Radiation reaction and vacuum fluctuations in spontaneous emission*

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A Heisenberg-picture treatment of spontaneous emission is given, and the origin of radiative line shifts and widths is discussed. It is shown that radiation reaction and vacuum fluctuations provide complementary conceptual bases for the interpretation of these radiative corrections. Alternative approaches are discussed, including the use of a random classical field to simulate the effects of the quantum-electrodynamical radiation field.

I. INTRODUCTION

The concept of spontaneous emission was introduced by Einstein[1] in 1917 in his classic derivation of Planck's radiation law. It was generally felt that this phenomenon could be attributed to the radiation reaction of the atomic-transition dipole moment back on itself, in complete analogy to the radiative damping of a classical oscillating dipole. For example, in a letter to *Nature* in 1924 Slater writes: "The part of the field originating from the given atom itself is supposed to induce a probability that that atom lose energy spontaneously, while radiation from external sources is regarded as inducing additional probabilities that it gain or lose energy, much as Einstein has suggested...."[2]

On the other hand, Welton's heuristic argument concerning the Lamb shift[3] influences many modern physicists to attribute spontaneous emission to the effect of vacuum-field fluctuations. Indirect support for this point of view comes from semiclassical radiation theories. Such theories[4] do not quantize the radiation field and typically do not allow spontaneous emission from an atom in any pure excited state. Thus the vacuum fluctuations of quantum electrodynamics are perceived to be the essential ingredient of a radiation theory, necessary to provide the triggering mechanism that releases the atom's stored energy in spontaneous emission.

Recent work[5] showing that the natural width and shift of an emission line may be attributed to radiation reaction has revitalized interest in the radiation-reaction interpretation of spontaneous emission. The purpose of this paper is to show to what extent the concepts of radiation reaction and vacuum-field fluctuations provide equally valid conceptual bases for the interpretation of spontaneous emission. A preliminary report of this work has been given in a recent letter.[6]

Before giving any detailed arguments, it is perhaps useful to review the context in which the idea of spontaneous emission arises. One studies the properties of isolated atoms in two steps. First, the interaction between the electron and the electromagnetic field of the nucleus is considered; at this stage one ignores completely the coupling of the electron to the free radiation field. The result of this analysis is that the atom has certain stationary states of well-defined energy. The second step is to refine this initial result by introducing the interaction between the electron and the free field as a perturbation. This refinement reveals that only the lowest-energy state is a true stationary state with a well-defined energy. All the excited states have a certain width in energy and decay—spontaneously—by releasing energy as electromagnetic radiation. Moreover, all energy levels are slightly shifted from the values obtained in the initial calculation.

In discussing spontaneous emission, then, we may regard the problem of the interaction of the electron with the nucleus as solved. We need only treat in detail the refinements produced by the electron's coupling to the radiation field. Moreover, since we are interested in a physical interpretation, not accurate numerical results for level shifts and widths, it is adequate to carry out our analysis within a nonrelativistic framework.

In Sec. II we describe the Hamiltonian and obtain the equations of motion for a one-electron atom coupled to the radiation field. In Sec. III we introduce an approximation technique and verify that a second-order calculation yields just the expected nonrelativistic results. In Sec. IV we note that a two-level model of the atom is adequate for our purposes. The two-level model is then used in Sec. V to show that radiation reaction and vacuum fluctuations provide equally valid bases for the physical interpretation of spontaneous emission. Section VI contrasts our results with those of other

workers and resolves a certain discrepancy. In Sec. VII we conclude by mentioning the connection of this work with other approaches.

II. EQUATIONS OF MOTION FOR AN ATOM INTERACTING WITH RADIATION FIELD

For our purposes, the nonrelativistic Hamiltonian for a one-electron atom[7] interacting with the electromagnetic radiation field may be written as

$$H=\frac{1}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}\right)^2+V(r)+\frac{1}{4\pi}\int d^3r\,\vec{E}^{\perp 2}, \tag{1}$$

where m and e (<0) are the electron mass and charge, respectively, $\vec{p}=m\,d\vec{r}/dt+(e/c)\vec{A}$ is the momentum conjugate to r, $\vec{A}$ is the Coulomb-gauge vector potential evaluated at the electronic position, and $\vec{E}^{\perp}=-(1/c)d\vec{A}/dt$. $\vec{A}(\vec{r},t)$ may be expanded in plane waves normalized in the volume V as[8]

$$\vec{A}(\vec{r},t)=\sum_{\vec{k},\lambda}\left(\frac{2\pi\hbar c^2}{\omega_k V}\right)^{1/2}[a_{\vec{k}\lambda}(t)e^{i\vec{k}\cdot\vec{r}}\hat{e}_{\vec{k}\lambda}+\text{H.c.}], \tag{2}$$

where $\vec{k}\cdot\hat{e}_{\vec{k}\lambda}=0$, $\lambda=1,2$ being the polarization index, $\hat{e}_{\vec{k}\lambda}\cdot\hat{e}^{*}_{\vec{k}\lambda'}=\delta_{\lambda\lambda'}$, and $\omega_k=kc$. $a_{\vec{k}\lambda}(t)$ and $a^{\dagger}_{\vec{k}\lambda}(t)$ are, respectively, the Heisenberg-picture photon annihilation and creation operators for mode $(\vec{k},\lambda)$:

$$[a_{\vec{k}\lambda}(t),a^{\dagger}_{\vec{k}',\lambda'}(t)]=\delta^3_{\vec{k},\vec{k}'}\delta_{\lambda\lambda'}. \tag{3}$$

The quantization volume V is assumed at this point to be finite, but eventually will be allowed to become infinite in order to admit all radiation modes. For definiteness, let V be a cube of side L, so that $(k_x,k_y,k_z)=(2\pi/L)(n_x,n_y,n_z)$, where n_x,n_y,n_z take on all integral values.

The Hamiltonian (1) may be written

$$H=H_A+H_F+H_{\text{int}}, \tag{4}$$

where

$$H_A=\frac{p^2}{2m}+V(r),\quad H_F=\sum_{\vec{k},\lambda}\hbar\omega_k a^{\dagger}_{\vec{k}\lambda}a_{\vec{k}\lambda},$$

and

$$H_{\text{int}}=-(e/mc)\vec{A}(0,t)\cdot\vec{p}+(e^2/2mc^2)\vec{A}^2(0,t).$$

We have adopted the dipole approximation and have evaluated the field at the position of the nucleus. This approximation is expected to be accurate if the orbital "radius" of the electron is much smaller than the relevant transition wavelengths. In what follows we neglect the small $\vec{A}^2(0,t)$ term.

The Schrödinger-picture stationary states of the atom will be labeled $|1\rangle,|2\rangle,|3\rangle,\ldots$, where

$$H_A|n(t)\rangle=E_n|n(t)\rangle=E_n e^{-iE_n t/\hbar}|n(0)\rangle. \tag{5}$$

The states $|n(t)\rangle$ span the Hilbert space of the atomic system. In this basis it is easily seen that the atomic Hamiltonian H_A may be written as

$$H_A=\sum_n E_n|n(t)\rangle\langle n(t)|=\sum_n E_n\sigma_{nn}(t), \tag{6}$$

where

$$\sigma_{nm}(t)=\exp[(i/\hbar)Ht]\sigma_{nm}(0)\exp[(-i/\hbar)Ht].$$

In the absence of any perturbation, $\sigma_{nn}(t)=\sigma_{nn}(0)$.

