

# 8.421: Module 3 Problem Set

Due: April 1, 2016

The resonant absorption rate for a two-level atom in an electromagnetic field is given by

$$\Gamma_{ab} = \frac{4\pi^2}{\hbar V} \left| \hat{\epsilon} \cdot \vec{d}_{ba} \right|^2 \omega_0 n(\omega_0)$$

where the field has frequency  $\omega_0$  and polarization  $\hat{\epsilon}$  and is confined to a volume  $V$ . The number of photons in a frequency interval  $d\omega$  is  $n(\omega)d\omega$ , and  $\vec{d}_{ba}$  is the dipole matrix element for the  $ba$  transition.

We want to study the spontaneous emission of an atom in a two-dimensional electromagnetic field. We do this by assuming that the atom is in a parallel plate capacitor with plate separation  $d \ll \lambda$ , the resonant wavelength. Define the direction of the plate separation to be  $\hat{z}$ .

## Problem 1

What are the relevant modes of the electromagnetic field, and what is their mode density?

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## Problem 2

For a simple transition with only one excited state and one ground state, what is the spontaneous emission rate in terms of  $\vec{d}_{ba}$ ? Compare  $\Gamma^{2D}$  to the 3D decay rate. Is it larger or smaller, and why?

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## Problem 3

Now find the excited state lifetimes for  $p \rightarrow s$  and  $s \rightarrow p$  transitions. How do your answers change when you change the quantization axis of the atoms from  $\hat{z}$  to  $\hat{x}$ ? In an experiment, this can be done by changing the direction of an external magnetic field. Write your answers in terms of  $\Gamma^{2D}$ , your answer from the previous problem.

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**Problem 4**

Suppose the atom has a  $J = 2 \rightarrow J' = 1$  transition with rates labeled as follows:

a:  $m = 2 \rightarrow m' = 1$

b:  $m = 1 \rightarrow m' = 1$

c:  $m = 0 \rightarrow m' = 1$

d:  $m = 1 \rightarrow m' = 0$

e:  $m = 0 \rightarrow m' = 0$

Find each of these rates in 2D, first in the case where the quantization axis is along  $\hat{z}$ , and second where the quantization axis is along  $\hat{x}$ . Write your answers in terms of  $\Gamma^{2D}$ . What are the closed transitions in each case?

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**Problem 5**

For the quantization axis along  $\hat{z}$  and along  $\hat{x}$ , what is the decay rate of an unpolarized sample in the  $J = 2$  excited state? Explain your result.

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