5/6/2013

Dipole Forces within the Dressed-Atom Picture

Ref: • J. Dalibard, C. C.-T., JOSA B 1985
• API

Provides insight for
· Dipole traps
· Blue molasses

Remember:

• Splitting between dressed states

$$\hbar\Omega(r) = \hbar\sqrt{\Omega_1^2(r) + \delta_L^2}$$

• Steady-state populations of dressed states

$$\pi_1^{st} = \frac{\sin^4\theta}{\cos^4\theta + \sin^4\theta} \quad (\tan 2\theta = -\Omega_1/\delta_L)$$

$$\pi_2^{st} = \frac{\cos^4\theta}{\sin^4\theta + \cos^4\theta}$$

• Relaxation rate for $\pi_{1/2}$ to reach steady state

$$\Gamma_{pop} = \Gamma(\cos^4\theta + \sin^4\theta)$$

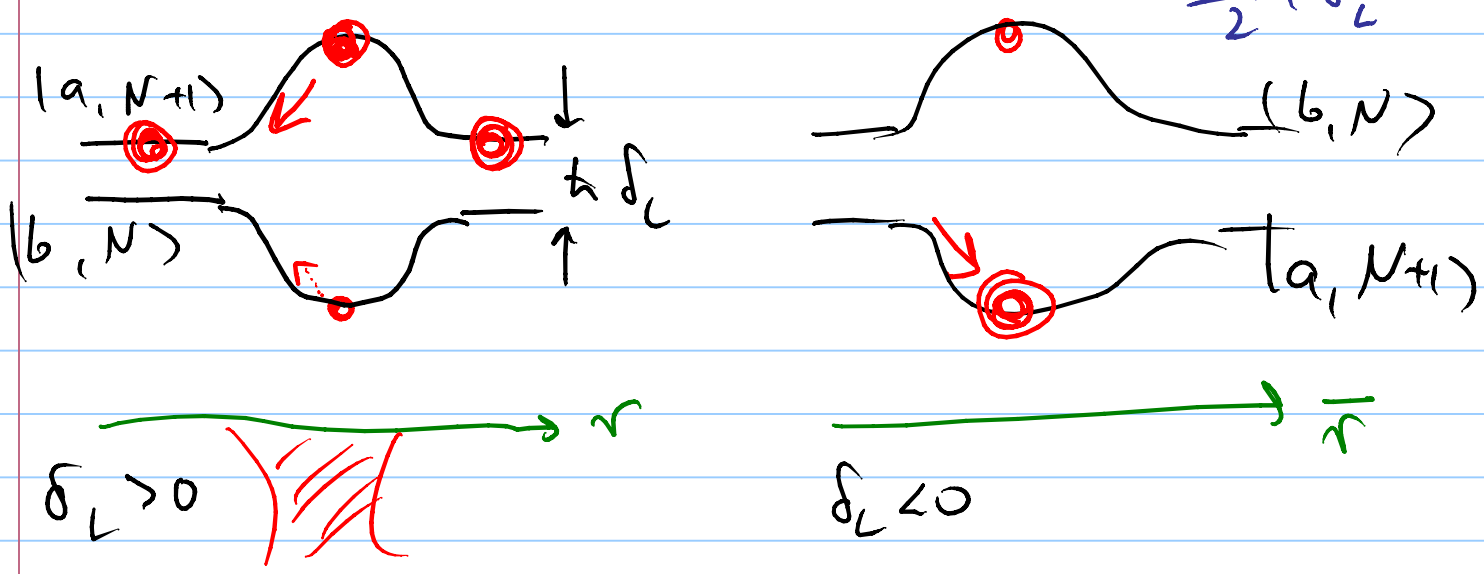
Mean dipole force, $v=0$

$$F_1 = -\frac{\hbar}{2}\nabla\Omega(r)$$

$$F_2 = +\frac{\hbar}{2}\nabla\Omega(r)$$

Force = derivative of position-dependent energy levels

$$\langle F_{dip}\rangle = F_1\Pi_1^{st} + F_2\Pi_2^{st} = -\frac{\hbar\delta_L}{2}\frac{\Omega_1^2}{\frac{\Omega_1^2}{2}+\delta_L^2}\vec{\alpha}$$



$$\delta_L = 0 \qquad \Pi_1^{st} = \Pi_2^{st} \Rightarrow \langle F_{dip}\rangle = 0$$

