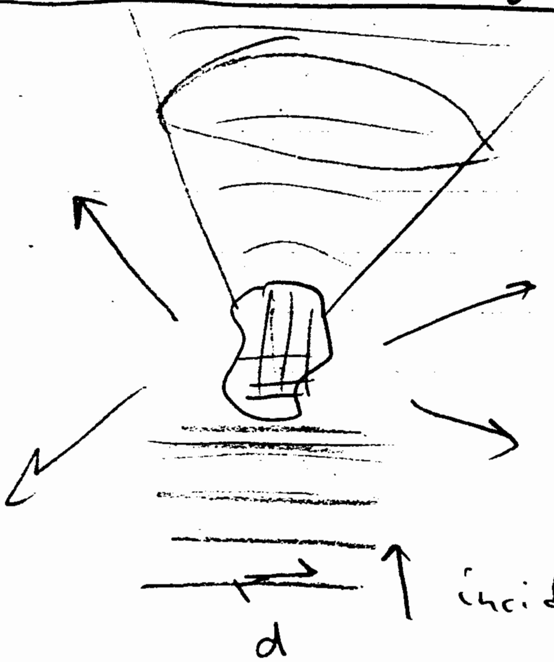
10.5.5. Coherent Forward scattering

Light scattering in Finite Size sample



N atoms

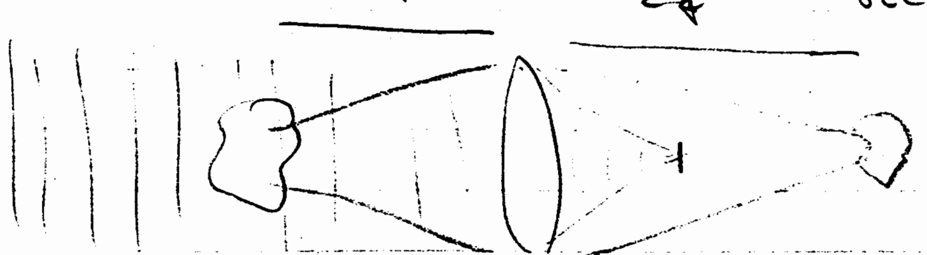
$$S_{incoh} = I_0 \, \sigma_{rayleigh} \cdot N$$ add intensities

$$S_{coh} = (\ ) \underbrace{I_0 \, \sigma_{rayleigh} \, N}^{} {}^2 \, \frac{\lambda^2}{d^2}$$ add amplitudes

N times enhanced, Dicke Superradiance

$$\frac{S_{coh}}{S_{incoh}} \sim \frac{N \lambda^2 \ell}{\ell \, d^2} = (\ ) \frac{N}{V} \, \ell \, \underset{\uparrow \atop \sigma_{res}}{\lambda^2} = (\ ) \underset{\uparrow \atop \text{resonant optical density}}{D_0}$$

BEC experiments ~ 200



Coherently image scattered light

200 x more

More quantitatively
using complex index of refraction

$$n_{ref} = \sqrt{1 + 4\pi n \alpha} \simeq 1 + 2\pi n \alpha$$

Polarizability α: complex Lorentzian

$$\frac{-1}{(\omega-\omega_0) + i(\Gamma/2)} = \frac{-1}{\delta + i} = \frac{i - \delta}{1+\delta^2}$$

$$\delta = (\omega - \omega_0) / (\Gamma/2)$$

$$\sigma_0 = 6\pi \lambda\!\!\!^-{}^2$$

$$n_{ref} = 1 + \frac{\sigma_0 n \lambda}{4\pi}\left[\frac{i-\delta}{1+\delta^2}\right]$$

transmitted electric field

$$E = E_0 e^{ik\int (n_{ref}-1)\,dz} = E_0 e^{i \frac{\sigma_0}{2} \frac{i-\delta}{1+\delta^2} \int n\,dz}$$

$$E = t E_0 e^{i\phi}$$

$$t = e^{-\tilde{D}/2}$$

$$\phi = -\delta \tilde{D}/2$$

$$\tilde{D} = \overbrace{\int n\,dz\,\sigma_0}^{\tilde{D}_0} / 1+\delta^2$$

↳ off resonant optical density

Absorbed light $1 - t^2 = 1 - e^{-\tilde{D}} \underset{\delta \text{ large}}{\approx} \tilde{D}$

$$E = t E_0 e^{i\phi} = E_0 + \Delta E$$

Scattered field

$$\left(\frac{\Delta E}{E_0}\right)^2 = \left|1 - e^{-\tilde{D}/2[1+i\delta]}\right|^2 \underset{\delta \text{ large}}{\approx} \left(\frac{\tilde{D}}{2}[1+i\delta]\right)^2$$

$$= \frac{\delta^2}{4}\tilde{D}^2 = \tilde{D} \cdot \frac{\tilde{D}_0}{4}$$

ratio $\frac{\tilde{D}_0}{4}$