Concept questions

Module 1
Topic 1

- For thermal light with average photon number $\bar{n}$, what is the most probable value
  - A 0 ✓
  - B $0.5\bar{n}$
  - C $\bar{n}$
  - D $2\bar{n}$
- For thermal light with large $\bar{n}$, the fluctuations Δn are
  - A $\sqrt{\bar{n}}$
  - B $\bar{n}$ ✓
  - C $\bar{n}^2$
- Same for coherent light

  $\bar{n}$

  $\sqrt{\bar{n}}$

- For coherent light with $|\alpha|^2 = 8.9$

  The most probable n is

  - A 8 ✓
  - B 9

reminder thermal $P_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}$

coherent light $P_n = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$

Let's look at very weak light, $\bar{n} \ll 1$

- In leading order, what are P_1 and P_2. Remember what $g^{(2)}(0)$ is for the two cases (2 thermal, 1 coherent).

Answer: P_1 $\bar{n}$

P_2 $\frac{1}{2} \bar{n}^2 g^{(2)}$

Let's now derive the relation between P_1, P_2 and $g^{(2)}$ in general:

- Express $\langle n^2 \rangle$ and $\langle n \rangle$ by P_1, P_2

$$\langle n \rangle = 1 \, P_1 + 2 \, P_2$$

$$\langle n^2 \rangle = 1 \, P_1 + 4 \, P_2$$

- Find $g^{(2)}(0) = \frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2} = \frac{2 P_2}{P_1^2}$

- $g^{(2)}$ is the normalized probability to detect 2 photons.

Explain the factor of 2 in

$$g^{(2)} = \frac{2p^2}{p_1^2}$$

use $|\psi\rangle = \sqrt{p_0}\,|0\rangle + \sqrt{p_1}\,|1\rangle + \sqrt{p_2}\,|2\rangle$

What is $aa|\psi\rangle$?

$$\sqrt{p_2}\,\sqrt{2}\,|0\rangle$$

What is $\dfrac{\langle a^\dagger a^\dagger a a\rangle}{\langle a^\dagger a\rangle^2} = \dfrac{2p_2}{p_1^2}$

- Now use

$$\rho = p_0\,|0\rangle\langle 0| + p_1\,|1\rangle\langle 1| + p_2\,|2\rangle\langle 2|$$

What is $a^\dagger a^\dagger a a \rho$

$$2p_2\,|2\rangle\langle 2|$$

What is $a^{\dagger} a$?

$\rho_1 \, |1\rangle\langle 1| + 2 \rho_2 \, |2\rangle\langle 2|$

What is $\dfrac{\langle a^{\dagger} a^{\dagger} a a \rangle}{\langle a^{\dagger} a \rangle^2}$?

$\dfrac{2\rho_2}{\rho_1^2}$

Now back to thermal and coherent states

- Work out the probabilities P_0, P_1, P_2 to find states with 0, 1, 2 photons to order $\bar{n}^2$

  - thermal

$$1 - \bar{n} + \bar{n}^2$$
$$\bar{n} - 2\bar{n}^2$$
$$\bar{n}^2$$

  - coherent

$$1 - \bar{n} + \bar{n}^2$$
$$\bar{n} - \bar{n}^2$$
$$\bar{n}^2 / 2$$

Plug those results into $g^{(2)} = \frac{2P_2}{P_1^2}$

2 thermal

1 coherent

If you absorb the whole state, you find the probability for a deposited energy of $\hbar\omega$ and $2\hbar\omega$, Γ_1 and Γ_2

What is Γ_2?

A $P_2/2$

B P_2 ✓

C $2P_2$

Reminder:

$$\Gamma_1 = P_1$$

$$P_2 = \frac{1}{2} P_1^2 g^{(2)}$$

$$\langle a^\dagger a^\dagger a a \rangle = P_1^2 g^{(2)} = 2P_2$$