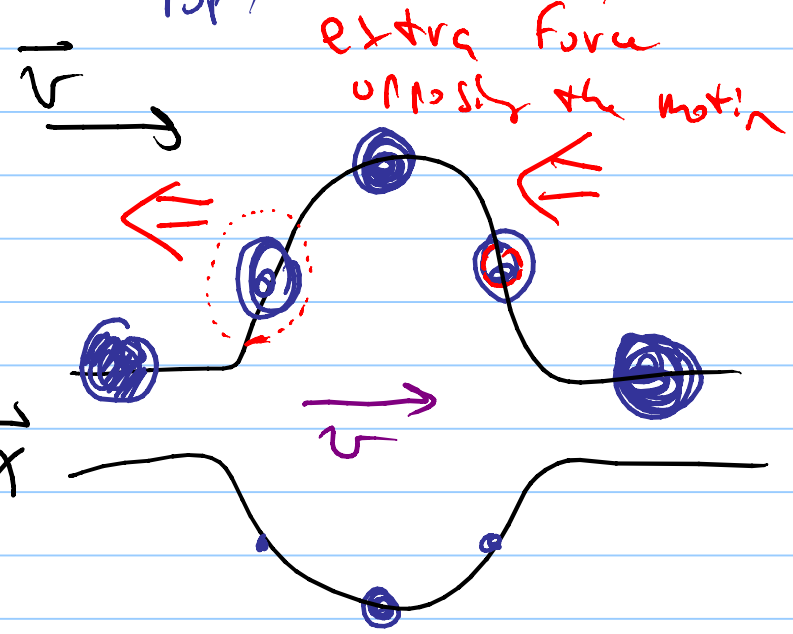
Mean dipole force for a slowly moving atom

$$\langle F_{dip}\rangle = F_1\Pi_1 + F_2\Pi_2 \qquad (1)$$

time lag by $\tau_{Pop} = 1/\Gamma_{Pop}$

$$\Pi_i \simeq \Pi_i^{st}\left(\vec{r} - \vec{v}\,\tau_{pop}\right)$$

(2)



$$\vec{F}_{dip}(\vec{r}, v) =$$

$$\vec{F}_{dip}^{st} - \frac{2\hbar\delta_L}{\Gamma}\left(\frac{\Omega_1^2(r)}{\Omega_1^2(r) + 2\delta_L^2}\right)^3 (\vec{\alpha}\,\vec{v})\,\vec{\alpha}$$

SW $\Omega_1(x) = 2\Omega\cos kx$ $\qquad \delta_L > 0$

λ-average $\Rightarrow \langle \vec{F}_{dip}^{st} \rangle = 0$

$$\langle F_{dip} \rangle = -\alpha v$$

For $\delta_L \gg \Omega$ $\qquad \vec{\alpha} \approx \vec{k}$

$$\alpha \simeq \frac{\hbar\delta_L}{\Gamma}\frac{\Omega_1^6}{\delta_L^6}k^2$$

← Will be rederived later

Atomic momentum diffusion

$$2D_p = \frac{d}{dt}\left(\langle p^2\rangle - \langle p\rangle^2\right)$$

$$= 2\left(\langle \vec{p}\,\vec{F}(0)\rangle - \langle\vec{p}\rangle\langle\vec{F}(0)\rangle\right) =$$

$$2\int_{-\infty}^{0}\left(\langle \vec{F}(t)\,\vec{F}(0)\rangle - \langle \vec{F}(t)\rangle\langle \vec{F}(0)\rangle\right)dt$$

$$D_p = \int_0^{\infty} dt\left[\langle F(0)\,F(\tau)\rangle - \langle F(0)\rangle^2\right]$$