In general any atomic operator $A(t)$ has the representation

$$A(t)=\sum_m\sum_n\langle n(0)|A(0)|m(0)\rangle\sigma_{nm}(t). \tag{7}$$

In particular the linear momentum operator $\vec{p}(t)$ has the representation

$$\vec{p}(t)=\sum_m\sum_n\langle n(0)|\vec{p}(0)|m(0)\rangle\sigma_{nm}(t)$$

$$=-\frac{im}{e}\sum_m\sum_n\omega_{mn}\vec{\mu}_{nm}\sigma_{nm}(t), \tag{8}$$

where $\hbar\omega_{mn}=E_m-E_n$ and $\vec{\mu}_{nm}=\langle n(0)|e\vec{r}(0)|m(0)\rangle$ is the electric-dipole-moment transition matrix element between states n and m; $\vec{\mu}_{nm}\neq 0$ if there is an allowed (electric-dipole) transition between states n and m.

The operators $\sigma_{nm}(t)$ will be referred to as the *atomic operators*. They provide the multilevel generalization of Dicke's[9] spin operators for two-level atoms. It is easily seen that the atomic operators obey the commutation rule

$$[\sigma_{ij}(t),\sigma_{kl}(t)]=\delta_{jk}\sigma_{il}(t)-\delta_{il}\sigma_{kj}(t). \tag{9}$$

Using the representations (6) and (8) for H_A and $\vec{p}$, respectively, the Hamiltonian (4) becomes

$$H=\sum_n E_n\sigma_{nn}(t)+\sum_{\vec{k},\lambda}\hbar\omega_k a^{\dagger}_{\vec{k}\lambda}(t)a_{\vec{k}\lambda}(t)$$

$$+i\hbar\sum_{\vec{k},\lambda}\sum_m\sum_n C_{\vec{k}\lambda nm}\sigma_{nm}(t)[a_{\vec{k}\lambda}(t)+a^{\dagger}_{\vec{k}\lambda}(t)], \tag{10}$$

where we define

$$C_{\vec{k}\lambda nm}=\frac{1}{\hbar}\left(\frac{2\pi\hbar}{\omega_k V}\right)^{1/2}\omega_{mn}\vec{\mu}_{nm}\cdot\hat{e}_{\vec{k}\lambda}. \tag{11}$$

The polarization vectors $\hat{e}_{\vec{k}\lambda}$ have been taken to be real (linear polarization basis).

Mathematically the problem of the interaction of the atom with the field is defined by the Heisenberg equations of motion based on the Hamiltonian (10), the commutation rule (3) for the field operators, and the commutation rule (9) for the atomic operators. One further commutation rule is required to define the problem unambiguously—that involving the atomic and field operators. It will be assumed that atomic and field operators

commute at $t=0$, or that the interaction is "switched on" at $t=0$.[10] Unitarity of the time evolution then requires that all equal-time commutators of atomic and field operators vanish. This renders the ordering chosen for atomic and field operators in (10) irrelevant for the calculation of numerical magnitudes relating to the atom-field interaction.

Choosing a normal ordering, in which photon annihilation operators are placed to the extreme right and creation operators to the extreme left, we may write the Heisenberg equations of motion

$$\dot{a}_{\vec{k}\lambda}(t) = -i\omega_k a_{\vec{k}\lambda}(t) + \sum_m \sum_n C_{\vec{k}\lambda nm}\sigma_{nm}(t), \tag{12}$$

$$\begin{aligned}\dot{\sigma}_{ij}(t) = {} & -i\omega_{ji}\sigma_{ij}(t) \\ & + \sum_{\vec{k},\lambda}\sum_m\sum_n C_{\vec{k}\lambda nm}[\delta_{jn}\sigma_{im}(t) - \delta_{im}\sigma_{nj}(t)]a_{\vec{k}\lambda}(t) \\ & + \sum_{\vec{k},\lambda}\sum_m\sum_n C_{\vec{k}\lambda nm}a^\dagger_{\vec{k}\lambda}(t)[\delta_{jn}\sigma_{im}(t) - \delta_{im}\sigma_{nj}(t)].\end{aligned} \tag{13}$$

III. APPROXIMATE SOLUTION OF EQUATIONS OF MOTION

In this section we describe a procedure for the approximate solution of the equations of motion (12), (13). Formally integrating Eq. (12), we have

$$a_{\vec{k}\lambda}(t) = a_{\vec{k}\lambda}(0)e^{-i\omega_k t} + \sum_m\sum_n C_{\vec{k}\lambda nm}\int_0^t dt_1\sigma_{nm}(t_1)e^{i\omega_k(t_1-t)}. \tag{14}$$

Assuming the atom-field interaction to be weak compared with the internal forces of the atom, we expect the atomic operators to evolve very nearly according to their free evolution $\dot{\sigma}_{ij}(t) = -i\omega_{ji}\sigma_{ij}(t)$. Hence we may use in Eq. (14) the approximation[5,6]

$$\sigma_{nm}(t_1) \cong \sigma_{nm}(t)e^{-i\omega_{mn}(t_1-t)}. \tag{15}$$

This will be referred to as the *adiabatic approximation*, and is shown to be equivalent to the Weisskopf-Wigner "pole" approximation in Appendix A.[11]

Using (15) in (14) and substituting into (13), one obtains the approximate atomic equations of motion,

$$\begin{aligned}\dot{\sigma}_{ij}(t) = {} & -i\omega_{ji}\sigma_{ij}(t) + \sum_{\vec{k},\lambda}\sum_m\sum_n C_{\vec{k}\lambda nm}a^\dagger_{\vec{k}\lambda}(0)e^{i\omega_k t}[\delta_{jn}\sigma_{im}(t) - \delta_{im}\sigma_{nj}(t)] \\ & + \sum_{\vec{k},\lambda}\sum_m\sum_n C_{\vec{k}\lambda nm}[\delta_{jn}\sigma_{im}(t) - \delta_{im}\sigma_{nj}(t)]a_{\vec{k}\lambda}(0)e^{-i\omega_k t} \\ & - \sum_m\sum_n\sum_l\sum_p \Gamma_{nmlp}(\omega_{pl}, t)[\delta_{jn}\sigma_{im}(t)\sigma_{lp}(t) - \delta_{im}\sigma_{nj}(t)\sigma_{lp}(t)] \\ & + \sum_m\sum_n\sum_l\sum_p \Gamma^*_{mnlp}(\omega_{pl}, t)[\delta_{jn}\sigma_{pl}(t)\sigma_{im}(t) - \delta_{im}\sigma_{pl}(t)\sigma_{nj}(t)],\end{aligned} \tag{16}$$

where

$$\Gamma_{nmlp}(\omega_{pl}, t) = \frac{1}{\hbar^2}\omega_{nm}\omega_{pl}\sum_{\vec{k},\lambda}(2\pi\hbar/\omega_k V)(\vec{\mu}_{nm}\cdot\hat{e}_{\vec{k}\lambda})(\vec{\mu}_{lp}\cdot\hat{e}_{\vec{k}\lambda})\int_0^t dt_1 e^{i(\omega_k-\omega_{pl})(t_1-t)}. \tag{17}$$

