

## The AC Stark effect

Due: Wednesday 3/11, drop off in class or in envelope outside 26-237 by 5 p.m.

For questions or assistance with this assignment, please contact:

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### 1. The AC Stark effect and Rayleigh Scattering

The AC Stark effect has been used to trap neutral atoms at the focus of one or more laser beams. This obviates the necessity of magnetic traps, which can be bulky and power guzzling. Scientifically more important is the fact that many interesting phenomena occur between states that are not necessarily all magnetically trappable. This has led to studies of, for example, spin domains in spinor Bose-Einstein condensates (Sadler *et al.*, *Nature* **443**, 312 (2006)), among many other examples. In the following exercise, you will explore the effects of both near-resonant and far off-resonant laser light used for trapping an ultracold atomic gas.

Let us consider an alkali atom with resonance frequency  $\omega_0$  on the principal  $nS \rightarrow nP$  transition. A sample of atoms in the ground state  $nS$  are exposed to monochromatic radiation of intensity  $I$  and frequency  $\omega < \omega_0$  (red-detuned from resonance). Use the fact that essentially all of the oscillator strength out of the ground state comes from the  $nS \rightarrow nP$  transition to treat this as an effective two-level system.

#### a) AC Stark Shift

Consider an electric field  $\vec{\epsilon}(t) = \epsilon \cos(\omega t) \hat{\mathbf{z}} = (\frac{1}{2}\epsilon \cdot e^{-i\omega t} + \frac{1}{2}\epsilon \cdot e^{i\omega t}) \hat{\mathbf{z}}$ , which couples the  $nS$  ground state denoted by  $|g\rangle$ , and the single excited state  $nP$ , denoted by  $|e\rangle$ . Determine how the wavefunction  $|\psi(t)\rangle = a_g(t)|g\rangle + a_e(t)e^{-i\omega_0 t}|e\rangle$  evolves in time under the dipole interaction Hamiltonian  $H' = -\vec{d} \cdot \vec{\epsilon}(t)$ . Do this by solving the time-dependent Schrodinger equation using time-dependent perturbation theory to leading non-zero order in the wavefunction coefficients (include both co-rotating and counter-rotating terms). Calculate the resulting dynamic polarizability  $\alpha(\omega)$  and the AC Stark shift  $U(\omega)$ . One can find a convenient expression for  $U(\omega)$  by writing the matrix element of the dipole moment via the oscillator strength, and using the fact that all of the oscillator strength comes from the  $nS$  to  $nP$  transition.

b) What is the probability of finding the atom in the excited state  $P_e = |a_e|^2$ ? Neglect rapidly oscillating terms and time-average the remaining time-dependent terms.

#### c) Rayleigh scattering

In this part you will calculate the photon scattering rate. Knowing the scattering rate for a particular atom and particular laser is important because it causes heating and may severely limit the holding time for an ultracold cloud. Here you are given a classical model to estimate the scattering rate, in which an atom can be treated as a dipole moment induced by the external light field, and it is oscillating at the same frequency. We only need to consider coherent scattering here, which means the scattered photons have the same frequency as the incident laser light. The scattered light can be regarded as radiation from the oscillating dipole. The scattering rate is defined as the total number of scattering events per second, where one scattering event corresponds to the scattering of one photon of  $\hbar\omega$ . Use the fact that, from classical electromagnetism, the power  $P$  radiated by an oscillating dipole of dipole moment  $\mathbf{d}(t) = \mathbf{d} \cos \omega t$  is  $P = ck^4 |\mathbf{d}|^2 / 3$ .

i. Consider the near resonant case  $\omega \approx \omega_0$  and apply the rotating wave approximation (RWA) by ignoring contributions arising from the counter-rotating term. What is the amplitude of oscillations of the induced dipole moment  $d$  and the probability of being in the excited state  $P_e$ ?

- ii. Express the scattering rate in terms of  $P_e$ , the spontaneous emission rate  $\Gamma$ , and other factors. For the spontaneous emission rate, which determines the lifetime of the atom in the excited state, you can use the expression:

$$\Gamma = \frac{4}{3} \frac{e^2 \omega_0^3}{\hbar c^3} |\langle b | \mathbf{r} | a \rangle|^2$$

You can interpret your result also as follows: By treating atoms as driven harmonic oscillators, you obtained the correct expression for the spontaneous emission rate.

