

4/29/2013

# LIGHT FORCES

API pp 370-378

$$H_{I,1} = -\vec{d} \cdot \vec{E}_{\perp}$$

Atomic  
dipole operator

operator of the  
quantized em. field  
or  $\hat{a} + \hat{a}^{\dagger}$

em. field coherent state  $|\alpha\rangle$

exercise 17 in API  
exact unitary transformation

$$|\alpha\rangle \longrightarrow |0\rangle \text{ vacuum state}$$
$$\vec{E}_{\perp} \longrightarrow \vec{E}_0 \cos \omega_L t + \vec{E}_{\perp}$$

vacuum fluctuations  
of the em field

Atomic dipole: OBE

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# Bloch Vector

$$\langle \hat{Y}_x \rangle = u = \frac{1}{2} (\hat{\sigma}_{ab} + \hat{\sigma}_{ba})$$

$$\langle \hat{Y}_y \rangle = v = \frac{1}{2i} (\hat{\sigma}_{ab} - \hat{\sigma}_{ba})$$

$$\langle \hat{Y}_z \rangle = w = \frac{1}{2} (\hat{\sigma}_{bb} - \hat{\sigma}_{aa})$$

$\hat{\sigma}$ : atomic density matrix in RWA and rotating frame

↳ Fictitious spin  $\frac{1}{2}$

[eq. A20]

$$\begin{aligned} \dot{u} &= \delta_L v - \frac{\Gamma}{2} u \\ \dot{v} &= -\delta_L u - \Omega_1 w - \frac{\Gamma}{2} v \\ \dot{w} &= \Omega_1 v - \Gamma w - \frac{\Gamma}{2} \end{aligned}$$

$$\delta_L = \omega_L - \omega_0$$

Case detuning

$\Omega_1$  Rabi Freq.

$$\hbar \Omega_1 = - \hat{d}_{ab} \vec{E}_0$$

$$\vec{E} = \vec{E}_0 \cos \omega_L t$$

Translation

$$\delta \rightarrow \delta_L$$

$$g \rightarrow \Omega_1$$

$$r_{x,y,z} \rightarrow 2u, 2v, 2w$$

## Lecture on solutions of OBE

$$\dot{r}_x = \delta r_y - \frac{\Gamma}{2} r_x$$

$$\dot{r}_y = -\delta r_x - g r_z - \frac{\Gamma}{2} r_y$$

$$\dot{r}_z = g r_y - \Gamma (r_z - 1)$$

Mechanical Force  $\vec{I} \vec{V} E$

$$\vec{I} = \vec{d}_{ab} (|b\rangle\langle a| + |a\rangle\langle b|) \quad \text{A.21}$$

$$\begin{aligned} \langle \dot{d} \rangle &= \text{Tr}(\dot{\sigma} d) = \vec{d}_{ab} (\dot{\sigma}_{ab} + \dot{\sigma}_{ba}) \\ &= \vec{d}_{ab} \left( \dot{\sigma}_{ab} e^{i\omega_L t} + \dot{\sigma}_{ba} e^{-i\omega_L t} \right) \\ &= 2 \vec{d}_{ab} \left( u \cos \omega_L t - v \sin \omega_L t \right) \end{aligned}$$

in phase
in quadrature  
with incident field

Absorbed <sup>power</sup> energy

$$\frac{dW}{dt} = q \epsilon_0 \cos \omega_L t \frac{dr}{dt} \quad \langle \dot{d} \rangle$$

$$\left\langle \frac{dW}{dt} \right\rangle = \hbar \Omega_1 \omega_L \nu \quad \div \hbar \omega_L$$

↑  
averaged over  
one cycle

$$\rightarrow \left\langle \frac{dN}{dt} \right\rangle = \Omega_1 \nu$$

# absorbed photons

# Radiative Force

$$\vec{F} = \dot{\vec{p}} \stackrel{\substack{\uparrow \\ \text{Heisenberg} \\ \text{eq of motion}}}{=} -\frac{\partial H}{\partial \vec{R}} = \sum_{j=x,y,z} d_j \nabla_{\vec{R}} \left[ E_{ej}(\vec{R}, t) + E_{\perp j}(\vec{R}) \right]$$

Center of  
Wavepacket  
 $\vec{r}_G$

Vacuum field does not  
contribute

Assumption that atoms can be well localized  
(Not valid when cooling to the recoil limit)

$$M \ddot{\vec{r}}_G = \sum_j \langle d_j \rangle \nabla E_{ej}(\vec{r}_G, t)$$

Use  $\omega_{st} \approx \nu_{st}$

Damping time for external motion  $\frac{\hbar}{E_{rec}}$

$\gg$  internal motion  $\Gamma^{-1}$

Na  $\times 400$   
He triplet comparable

## Forces For $v=0$ at $r=0$

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$$\vec{E}_e(\vec{r}, t) = \hat{e} \epsilon_0 \cos(\omega_L t + \phi(r))$$