This expression for $\Gamma_{nmlp}(\omega_{pl}, t)$ can be simplified by performing the summation over the field modes. As $V\to\infty$, the summation becomes an integral, and we have

$$\Gamma_{nmlp}(\omega_{pl}, t) = \frac{2\vec{\mu}_{nm}\cdot\vec{\mu}_{lp}}{3\pi\hbar c^3}\omega_{nm}\omega_{pl}\times\int_0^\infty d\omega\,\omega\int_0^t dt_1 e^{i(\omega-\omega_{pl})(t_1-t)}. \tag{18}$$

Since we are only interested in times long compared with any of the $|\omega_{pl}|^{-1}$, we may use the well-known approximation[12]

$$\int_0^t dt_1 e^{i(\omega-\omega_{pl})(t_1-t)} \cong \pi\delta(\omega-\omega_{pl}) - iP[1/(\omega-\omega_{pl})], \tag{19}$$

which leads to

$$\begin{aligned}\Gamma_{nmlp}(\omega_{pl}, t) & \cong \frac{2\omega_{nm}\omega_{pl}^2}{3\hbar c^3}\vec{\mu}_{nm}\cdot\vec{\mu}_{lp}U(\omega_{pl}) \\ & \quad - \frac{2i\omega_{nm}\omega_{pl}}{3\pi\hbar c^3}\vec{\mu}_{nm}\cdot\vec{\mu}_{lp}P\int_0^\infty \frac{\omega d\omega}{(\omega-\omega_{pl})} \\ & = \beta_{nmlp} - i\gamma_{nmlp},\end{aligned} \tag{20}$$

where U is the unit step function.

The final two terms of Eq. (16) can also be simplified by using $\sigma_{im}(t)\sigma_{lp}(t) = \delta_{ml}\sigma_{ip}(t)$ to yield

$$\begin{aligned}\dot{\sigma}_{ij}(t) = {} & -i\left[\omega_{ji} - \sum_m(\gamma_{jmmj} - \gamma_{immi})\right]\sigma_{ij}(t) - \sum_m(\beta_{immi} + \beta_{jmmj})\sigma_{ij}(t) + X_{ij}(t) - \sum_m\sum_{p\neq j}(\beta_{jmmp} - i\gamma_{jmmp})\sigma_{ip}(t) \\ & - \sum_m\sum_{p\neq i}(\beta^*_{immp} + i\gamma^*_{immp})\sigma_{pj}(t) + \sum_m\sum_p[(\beta_{mijp} - i\gamma_{mijp})\sigma_{mp}(t) + (\beta^*_{mjip} + i\gamma^*_{mjip})\sigma_{pm}(t)],\end{aligned} \tag{21}$$

where we have moved all the nonvanishing contributions of the fourth and fifth terms of Eq. (16) which are proportional to σ_{ij} into the first two terms of (21), and have abbreviated the second and third terms of (16) by

$$X_{ij}(t) = \sum_{\vec{k},\lambda} \sum_m \sum_n C_{\vec{k}\lambda nm}\, a^{\dagger}_{\vec{k}\lambda}(0)\, e^{i\omega_k t}[\delta_{jn}\sigma_{im}(t) - \delta_{im}\sigma_{nj}(t)]$$
$$+ \sum_{\vec{k},\lambda} \sum_m \sum_n C_{\vec{k}\lambda nm}[\delta_{jn}\sigma_{im}(t) - \delta_{im}\sigma_{nj}(t)]\, a_{\vec{k}\lambda}(0)\, e^{-i\omega_k t}\,. \tag{22}$$

It is important to bear in mind the two basic approximations made in obtaining Eq. (21) from the Heisenberg equations of motion. First we made the adiabatic approximation (15) based on the assumption of a weak interaction of the atom with the field. Then the approximation (19) followed from an interest in long-time nontransient dynamics.

With the derivation of Eq. (21), the computational portion of this section is complete. We next discuss some implications of this equation.

For $i \neq j$ we see, in the first term of Eq. (21), that

$$\Delta_{ji} = \sum_m (\gamma_{jmmj} - \gamma_{immi})$$
$$= \frac{2}{3\pi\hbar c^3} \sum_m \left(\omega_{jm}^2 |\vec{\mu}_{jm}|^2 P \int_0^\infty \frac{\omega\, d\omega}{\omega - \omega_{jm}} - \omega_{im}^2 |\vec{\mu}_{im}|^2 P \int_0^\infty \frac{\omega\, d\omega}{\omega - \omega_{im}} \right) \tag{23}$$

represents a frequency shift for the $j \to i$ transition. Writing

$$\Delta_{ji} = -\Delta E_j + \Delta E \tag{24}$$

enables us to identify

$$\Delta E_j = -\frac{2}{3\pi\hbar c^3} \sum_m \omega_{jm}^2 |\vec{\mu}_{jm}|^2 P \int_0^\infty \frac{\omega\, d\omega}{\omega - \omega_{jm}} \tag{25}$$

as the level shift of state $|j\rangle$. This is just the divergent expression to which Bethe[13] applied his famous renormalization. Recovering this result verifies that the approximations used to derive Eq. (21) are valid in the sense of second-order perturbation theory.

Again for $i \neq j$, we see that in the second term of Eq. (21),

$$\beta_{ij} = \sum_m (\beta_{immi} + \beta_{jmmj})$$
$$= \frac{2}{3\hbar c^3} \sum_{j>m} |\vec{\mu}_{jm}|^2 \omega_{jm}^3 + \frac{2}{3\hbar c^3} \sum_{i>m} |\vec{\mu}_{im}|^2 \omega_{im}^3\,, \tag{26}$$

where $j > m$ means $E_j > E_m$, represents the natural linewidth of the $j \to i$ transition. This again is the result of second-order perturbation theory.[14]

The fourth, fifth, and sixth terms on the right-hand side of Eq. (21) oscillate at frequencies different from that of σ_{ij} and produce very small changes in these shifts and widths. Their effect is analogous to that of the counter-rotating term in the Bloch-Siegert problem.[15]

The third term on the right-hand side of Eq. (21), $X_{ij}(t)$, is the only term which explicitly contains the vacuum or source-free part of the field operator. All of the other terms in the equation contain no explicit reference to the vacuum radiation field, and can be interpreted as constituting the effects of the radiation reaction of the atom back on itself. The essential result of the adiabatic approximation (15) is that the atomic equations are simplified to the extent that the effects of the vacuum radiation field are contained entirely in the term $X_{ij}(t)$.

In the study of spontaneous emission it is of interest to consider the evolution of the system from an initial state

$$|\psi(0)\rangle = |\phi_A(0)\rangle \otimes |\{0\}\rangle\,, \tag{27}$$

where $|\phi_A(0)\rangle$ is an arbitrary initial atomic state and $|\{0\}\rangle$ the vacuum state of the field. When expectation values are taken on both sides of Eq. (21) in the state (27), $\langle X_{ij}(t)\rangle = 0$, and no role seems to be played by the vacuum field. Based on this result for the two-level atom, Ackerhalt, Knight, and Eberly[5] have concluded that the radiative corrections can be understood solely on the basis of radiation reaction: The adiabatic approximation (15) has given unambiguously a separation of the field operator into two parts, the vacuum or free-field part and a source part,

$$a_{\vec{k}\lambda}(t) = a_{\vec{k}\lambda}(0)\, e^{-i\omega_k t} + a^{(s)}_{\vec{k}\lambda}(t)\,, \tag{28}$$

and only the source part $a^{(s)}_{\vec{k}\lambda}(t)$ seems to be responsible for the line width and shift. The relation between the source term and the usual notion of the radiation reaction field is discussed in Appendix B.[16]

It must be emphasized, of course, that greater prominence is imparted to the effect of radiation reaction only after expectation values are taken. The vacuum field certainly plays a crucial role, even if it is hidden by taking vacuum expectation values. That role is to assure the preservation of equal-time commutation relations, without which

the equations of motion are meaningless. The importance of retaining operators with vanishing expectation values in operator equations of motion has been emphasized by Senitzky[17] in a related problem.