$$\langle \dot{E}_{Heat}\rangle = D_p/M \qquad k_B T = D_p/\alpha$$

Radiative cascades

Force switches between F_1 and $F_2 = -F_1$

$$|F_1| = |F_2| = \frac{\hbar}{2}\nabla\Omega$$

on resonance: correlation time $\tau = 2/\Gamma$

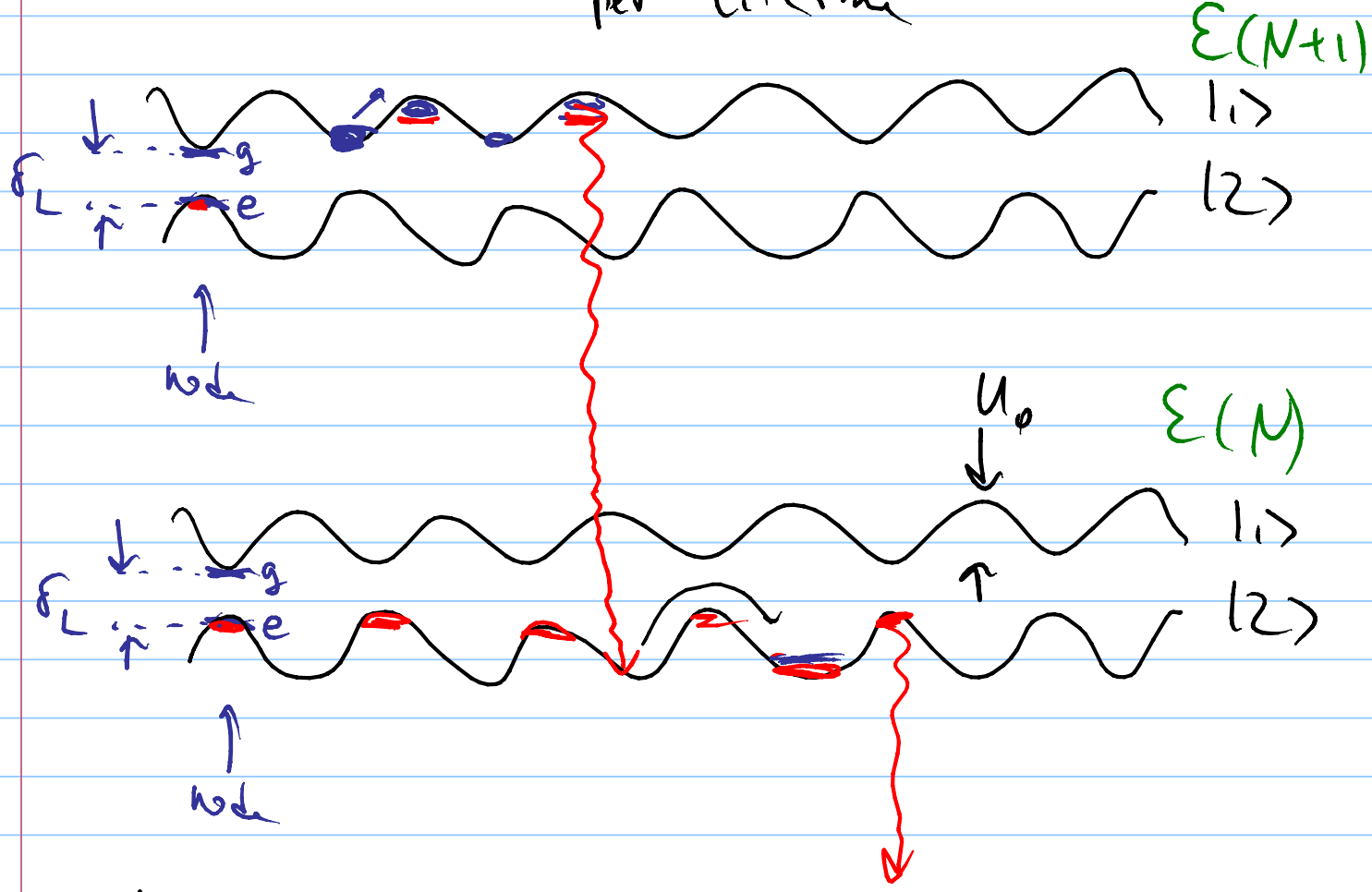
$$D_{dip} \simeq \frac{\hbar^2(\nabla\Omega_1)^2}{2\Gamma}$$

result for arbitrary detuning δ

$$D_{dip} \simeq \frac{\hbar^2(\nabla\Omega_1)^2}{2\Gamma}\left(\frac{\Omega_1^2}{\Omega_1^2+2\delta^2}\right)^3$$