- iii. What if the drive frequency  $\omega$  is off-resonant and the RWA no longer applies? Find an expression for the scattering rate, and show that it is no longer proportional to the fraction of population in the excited state,  $P_e = |a_e|^2$ . Optional: Try to explain in words why this is no longer the case by considering how the rotating and counter-rotating terms add up.

In this part, you have shown that the description of Rayleigh scattering as absorption followed by spontaneous emission is correct only in the near-resonant case.

## 2. Magic Wavelength Optical Trap

The official time standard is currently based on a cesium microwave transition, but major research efforts are underway to improve on that by using an optical transition (see for example Ludlow *et al.*, Science **319**, 1805 (2008)). In particular, one path is to use neutral atoms trapped in optical lattices that have only one atom per lattice site. This can eliminate collisional effects that can limit the precision of such clocks. However, AC Stark shifts of the two states involved in the relevant transition due to the lattice lasers can cause systematic uncertainties. This can be alleviated by choosing a lattice laser wavelength where the AC Stark shift of the two states are exactly the same, thereby eliminating the systematic uncertainty. This wavelength is called the magic wavelength. You will find the magic wavelength for a model system in this problem.

Consider a laser of frequency  $\omega$  and intensity  $I$  which creates a dipole trap for an atom.

- a) Consider a two-level system with lower state  $|S\rangle$  and upper state  $|P\rangle$  with ‘bare’ energy separation  $\hbar\omega_{PS}$ . Also let the dipole moment be  $d_{PS}$ .
  - i. Find the AC Stark shift for the state  $|S\rangle$  in terms of the dipole moment, the frequencies, and an overall constant factor. Do not make the rotating wave approximation!
  - ii. Find the AC Stark shift for the state  $|P\rangle$  in the same way. What is the relationship between this and the AC Stark shift found in *i*?
  - iii. Consider the difference in transition frequency  $E_P - E_S$  with and without the laser. Is there any way this can be zero with the current setup (while still having a trapping laser, of course)?
- b) Now consider adding a third, higher level to the system to make it a three-level system with the third, highest state  $|D\rangle$  with energy separation between  $|D\rangle$  and  $|P\rangle$  as  $\hbar\omega_{DP}$ . Also let the dipole moment be  $d_{DP}$ . Assume that  $\omega_{PS} = f\omega_{DP}$  and  $d_{PS} = nd_{DP}$ , where  $f$  and  $n$  are some numbers, and that the states  $|S\rangle$  and  $|D\rangle$  do not couple directly.
  - i. Find the AC Stark shift for the state  $|P\rangle$  taking into account both the higher state  $|D\rangle$  and lower state  $|S\rangle$ .
  - ii. What is the laser frequency needed to make the  $S \rightarrow P$  transition frequency independent of trap laser power? Express this in terms of  $\omega_{DP}$  and  $d_{DP}$ . This is the so-called magic wavelength.

## 3. Species-Dependent and Spin-Dependent AC Stark Shift

Now consider how multiple species or multiple sub-states of atoms can be selectively manipulated using AC Stark shifts. We can consider putting more than one type of atom in the laser field. Also, in reality atoms do have sub-states, and the different sub-states can couple differently to the laser field. We will explore these situations in this problem. For these cases, the polarization of the light with respect to the quantization axis also begins to matter.

Consider a three-level system with fine structure, *i.e.* an alkali atom. The lowest level is  $S_{1/2}$  and the upper  $P$  state splits into two levels,  $P_{3/2}$  and  $P_{1/2}$ . Assume the state  $P_{3/2}$  has higher energy than  $P_{1/2}$ . Let the energy difference be  $\hbar\omega_1 = E_{P_{1/2}} - E_{S_{1/2}}$  and  $\hbar\omega_2 = E_{P_{3/2}} - E_{S_{1/2}}$ . (As a side note, the labels 1 and 2 for the frequencies correspond to the fact that the transitions are called the D1 and D2 lines, respectively.) Assume the atom is quantized along the  $z$  direction. You also need to know the dipole coupling of the form  $\langle S_{1/2}, m_J | \hat{\epsilon} \cdot \mathbf{r} | P_{3/2, 1/2}, m'_J \rangle$  between the relevant states, where  $\hat{\epsilon}$  is the polarization vector of the electric field. The relevant dipole matrix elements are as follows:

$$|e \langle S_{1/2}, +1/2 | \hat{\sigma}_+ \cdot \mathbf{r} | P_{3/2}, +3/2 \rangle|^2 = d_2^2 \quad (1)$$

