8.421: Module 3 Problem Set

Due: April 13, 2020

The spontaneous emission rate for a two-level atom in an electromagnetic field is given by

$$\Gamma_{ab} = \frac{4\pi^2}{\hbar V} \left| \hat{\epsilon} \cdot \vec{d}_{ba} \right|^2 \omega_0 n(\omega_0)$$

where the field has frequency ω_0 and polarization $\hat{\epsilon}$ and is confined to a volume V. The number of photons in a frequency interval $d\omega$ is $n(\omega)d\omega$, and $\vec{d}_{ba}$ is the dipole matrix element for the ba transition.

We want to study the spontaneous emission of an atom in a two-dimensional electromagnetic field. We do this by assuming that the atom is in a parallel plate capacitor with plate separation $d \ll \lambda$, the resonant wavelength. Define the direction of the plate separation to be $\hat{z}$.

Problem 1

What are the relevant modes of the electromagnetic field, and what is their mode density?

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Problem 2

For a simple transition with only one excited state and one ground state, what is the spontaneous emission rate in terms of $\vec{d}_{ba}$? Compare Γ^{2D} to the 3D decay rate. Is it larger or smaller, and why?

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Problem 3

Now find the excited state lifetimes for $p \to s$ and $s \to p$ transitions. How do your answers change when you change the quantization axis of the atoms from $\hat{z}$ to $\hat{x}$? In an experiment, this can be done by changing the direction of an external magnetic field. Write your answers in terms of Γ^{2D}, your answer from the previous problem.

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Problem 4

Suppose the atom has a $J = 2 \rightarrow J' = 1$ transition with rates labeled as follows:

a: $m = 2 \rightarrow m' = 1$
b: $m = 1 \rightarrow m' = 1$
c: $m = 0 \rightarrow m' = 1$
d: $m = 1 \rightarrow m' = 0$
e: $m = 0 \rightarrow m' = 0$

Find each of these rates in 2D, first in the case where the quantization axis is along $\hat{z}$, and second where the quantization axis is along $\hat{x}$. Write your answers in terms of Γ^{2D}. When excited by light with a particular polarization, a closed (aka cycling) transition will always return to it's initial state. What are the closed transitions in each case?

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Problem 5

For the quantization axis along $\hat{z}$ and along $\hat{x}$, what is the decay rate of an unpolarized sample in the $J = 2$ excited state? Explain your result.

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