that these “virtual” terms can affect the physics of an atom interacting with light? Do these terms ever cause transitions? Why or why not?

A1: In perturbation theory we get energy shifts by summing terms over all involved states. Energy shifts due to the counter-rotating terms show up when we include “virtual” states in the sum. The system is allowed to explore these states for a short amount of time*, but is never allowed to actually make the transition because after the system has had time to resolve the energy mismatch, conservation of energy must be enforced.

(*) Given by the uncertainty principle $\Delta t = h / \Delta E$. The weaker the energy mismatch, the longer the system can “explore” these states which don’t conserve energy. An equivalent way to see this: how long does it take for the system to resolve the energy (frequency) mismatch, in this case given by Fourier $\Delta t = 1 / \Delta \omega$.

Q2: Consider a two-level atom with three transition rates (described by the Einstein rate equations). One of the transition rates is fixed (Γ , a decay rate). The other two, one for absorption and the other for stimulated emission, are equal and opposite and depend on the laser/drive power. Give a simple argument for how the equilibrium population approaches its asymptotic value, as we increase the laser/drive power.

A2: In a system with three rates, the rate constants add together to give the total transition rate. Since Γ is fixed, it eventually becomes negligible compared with the stimulated emission/absorption rates. This means we “push” the population equally from above and below, and the excited state population approaches 50%. But Γ never actually goes away, so the total decay rate is a little bit more than the total absorption rate, which is why we approach 50% asymptotically from below.

Q3: Give a simple physical argument explaining the phenomenon of power broadening, for the case of a transition rate with Lorentzian lineshape.

A3: We know two facts: 1) the rate cannot increase without bound, because of saturation. 2) the rate increases most quickly in the center of the Lorentzian curve (at resonance). The height of a point in the wings of the Lorentzian (away from resonance) increases too, but not as quickly. This means the weight of the curve increases in the wings relative to the peak as we increase the power, broadening the curve.

Q4: Describe the behavior of the scattering cross section and the scattering rate as the light intensity is increased? What are their asymptotic limits?

A4: The cross-section goes to zero as the intensity becomes much stronger than the saturation intensity, while the scattering rate initially increases, then saturates. Since the scattering rate is proportional to the cross section, these seem like conflicting conclusions. The resolution is that scattering cross section measures the *ratio* of scattered to unscattered light, making it vanish as we increase intensity far beyond the saturation intensity.