Obviously the vacuum field did not contribute to the expectation values above because of the use of normal ordering. This suggests that in a different ordering we may find an explicit contribution from the free-field part to vacuum expectation values, and this is indeed the case.[6,18] Naturally the final result must be the same as that obtained using the more convenient normal ordering, since equal-time atomic and field operators commute. Thus the problem becomes one of interpretation.

In a certain sense, the above considerations are forced upon us by the necessity, quantum mechanically, of retaining both terms on the right-hand side of Eq. (28). In a classical description of a vacuum-field problem the homogeneous part $a_{\vec{k}\lambda}(0)e^{-i\omega_k t}$ is taken to be zero. Quantum mechanically, however, the homogenous part must be retained for the fundamental purpose of preserving commutation relations; initial conditions are relevant only to expectation values. In this connection it is interesting to note the surprising successes of the classical radiation theory which departs from the usual (classical) boundary condition of equating to zero the homogeneous solutions of the field equations.[19] As Boyer[19] clearly demonstrates, this theory is completely within the framework of the classical Maxwell theory, and represents a departure only from an arbitrary but conventional choice of boundary condition. We discuss this theory in the context of spontaneous emission in Sec. VII.

In the sense in which we use the term "radiation reaction," it may be said that when normal ordering is used the only part of the field which acts on the atom is its own radiation-reaction field, at least insofar as vacuum expectation values are involved. The result is a complicated expression [Eq. (21)] in which each transition dipole is acted upon by its own radiation-reaction field as well as the reaction fields from all the remaining possible transition dipoles.

IV. TWO-LEVEL MODEL

In order to discuss more fully the connection between radiation reaction and vacuum-field fluctuations in the interpretation of radiative corrections, we consider for simplicity the two-level model of the atom. Only in the two-level model is there the simple interpretation of a single transition-dipole moment damped by its own radiation-reaction field. We emphasize, however, that our main points are equally demonstrable in the multilevel formalism.

From Eq. (21) the approximate two-level-atom equations of motion are

$$\dot{\sigma}_{12}(t) = -i(\omega_{21} - \Delta_{21} - i\beta_{21})\sigma_{12}(t) + \sum_{\vec{k},\lambda} C_{\vec{k}\lambda 21}[\sigma_{11}(t) - \sigma_{22}(t)]\, a_{\vec{k}\lambda}(0)e^{-i\omega_k t}$$

$$+ \sum_{\vec{k},\lambda} C_{\vec{k}\lambda 21} a^{\dagger}_{\vec{k}\lambda}(0)e^{i\omega_k t}[\sigma_{11}(t) - \sigma_{22}(t)] + (\beta_{2121} - i\gamma_{2121} + \beta^{*}_{1212} + i\gamma^{*}_{1212})\sigma_{21}(t), \tag{29}$$

$$\dot{\sigma}_{11}(t) = -\beta_{11}\sigma_{11}(t) + 2\beta_{2112}\sigma_{22}(t) + \sum_{\vec{k},\lambda}[C_{\vec{k}\lambda 12}\sigma_{12}(t) - C_{\vec{k}\lambda 21}\sigma_{21}(t)]\, a_{\vec{k}\lambda}(0)e^{-i\omega_k t}$$

$$+ \sum_{\vec{k},\lambda} a^{\dagger}_{\vec{k}\lambda}(0)e^{i\omega_k t}[C_{\vec{k}\lambda 12}\sigma_{12}(t) - C_{\vec{k}\lambda 21}\sigma_{21}(t)], \tag{30}$$

$$\dot{\sigma}_{22}(t) = -\beta_{22}\sigma_{22}(t) + 2\beta_{1221}\sigma_{11}(t) + \sum_{\vec{k},\lambda}[C_{\vec{k}\lambda 21}\sigma_{21}(t) - C_{\vec{k}\lambda 12}\sigma_{12}(t)]a_{\vec{k}\lambda}(0)e^{-i\omega_k t}$$

$$+ \sum_{\vec{k},\lambda} a^{\dagger}_{\vec{k}\lambda}(0)e^{i\omega_k t}[C_{\vec{k}\lambda 21}\sigma_{21}(t) - C_{\vec{k}\lambda 12}\sigma_{12}(t)], \tag{31}$$

and the equation for $\sigma_{21}(t) = \sigma^{\dagger}_{12}(t)$ is just the Hermitian adjoint of Eq. (29).

If level 1 is taken to be the ground state and the dipole matrix element $\vec{\mu}_{12}$ is taken to be real, a substantial simplification of notation results, since $\beta_{11} = \beta_{1221} = \beta_{2121} = 0$, $\beta_{22} = 2\beta_{2112} = -2\beta^{*}_{1212} = 2\beta_{21} = 2\beta$, and $\gamma_{2121} - \gamma^{*}_{1212} = \Delta$,

where

$$\beta = \frac{2\mu_{12}^2\omega_0^3}{3\hbar c^3}, \tag{32}$$

$$\Delta = \frac{2\mu_{12}^2\omega_0^2}{3\pi\hbar c^3} P\int_0^{\infty} d\omega \left(\frac{\omega}{\omega - \omega_0} - \frac{\omega}{\omega + \omega_0}\right), \tag{33}$$

and

$\omega_0 = \omega_{21}$.

We can then write Eqs. (29)–(31) in terms of the usual notation for the atomic inversion

$$\sigma_z(t) = \sigma_{22}(t) - \sigma_{11}(t) = 2\sigma_{22}(t) - 1 \tag{34}$$

and for the atomic polarization

$$\sigma(t) = \sigma_{12}(t), \quad \sigma^\dagger(t) = \sigma_{21}(t). \tag{35}$$

The two-level analogs to Eq. (21) are then

$$\dot{\sigma}(t) = -i(\omega_0 - \Delta)\sigma(t) - \beta\sigma(t) - \Big(\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}\sigma_z(t)a_{\vec{k}\lambda}(0)e^{-i\omega_k t} + \sum_{\vec{k},\lambda} C_{\vec{k}\lambda}a^\dagger_{\vec{k}\lambda}(0)e^{i\omega_k t}\sigma_z(t)\Big) - i(\Delta - i\beta)\sigma^\dagger(t), \tag{36}$$

$$\dot{\sigma}_z(t) = -2\beta[1 + \sigma_z(t)] + 2\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}[\sigma^\dagger(t) + \sigma(t)] \times a_{\vec{k}\lambda}(0)e^{-i\omega_k t} + 2\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}a^\dagger_{\vec{k}\lambda}(0)e^{i\omega_k t}[\sigma(t) + \sigma^\dagger(t)], \tag{37}$$

where $C_{\vec{k}\lambda} = C_{\vec{k}\lambda 21}$ is real.

The first two terms of (36) contain the frequency shift and width and the final term gives the very small antiresonant corrections to them, while the third term is the only one which contains explicit reference to the vacuum radiation field.