$$|e \langle S_{1/2}, +1/2 | \hat{\sigma}_- \cdot \mathbf{r} | P_{3/2}, -1/2 \rangle|^2 = d_2^2/3 \quad (2)$$

$$|e \langle S_{1/2}, -1/2 | \hat{\sigma}_+ \cdot \mathbf{r} | P_{3/2}, +1/2 \rangle|^2 = d_2^2/3 \quad (3)$$

$$|e \langle S_{1/2}, -1/2 | \hat{\sigma}_- \cdot \mathbf{r} | P_{3/2}, -3/2 \rangle|^2 = d_2^2 \quad (4)$$

$$|e \langle S_{1/2}, +1/2 | \hat{\sigma}_- \cdot \mathbf{r} | P_{1/2}, -1/2 \rangle|^2 = 2d_1^2/3 \quad (5)$$

$$|e \langle S_{1/2}, -1/2 | \hat{\sigma}_+ \cdot \mathbf{r} | P_{1/2}, +1/2 \rangle|^2 = 2d_1^2/3, \quad (6)$$

where  $e$  is the electron charge. All other matrix elements are irrelevant for this problem (which should become clear if you consider the selection rules and the photon polarizations considered in this problem). You should also notice that the interference terms between the different polarizations vanish for this problem. Note that  $\hat{\sigma}_\pm = (\hat{x} \pm i\hat{y})/\sqrt{2}$  correspond to  $\sigma_\pm$  polarized light where absorbing a photon with such a polarization leads to  $\Delta m_J = \pm 1$ . The total AC Stark shift for a given sub-state is then just a sum over all of the AC Stark shifts due to the coupled states. Do not use the rotating wave approximation in this problem.

- a) Consider a linearly polarized dipole trap laser with electric field polarization along the  $x$  direction propagating along the  $z$  direction.
  - i.* At what laser wavelength will the atom in state  $S_{1/2}$  not feel the dipole trap anymore? Use the following parameters to numerically find the wavelength specifically for rubidium:  $\lambda_1 = 795$  nm,  $\lambda_2 = 780$  nm,  $d_1 = 3.0ea_0$ ,  $d_2 = 4.2ea_0$ , where  $a_0$  is the Bohr radius.
  - ii.* This has been used to create a system where one type of atom ( $^{41}\text{K}$ ) feels a dipole trap, but the other type ( $^{87}\text{Rb}$ ) does not, in order to demonstrate entropy exchange between the two species (Catani *et al.*, *Phys. Rev. Lett.* **103**, 140401 (2009)). They used a laser at a wavelength of 790 nm for this purpose. Does the wavelength you found agree with what was used in their paper? Is there anything that we neglected to take into account?
- b) Now consider a  $\sigma_+$  polarized light propagating along the  $z$  direction, with laser frequency much closer to the  $S_{1/2} \rightarrow P_{3/2}$  transition than the  $S_{1/2} \rightarrow P_{1/2}$  transition so that you can neglect coupling to the  $P_{1/2}$  state.
  - i.* What is the AC Stark shift for the two states  $|S_{1/2}, m_J = +1/2\rangle$  and  $|S_{1/2}, m_J = -1/2\rangle$ ?
  - ii.* Rewrite the AC Stark shift you found in *i* in the form  $U = U_S + U_V$ , where  $U_S$  is the spin-independent, scalar light shift, and  $U_V = -\boldsymbol{\mu} \cdot \mathbf{B}_{\text{eff}} = g_J m_J \mu_B B_{\text{eff}}$  is the spin-dependent, vector light shift. Note  $g_J(S_{1/2}) = 2$ . What is  $\mathbf{B}_{\text{eff}}$ ? We see that the spin-dependence of the AC Stark shift can be expressed in terms of a fictitious magnetic field acting on the spin of the atoms.

This configuration is relevant to situations where you want to induce spin-dependent interactions. This can be used to generate entanglement for quantum information processing (Mandel *et al.*, *Nature* **425**, 937 (2003)), or realizing, for example, anti-ferromagnetic many-body ground states to study quantum magnetism (Duan *et al.*, *Phys. Rev. Lett.* **91**, 090402 (2003)).

#### 4. Angular Momenta, Magnetic Moments, and g Factors

For a one-electron atom, what are

- i.* The largest and smallest possible angular momenta  $J$ ?
- ii.* The largest and smallest possible magnetic moments?
- iii.* The largest and smallest possible g factors?