Atoms moving in a standing wave

$kv \geq \Gamma$ atoms move several λ per lifetime



both $|1\rangle$, $|2\rangle$: preferential spont. em at the top of the hill

$\rightarrow$ SISYPHUS cooling

Cooling rate $\simeq U_0 \Gamma_{1\to 2}$

Cooling Atoms with Stimulated Emission

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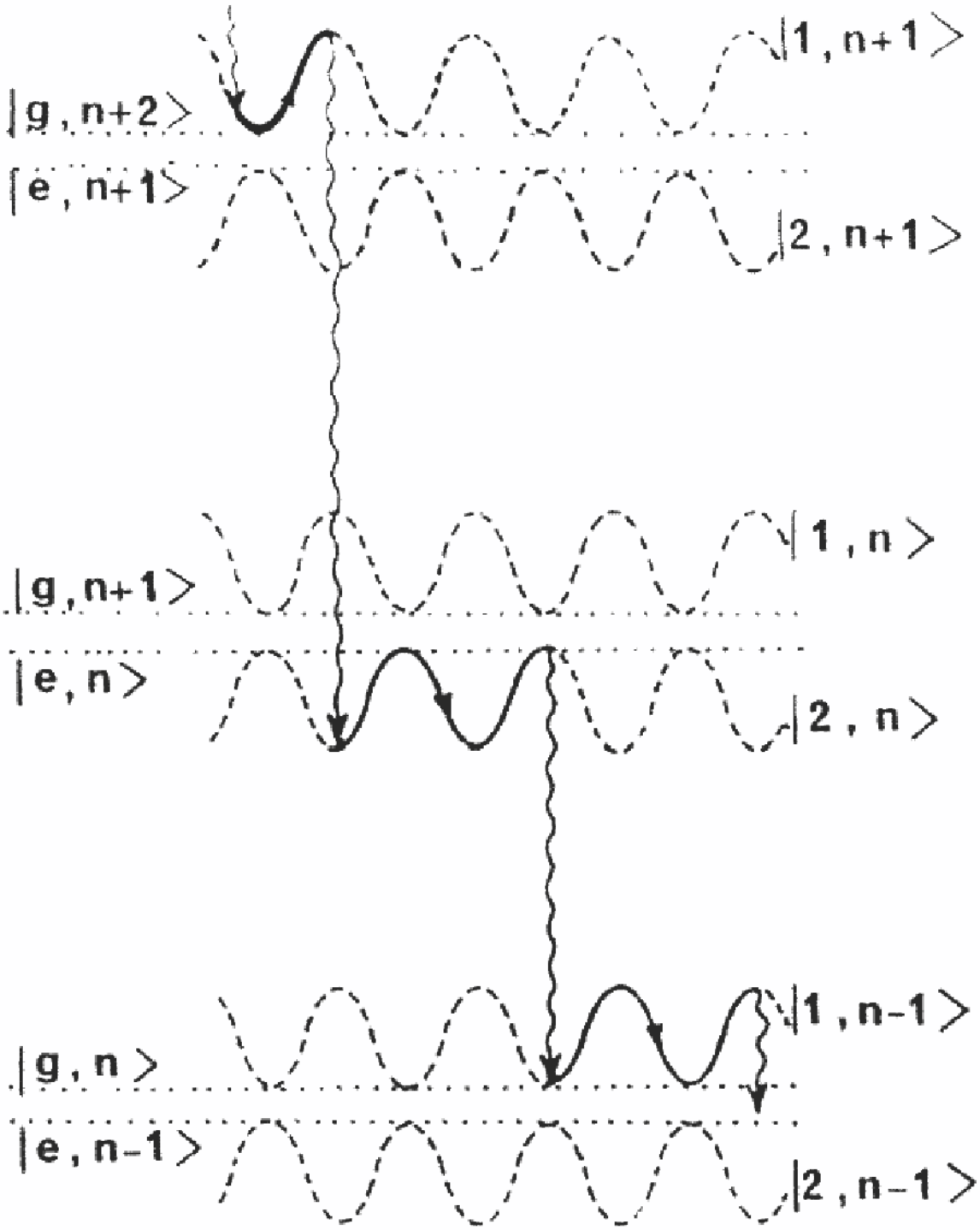