Taking the expectation value in a state such as (27) causes the vacuum-field contribution to disappear from these equations:

$$\langle\dot{\sigma}(t)\rangle = -i(\omega_0 - \Delta)\langle\sigma(t)\rangle - \beta\langle\sigma(t)\rangle - i(\Delta - i\beta)\langle\sigma^\dagger(t)\rangle, \tag{38}$$

$$\langle\dot{\sigma}_z(t)\rangle = -2\beta(1 + \langle\sigma_z(t)\rangle). \tag{39}$$

Thus the radiation-reaction interpretation also carries through into the two-level model.

In general, the two-level model may be a good first approximation for problems involving a quasimonochromatic external field tuned to an atomic transition, as illustrated, for example, by the theory of self-induced transparency.[20] In vacuum-field problems, however, the two-level model is much less successful as an accurate description of a real atom. In particular, it is seen in comparing Δ with Δ_{ji} that the two-level radiative frequency shift is a poor representation of the multilevel result, since all but one of the virtual transitions are excluded. However, since it mirrors in some fashion all the essential features of the realistic multilevel atom, the two-level model is adequate for our purposes.

V. RADIATION REACTION AND VACUUM FLUCTUATIONS

The results (38) and (39) were derived using normal ordering. In order to show that the radiation-reaction interpretation of these equations is dependent on this seemingly innocuous choice of orderings, we carry through, in this section, a similar calculation using an "antinormal ordering" (photon creation operators to the extreme right and annihilation operators to the extreme left). In this calculation we use the simple notation of the two-level model introduced in Sec. IV. We emphasize again that a complete multilevel calculation, leading to results analogous to Eq. (21), produces the same conclusions.

The two-level-atom Heisenberg equations in "antinormal" order are

$$\dot{a}_{\vec{k}\lambda}(t) = -i\omega_k a_{\vec{k}\lambda}(t) + C_{\vec{k}\lambda}\sigma^\dagger(t) - C_{\vec{k}\lambda}\sigma(t), \tag{40}$$

$$\dot{\sigma}(t) = -i\omega_0\sigma(t) - \sum_{\vec{k},\lambda} C_{\vec{k}\lambda}[\sigma_z(t)a^\dagger_{\vec{k}\lambda}(t) + a_{\vec{k}\lambda}(t)\sigma_z(t)], \tag{41}$$

$$\dot{\sigma}_z(t) = 2\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}a_{\vec{k}\lambda}(t)[\sigma(t) + \sigma^\dagger(t)] + 2\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}[\sigma(t) + \sigma^\dagger(t)]a^\dagger_{\vec{k}\lambda}(t). \tag{42}$$

Formally integrating Eqs. (40) and (42), we have

$$a_{\vec{k}\lambda}(t) = a_{\vec{k}\lambda}(0)e^{-i\omega_k t} + C_{\vec{k}\lambda}\int_0^t dt_1\sigma^\dagger(t_1)e^{i\omega_k(t_1 - t)} - C_{\vec{k}\lambda}\int_0^t dt_1\sigma(t_1)e^{i\omega_k(t_1 - t)}, \tag{43}$$

$$\sigma_z(t) = \sigma_z(0) + 2\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}\Big(\int_0^t dt_1\, a_{\vec{k}\lambda}(t_1)\sigma^\dagger(t_1) + \int_0^t dt_1\, a_{\vec{k}\lambda}(t_1)\sigma(t_1) + \int_0^t dt_1\,\sigma^\dagger(t_1)a^\dagger_{\vec{k}\lambda}(t_1) + \int_0^t dt_1\,\sigma(t_1)a^\dagger_{\vec{k}\lambda}(t_1)\Big). \tag{44}$$

The adiabatic approximation gives, in the two-level model,

$$a_{\vec{k}\lambda}(t_1) \cong a_{\vec{k}\lambda}(t)e^{-i\omega_k(t_1 - t)}, \quad \sigma(t_1) \cong \sigma(t)e^{-i\omega_0(t_1 - t)}. \tag{45}$$

We next make this adiabatic approximation in the final terms of Eqs. (43) and (44) and substitute those equations into (41). Retaining only terms to second order in the coupling constant we obtain the analog to Eq. (36),

$$\dot{\sigma}(t) = -i\omega_0\sigma(t) - \Bigg(\sum_{\vec{k},\lambda} \Big[C_{\vec{k}\lambda}\sigma_z(0)a^\dagger_{\vec{k}\lambda}(0)e^{i\omega_k t} - C^2_{\vec{k}\lambda}\sigma(t)\int_0^t dt_1\, e^{-i(\omega_k+\omega_0)(t_1-t)}\Big] - \sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda}\sigma^\dagger(t)\int_0^t dt_1\, e^{-i(\omega_k-\omega_0)(t_1-t)}$$
$$+2\sum_{\vec{k},\lambda}\sum_{\vec{\mu},v} C_{\vec{k}\lambda}C_{\vec{\mu}v}\sigma^\dagger(t)a_{\vec{\mu}v}(0)a^\dagger_{\vec{k}\lambda}(0)e^{i(\omega_k-\omega_\mu)t}\int_0^t dt_1\, e^{-i(\omega_\mu-\omega_0)(t_1-t)}$$
$$+2\sum_{\vec{k},\lambda}\sum_{\vec{\mu},v} C_{\vec{k}\lambda}C_{\vec{\mu}v}\sigma(t)a_{\vec{\mu}v}(0)a^\dagger_{\vec{k}\lambda}(0)e^{i(\omega_k-\omega_0)t}\int_0^t dt_1\, e^{-i(\omega_\mu+\omega_0)(t_1-t)}$$
$$+2\sum_{\vec{k},\lambda}\sum_{\vec{\mu},v} C_{\vec{k}\lambda}C_{\vec{\mu}v}a^\dagger_{\vec{\mu}v}(0)\sigma^\dagger(t)a^\dagger_{\vec{k}\lambda}(0)e^{i(\omega_k+\omega_\mu)t}\int_0^t dt_1 e^{i(\omega_\mu+\omega_0)(t_1-t)}$$
$$+2\sum_{\vec{k},\lambda}\sum_{\vec{\mu},v} C_{\vec{k}\lambda}C_{\vec{\mu}v}a^\dagger_{\vec{\mu}v}(0)\sigma(t)a^\dagger_{\vec{k}\lambda}(0)e^{i(\omega_k+\omega_\mu)t}\int_0^t dt_1\, e^{i(\omega_\mu-\omega_0)(t_1-t)} + \text{H.c.}\Bigg), \tag{46}$$

where we have used the fact that $\sigma_z(0)\sigma(t)$ may be replaced by $\sigma_z(t)\sigma(t) = -\sigma(t)$ to remain in second order. Taking expectation values in the state (27) yields, in second order,

$$\langle\dot{\sigma}(t)\rangle = -i\omega_0\langle\sigma(t)\rangle - \langle\sigma(t)\rangle\sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda}\left[2\langle a_{\vec{k}\lambda}(0)a^\dagger_{\vec{k}\lambda}(0)\rangle - 1\right]\left[\int_0^t dt_1\, e^{-i(\omega_k+\omega_0)(t_1-t)} + \int_0^t dt_1\, e^{i(\omega_k-\omega_0)(t_1-t}\right]$$
$$-\langle\sigma^\dagger(t)\rangle\sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda}\left[2\langle a_{\vec{k}\lambda}(0)a^\dagger_{\vec{k}\lambda}(0)\rangle - 1\right]\left[\int_0^t dt_1\, e^{i(\omega_k+\omega_0)(t_1-t)} + \int_0^t dt_1\, e^{-i(\omega_k-\omega_0)(t_1-t)}\right]. \tag{47}$$