↑

Pol. independent of  $\vec{r}$

$$\nabla E e_j = e_j [\cos \omega_L t \nabla \epsilon_0 - \sin \omega_L t \epsilon_0 \nabla \phi]$$

$$\langle d_j \rangle = 2(d_{ab})_j \left[ \frac{u}{s} \cos \omega_L t - \frac{v}{s} \sin \omega_L t \right]$$

$$\vec{F} = (\hat{e} d_{ab}) \left[ \frac{u}{s} \nabla \epsilon_0 + \frac{v}{s} \epsilon_0 \nabla \phi \right]$$

cycle average

$\vec{F}_{react}$

$\vec{F}_{diss}$

$$\Omega_1 = -\frac{1}{\hbar} \hat{e} \epsilon_0 / t \quad \text{Rabi Freq.}$$

$$\vec{\alpha} = \frac{\nabla \Omega_1}{\Omega_1}$$

$$\vec{\beta} = \nabla \phi$$

$$\vec{F}_{react} = -\hbar \Omega_1 \frac{u}{s} \vec{\alpha}$$

$$\vec{F}_{diss} = -\hbar \Omega_1 \frac{v}{s} \vec{\beta}$$

# Radiation pressure force

Plane travelling wave  $\vec{\alpha} = 0$

$$\vec{E}_e = \hat{e} \Sigma_0 \cos(\omega_L t - \vec{k}_L \cdot \vec{r})$$

$$\vec{\beta} = -\vec{k}_L$$

$$F_{\text{diss}} = \underbrace{\Omega_1 \nu_{st}}_{\substack{\text{St} \\ \sigma_{bb}}} \hbar k_L = \left\langle \frac{dN}{dt} \right\rangle_{st} \hbar k_L$$

# of absorbed atoms

$$F_{\text{diss}} = \hbar k_L \frac{\Gamma}{2} \frac{\Omega_1^2 / 2}{(\omega_L - \omega)^2 + (\Gamma^2 / 4) + (\Omega_1^2 / 2)}$$

$$F_{\text{diss, max}} = \hbar k_L \cdot \frac{\Gamma}{2}$$

## Reactive Forces

simplest ex. standing wave

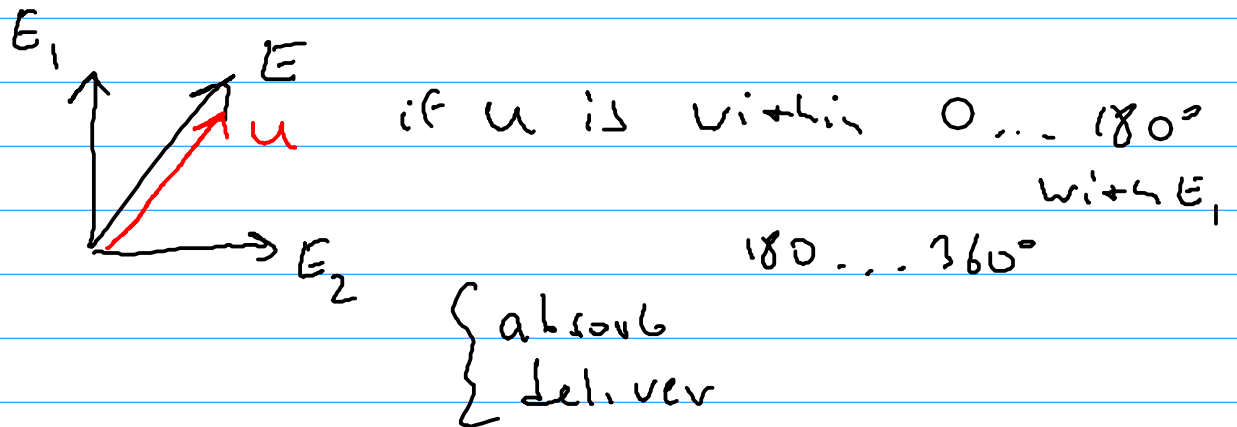
$$\vec{E}_e(\vec{r}, t) = \hat{e}_x \Sigma_0 \cos k_L z \cos \omega_L t$$

depends only on  $z \Rightarrow$  no exchange of energy

BUT: redistribution of energy and mom

SW as superposition of two TWs

pick a point where  $E_1$  and  $E_2$  are  $90^\circ$  phase shifted



$E_1$  loses energy  
 $E_2$  gains energy

$$\vec{F}_{\text{react}} = -\frac{\hbar(\omega_L - \omega_0)}{4} \nabla \left( \frac{\Omega_L^2}{(\omega_L - \omega_0)^2 + \frac{\Gamma^2}{4} + \frac{\Omega_L^2}{2}} \right)$$

Dispersive  
 red-detuning      attractive  
 blue                  repulsive

$$\vec{F}_{\text{react}} = -\nabla U$$

$$U = \frac{\hbar(\omega_L - \omega_0)}{2} \ln \left[ 1 + \frac{\Omega_L^2/2}{(\omega_L - \omega_0)^2 + \Gamma^2/4} \right]$$

dipole potential  $\Rightarrow$  dipole traps

Max Force?

For given  $\Omega_1^2$ , optimum  $|\delta_L| \sim |\Omega_1|$   
 $|\mathcal{F}_{\text{react}}| \sim \frac{\hbar |\nabla \Omega_1^2|}{\Omega_1} \sim \hbar |\nabla \Omega_1| \sim \hbar k_L \Omega_1$

momentum exchanges of  $R_L$  at rate  $\Omega_1$