We have deliberately not made use of the fact that $\langle a_{\vec{k}\lambda}(0)a^\dagger_{\vec{k}\lambda}(0)\rangle = 1$ at this stage. If we use that fact, we can readily perform the sum over modes, using the approximation (19), to give

$$\langle\dot{\sigma}(t)\rangle = -i(\omega_0-\Delta)\langle\sigma(t)\rangle - \beta\langle\sigma(t)\rangle$$
$$-i(\Delta - i\beta)\langle\sigma^\dagger(t)\rangle. \tag{48}$$

This is once again just Eq. (38), as is to be expected. But looking back to Eq. (47), we see that the contribution from the vacuum fluctuations does not vanish, as it did in Eq. (38); rather, we may now interpret Δ (and similarly β) as $\Delta = 2\Delta_{\mathrm{VF}} - \Delta_{\mathrm{RR}}$, where $\Delta_{\mathrm{VF}} = \Delta$ is the contribution from the vacuum field fluctuations and $\Delta_{\mathrm{RR}} = \Delta$ is the contribution from the radiation reaction of the source back on itself.

Thus we have found that when normal ordering is used the entire contribution to the shift comes from the radiation reaction, whereas when antinormal ordering is used 2Δ comes from the vacuum field fluctuations with another $-\Delta$ from the radiation reaction.[6,18] If a symmetric ordering (half normal and half antinormal) is used, the entire shift stems from vacuum-field fluctuations.[6] For other orderings neither the radiation-reaction nor vacuum-fluctuations term produces the entire shift; rather, each contributes a portion complementary to the other. The two interpretations of the shift of the emitted line are equivalent, the interpretation depending as it does on the particular ordering chosen for commuting atomic and field operators. The two interpretations "are merely two sides of the same quantum-mechanical coin, with each ... being an oversimplification motivated by the ordering scheme adopted."[18] The situation with regard to the width of the shifted line is somewhat subtler, and in Sec. VI we consider the linewidth question by investigating the two-level atom's lifetime.

VI. INTERPRETATION OF ATOMIC ENERGY LOSS

Regarding the decay of $\langle\sigma_z(t)\rangle$, it was stated in Ref. 6 that "there is no ordering which would attribute the decay entirely to a vacuum-fluctuation effect." This appears to contradict a conclusion of Senitzky,[18] namely, that both the $\langle\sigma(t)\rangle$ and $\langle\sigma_z(t)\rangle$ equations can be interpreted on the basis of vacuum-field fluctuations. In order to resolve this contradiction, a brief analysis of the decay of $\langle\sigma_z(t)\rangle$ will be given here.

Since it does not affect our results, we take the rotating-wave approximation version of the equations of motion,

$$\dot{\sigma}_z(t) = 2\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}[a_{\vec{k}\lambda}(t)\sigma^\dagger(t) + \sigma(t)a^\dagger_{\vec{k}\lambda}(t)], \tag{49}$$

$$\dot{\sigma}(t) = -i\omega_0\sigma(t) - \sum_{\vec{k},\lambda} C_{\vec{k}\lambda}a_{\vec{k}\lambda}(t)\sigma_z(t), \tag{50}$$

$$\dot{a}_{\vec{k}\lambda}(t) = -i\omega_k a_{\vec{k}\lambda}(t) - C_{\vec{k}\lambda}\sigma(t), \tag{51}$$

in antinormal order as in Ref. 18.

Formally integrating Eqs. (50) and (51), one finds

$$\sigma(t) = \sigma(0)e^{-i\omega_0 t} - \sum_{\vec{k},\lambda} C_{\vec{k}\lambda}\int_0^t dt_1\, a_{\vec{k}\lambda}(t_1)$$
$$\times\sigma_z(t_1)e^{i\omega_0(t_1-t)}, \tag{52}$$

$$a_{\vec{k}\lambda}(t) = a_{\vec{k}\lambda}(0)e^{-i\omega_k t} - C_{\vec{k}\lambda}\int_0^t dt_1\,\sigma(t_1)e^{i\omega_k(t_1-t)}\,. \tag{53}$$

Using now the adiabatic approximation $\sigma(t_1)\cong\sigma(t)e^{-i\omega_0(t_1-t)}$ and $a_{\vec{k}\lambda}(t_1)\sigma_z(t_1)\cong a_{\vec{k}\lambda}(t)\sigma_z(t)e^{-i\omega_k(t_1-t)}$ in Eqs. (52) and (53), and substituting these equations into (49), we have

$$\dot{\sigma}_z(t) = 2\sum_{\vec{k}\lambda} C_{\vec{k}\lambda}a_{\vec{k}\lambda}(0)\sigma^\dagger(0)e^{-i(\omega_k-\omega_0)t} - 2\sum_{\vec{k},\lambda}\sum_{\vec{\mu}\nu} C_{\vec{k}\lambda}C_{\vec{\mu}\nu}\sigma_z(t)a_{\vec{k}\lambda}(0)\,a^\dagger_{\vec{\mu}\nu}(t)e^{-i\omega_k t}\int_0^t dt_1\,e^{i(\omega_\mu-\omega_0)(t_1-t)}$$
$$-2\sum_{\vec{k},\lambda}\sum_{\vec{\mu},\nu} C_{\vec{k}\lambda}C_{\vec{\mu}\nu}\sigma(t)\sigma^\dagger(0)e^{i\omega_0 t}\int_0^t dt_1\,e^{i(\omega_k-\omega_0)(t_1-t)} + \text{H.c.}\,, \tag{54}$$

where we may replace $a_{\vec{\mu}\nu}(t)$ by $a_{\vec{\mu}\nu}(0)e^{-i\omega_\mu t}$ to remain in second order.

The final step is to take the expectation value of (54) in a state such as (27):

$$\langle\dot{\sigma}_z(t)\rangle = -2\sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda}\langle\sigma_z(t)\rangle\langle a_{\vec{k}\lambda}(0)a^\dagger_{\vec{k}\lambda}(0)\rangle\left(\int_0^t dt_1\,e^{i(\omega_k-\omega_0)(t_1-t)} + \text{c.c.}\right)$$
$$-\sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda}\langle\sigma(t)\sigma^\dagger(0)\rangle e^{i\omega_0 t}\left(\int_0^t dt_1\,e^{i(\omega_k-\omega_0)(t_1-t)} + \text{c.c.}\right), \tag{55}$$

where again we have written $\langle a_{\vec{k}\lambda}(0)a^\dagger_{\vec{k}\lambda}(0)\rangle$ rather than unity to show explicitly the effect of the vacuum field. If we replace $\langle a_{\vec{k}\lambda}(0)a^\dagger_{\vec{k}\lambda}(0)\rangle$ with unity, we can again perform the sum over modes using the approximation (19) to give

$$\langle\dot{\sigma}_z(t)\rangle = -2\sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda}\langle\sigma(t)\sigma^\dagger(0)\rangle$$
$$\times e^{i\omega_0 t}\left(\int_0^t dt_1 e^{i(\omega_k-\omega_0)(t_1-t)} + \text{c.c.}\right) - 4\beta\langle\sigma_z(t)\rangle\,, \tag{56}$$

where now we must remember that the final term is the one which stems from vacuum fluctuations. Senitzky[18] now considers the initial atomic state to be $|+\rangle$, the excited state. Then from Eq. (56) it is seen that all but the vacuum-fluctuation term in (55) vanishes to give

$$\langle\dot{\sigma}_z(t)\rangle \cong -4\beta\langle\sigma_z(t)\rangle\,. \tag{57}$$

This gives the correct Einstein A coefficient for the decay of the excited atomic state, resulting entirely from vacuum field fluctuations. But this is not the whole story. If the atomic state is anything other than a pure excited state, the radiation-reaction terms give an essential contribution to the motion of $\langle\sigma_z(t)\rangle$.[6] This is to be expected. Equation (57) cannot be valid, for example, for the atom in the ground state; it would predict spontaneous absorption! For the consistency of our second-order approach we should replace the term $\langle\sigma(t)\sigma^\dagger(0)\rangle e^{i\omega_0 t}$ in Eq. (56) by $\langle\sigma^\dagger(t)\sigma(t)\rangle$. Then the correct equation for $\langle\sigma_z(t)\rangle$ for an arbitrary initial atomic state contains this radiation reaction contribution:

$$\langle\dot{\sigma}_z(t)\rangle = -2\beta(1+\langle\sigma_z(t)\rangle)\,. \tag{58}$$

Thus only when vacuum fluctuations and radiation reaction are both accounted for is an antinormally ordered calculation free of spontaneous absorption difficulties; the solution of Eq. (57) when the atom is initially in the ground state is $\langle\sigma_z(t)\rangle = -1$, and the atom remains in the ground state as it should.

We have shown, therefore, that for a proper antinormally ordered calculation of the decay of $\langle\sigma_z(t)\rangle$ we need both the vacuum field and the source field. Vacuum field considerations alone do not suffice; it may be shown that this is true regardless of the ordering scheme used.

In a different framework, one concerned with atomic fluctuations as well as vacuum field fluctuations, this point has already been emphasized by Fain.[21] He has shown that for the initial state $|-\rangle\otimes|\{0\}\rangle$ (atomic ground state, vacuum field) the vacuum field fluctuations, which would lead to unphysical spontaneous absorption, are exactly canceled by atomic fluctuations.

VII. CONNECTION WITH OTHER APPROACHES

The essential results of this paper concern the interpretation of radiative corrections as contained in Secs. V and VI. Here we simply mention some connections between this work and other approaches.

The atomic operator formalism can provide more physical insight than the usual perturbation-theory approach; in some cases it is computationally simpler. For example, it is a trivial matter to derive the Ladenburg dispersion formula under the approximation $\langle\sigma_{nn}(t)\rangle\cong\delta_{n0}$, where $|0\rangle$ is the atomic ground state; the Kramers-Heisenberg formula follows by relaxing this restriction slightly to allow for Kramer's "negative oscillators,"[22] i.e.,

the possibility of transitions *to* the ground state.

The operator formalism also provides better insight into semiclassical radiation theories. For example, the neoclassical equations[4] for spontaneous emission are easily derived by decorrelating expectation values of the type $\langle A(t)F(t)\rangle$ into $\langle A(t)\rangle\langle F(t)\rangle$, where $A(t)$ and $F(t)$ are, respectively, atomic and field operators. The polarization correlations in the atomic cascade discussed by Clauser[23] are easily found by considering a correlation function of the type $\langle a_A^\dagger(t)a_B^\dagger(t)a_B(t)a_A(t)\rangle$.[24] The corresponding decorrelation (c-number field) gives the neoclassical prediction.

Although it is not our purpose here to discuss the various theories of spontaneous emission,[4] some brief comments are in order regarding the use of classical zero-point radiation to mimic the effects of the quantum vacuum field. In particular, we have in mind using Boyer's concept of zero-point fluctuations in the classical electromagnetic field.[19] Where vacuum expectation values are to be taken in the quantum-mechanical case, one would instead average over Boyer's statistical distribution of the phases of the fluctuating zero-point classical electromagnetic field. It appears that such an approach will give plausible results whenever the final expressions (whose expectation values are evaluated) are of a particular structure. Namely, there are terms bilinear in the field and the positive- and negative-frequency parts appear additively as the total field in these terms. This occurs in the linearization approximation in which the atom is essentially replaced by a harmonic oscillator.[25] For example, the quantum-mechanical operator equations in the derivation of the van der Waals force between neutral atoms are formally the same as Boyer's classical equations[19]; Boyer's classical fluctuating zero-point field plays the role of the quantum-electrodynamical vacuum field.

For the spontaneous-emission problem, we have noted above that the line shift and linewidth may be attributed totally to the vacuum field fluctuations when a symmetric ordering is used. Precisely in the symmetric-ordering case does the total radiation field appear as the sum of its positive- and negative-frequency parts. This suggests that we can derive the line shift and linewidth by incorporating Boyer's concept of a classical zero-point radiation field into the standard semiclassical model (quantum matter, classical electromagnetic fields). Consider the Bloch equations for a two-level atom interacting with a classical zero-point field[26]:

$$\dot{x}(t) = -i\omega_0 x(t) + (\omega_0/\hbar c)\vec{\mu}\cdot\vec{A}_0(0,t)z(t)\,, \tag{59}$$

$$\dot{z}(t) = -(2\omega_0/\hbar c)\vec{\mu}\cdot\vec{A}_0(0,t)[x(t)+x^*(t)]\,, \tag{60}$$

where

$$\vec{A}_0(\vec{r},t) = \sum_{\vec{k},\lambda}\left(\frac{\pi\hbar c^2}{\omega_k V}\right)^{1/2} \times\{\exp[i(\vec{k}\cdot\vec{r}-\omega_k t+\Theta_{\vec{k}\lambda})]\hat{e}_{\vec{k}\lambda}+\text{c.c.}\} \tag{61}$$

and the constants are adjusted so that

$$\frac{1}{4\pi}\langle\vec{E}_0^2(\vec{r},t)\rangle_\Theta = \frac{1}{V}\sum_{\vec{k},\lambda}\tfrac{1}{2}\hbar\omega_k\,, \tag{62}$$

where $\langle\ \rangle_\Theta$ denotes an average over the random phases. Now we integrate Eq. (60) and substitute it into Eq. (59), following the same method as in the quantum-mechanical case, but replacing vacuum expectation values by $\langle\ \rangle_\Theta$. We find after phase averaging

$$\dot{x}(t) = -i(\omega_0-\Delta)x(t) - \beta x(t) - i(\Delta - i\beta)x^*(t) \tag{63}$$

which is the exact analog of Eq. (48). Thus we have a successful extension of Boyer's analysis (*classical* matter interacting with a *classical* electromagnetic field that contains zero-point fluctuations) to *quantum* matter.[27] The *classical* zero-point fluctuations supply the effects of the quantum vacuum field which are absent in the usual semiclassical calculations. In fact, it is clear that Welton's argument[3] is already formulated in the language of a classical zero-point field.

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APPENDIX A

Our basic approximation has been the adiabatic approximation, which we now show to be equivalent to the Weisskopf-Wigner "pole" or "on-shell" approximation.[28-30] For a two-level atom initially in the upper state and no photons in the field, the "essential states" are $|\phi_0\rangle = |+\rangle\otimes|\{0\}\rangle$ and $|\phi_{\vec{k}\lambda}\rangle = |-\rangle\otimes|1_{\vec{k}\lambda}\rangle$, respectively, the initial state and the state in which the atom is in the ground state and one photon is in the field. The state vector at time t may be written

$$|\psi(t)\rangle = b(t)\,|\phi_0\rangle + \sum_{\vec{k},\lambda} b_{\vec{k}\lambda}(t)\,|\phi_{\vec{k}\lambda}\rangle\,, \tag{A1}$$

and the Schrödinger equation becomes

$$\dot{b}(t) = -\sum_{\vec{k},\lambda} C_{\vec{k}\lambda}\, b_{\vec{k}\lambda}(t)\,, \tag{A2}$$

$$\dot{b}_{\vec{k}\lambda}(t) = -i(\omega_k - \omega_0)b_{\vec{k}\lambda}(t) + C_{\vec{k}\lambda}b(t), \tag{A3}$$

with $C_{\vec{k}\lambda}$ defined as in Sec. IV and with the energy scale defined by taking the unperturbed energy of state $|\phi_0\rangle$ to be zero.

We have for $b(t)$ the equation

$$\dot{b}(t) = -\sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda} \int_0^t dt_1\, b(t_1) e^{i(\omega_k - \omega_0)(t_1 - t)}$$

$$\cong -b(t) \sum_{\vec{k},\lambda} C^2_{\vec{k}\lambda} \int_0^t dt_1\, e^{i(\omega_k - \omega_0)(t_1 - t)} \tag{A4}$$

in the adiabatic approximation. Following the same procedure used throughout this work, Eq. (A4) is replaced by

$$\dot{b}(t) = -(\beta - i\Delta^{(+)})b(t), \tag{A5}$$

where

$$\Delta^{(\pm)} = \frac{2\mu_{12}^2\omega_0^2}{3\pi\hbar c^3} P \int_0^\infty \frac{\omega\, d\omega}{\omega \mp \omega_0}. \tag{A6}$$

$-\hbar\Delta^{(+)}$ represents the *level shift* of the initial state $|\phi_0\rangle$. It is easily shown that the level shift in the final state $|\phi_{\vec{k}\lambda}\rangle$ is $-\hbar\Delta^{(-)}$; thus the *frequency shift* of the transition is $-(\Delta^{(+)} - \Delta^{(-)}) = -\Delta$. The atomic-operator formalism we use gives frequency shifts directly.

Within the Schrödinger picture, therefore, we have shown that the adiabatic approximation yields results identical to those of the usual "pole" approximation.[11]

APPENDIX B

To justify our use of the term "radiation reaction" in this work, we now show that the source part of the field operator, $a^{(s)}_{\vec{k}\lambda}(t)$, gives an expression for the total (operator) reaction field of the form well known from classical electrodynamics. The source part of the vector potential operator, evaluated at $\vec{r} = 0$, is

$$\vec{A}_{RR}(t) = \sum_{\vec{k},\lambda} \left(\frac{2\pi\hbar c^2}{\omega_k V}\right)^{1/2} a^{(s)}_{\vec{k}\lambda}(t)\hat{e}_{\vec{k}\lambda} + \text{H.c.}$$

$$= \sum_{\vec{k},\lambda} \sum_m \sum_n \left(\frac{2\pi\hbar c^2}{\omega_k V}\right)^{1/2} C_{\vec{k}\lambda nm}\hat{e}_{\vec{k}\lambda} \int_0^t dt_1\, \sigma_{nm}(t_1)$$

$$\times e^{i\omega_k(t_1 - t)} + \text{H.c.} \tag{B1}$$

Letting $V \to \infty$, Eq. (B1) may be written as

$$\vec{A}_{RR}(t) = \frac{4i}{3\pi c^3} \sum_m \sum_n \omega_{mn}\vec{\mu}_{nm} \int_0^\Omega d\omega\, \omega$$

$$\times \int_0^t dt_1\, \sigma_{nm}(t_1) \sin\omega(t_1 - t), \tag{B2}$$

where Ω is the usual high-frequency cutoff required in nonrelativistic calculations. All the integrations over frequency in this paper must be understood as having this cutoff.

Using the representation (8) of $\vec{p}(t)$, we have

$$\vec{A}_{RR}(t) = -\frac{4e}{3\pi mc^2} \int_0^\Omega d\omega\, \omega \int_0^t dt_1\, \vec{p}(t_1) \sin\omega(t_1 - t)$$

$$= \frac{4e}{3\pi mc^2} \int_0^t dt_1\, \vec{p}(t_1) \frac{\partial}{\partial t_1}\left(\frac{\sin\Omega(t_1 - t)}{t_1 - t}\right)$$

$$= \frac{4e}{3\pi mc^2}\left(\Omega\vec{p}(t) - \frac{\pi}{2}\frac{d\vec{p}(t)}{dt}\right) \tag{B3}$$

for $\Omega \to \infty$. This is the form of the radiation-reaction field well known from classical electrodynamics. For example, if we write $\vec{p}(t) = m d\vec{r}(t)/dt$ we have for the radiation-reaction field the familiar result

$$\vec{E}^{\perp}_{RR}(t) = -\frac{1}{c}\frac{d\vec{A}_{RR}(t)}{dt} = \frac{2e}{3c^3}\frac{d^3\vec{r}(t)}{dt^3} - \frac{4e\Omega}{3\pi c^3}\frac{d^2\vec{r}(t)}{dt^2} \tag{B4}$$

for a point particle. Note, however, that our field variables are quantum-mechanical operators.

These results justify our use of the term "radiation reaction" as the quantum-mechanical generalization of the well-known classical concept. For a more detailed discussion we refer the reader to the paper by Ackerhalt and Eberly.[5]

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[25]This approximation is useful when with high probability the atom will remain in its ground state. Then, in the two-level model, for example, we may effectively take $\sigma_z(t)=-1$; the fermion commutation relation $[\sigma, \sigma^\dagger] = -\sigma_z(t)$ becomes $[\sigma, \sigma^\dagger]=1$, so that σ and $\sigma^\dagger$ become effectively boson-lowering and -raising operators, respectively. This is the quantum-mechanical "justification" of the Lorentz model.

[26]The optical Bloch equations are obtained in their more familiar form by defining $x=\frac{1}{2}(u-iv)$, u and v real.

[27]If we consider the z equation, we encounter again the "spontaneous absorption" difficulty. We do not have a fair comparison with Boyer's program, since Eqs. (59) and (60) are quantum mechanical. Boyer uses the classical zero-point energy method in the context of purely classical material equations. It should also be mentioned that the use of such a zero-point field in the spontaneous emission problem has been alluded to by G. W. Series [Bull. Am. Phys. Soc. II 18, 1523 (1974)]. Series suggests that the consequent spontaneous absorption may be connected with the Feynman-Wheeler absorber theory.

[28]V. F. Weisskopf and E. Wigner, Z. Phys. 63, 54 (1930).

[29]G. Källen, *Encyclopedia of Physics* (Springer, Berlin, 1958), Vol. VII, p. 274; see also V. Ernst and P. Stehle, Phys. Rev. 176, 1456 (1968).

[30]A treatment of *two-atom* spontaneous emission which goes beyond the usual pole approximation is given by P. W. Milonni and P. L. Knight [Phys. Rev. A 10, 1096 (1974)].