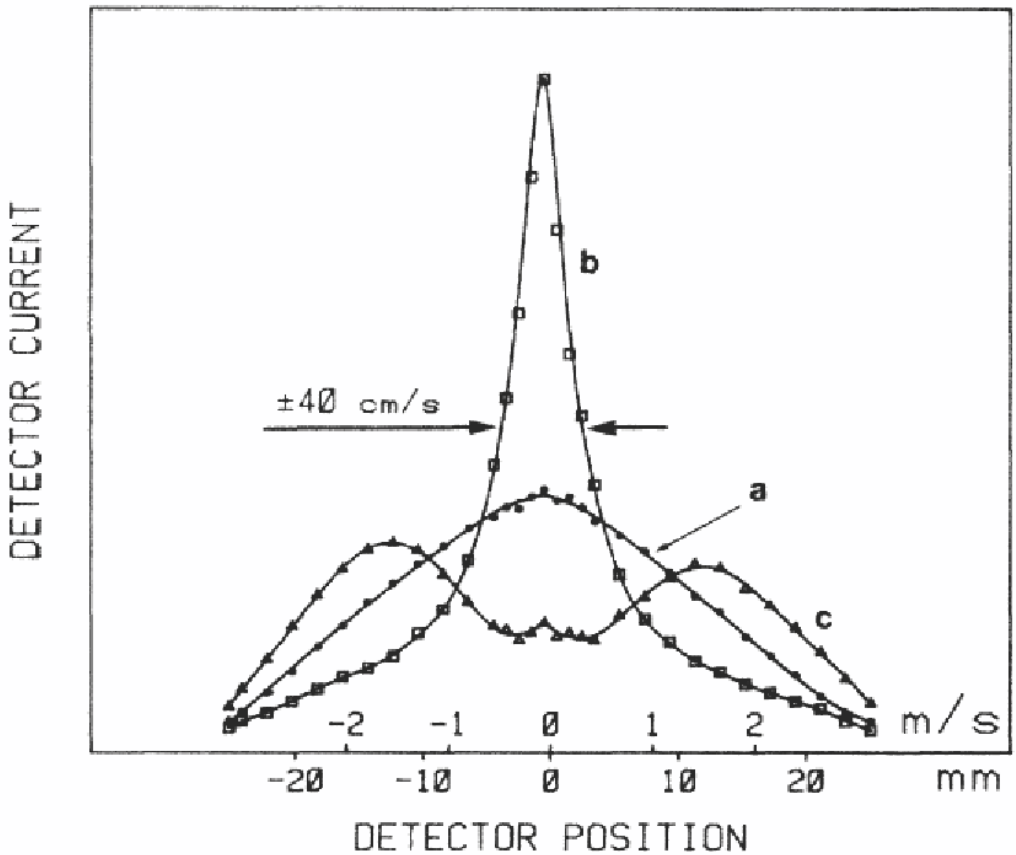


FIG. 3. Detector current vs position of the hot-wire detector. The corresponding transverse atomic velocities are given in m/s. Peak current is 2.2×10^9 atoms/s. The full lines are intended merely as visual aids. Curve *a*, laser beam off (HWHM 2 m/s); curve *b*, laser beam on with a positive detuning ($\delta/2\pi = +30$ MHz); curve *c*, laser beam on with a negative detuning ($\delta/2\pi = -30$ MHz).

Cooling in an intense standing wave

Summary of concepts in the simplest limit $\delta_L \gg \Omega \gg \Gamma$

neglect factors ≈ 1

$\hbar = 1$

$$\Omega_1(x) = 2\Omega_1 \cos kx \qquad \text{SW} \qquad \text{— b}$$

$$|1\rangle = |a\rangle + \frac{\Omega_1(x)}{\delta_L}|b\rangle \qquad \text{— a}$$

$$|2\rangle = |b\rangle - \frac{\Omega_1(x)}{\delta_L}|a\rangle$$

$$\Gamma_{1\to 2} = \Gamma\left(\frac{\Omega_1(x)}{\delta_L}\right)^4$$

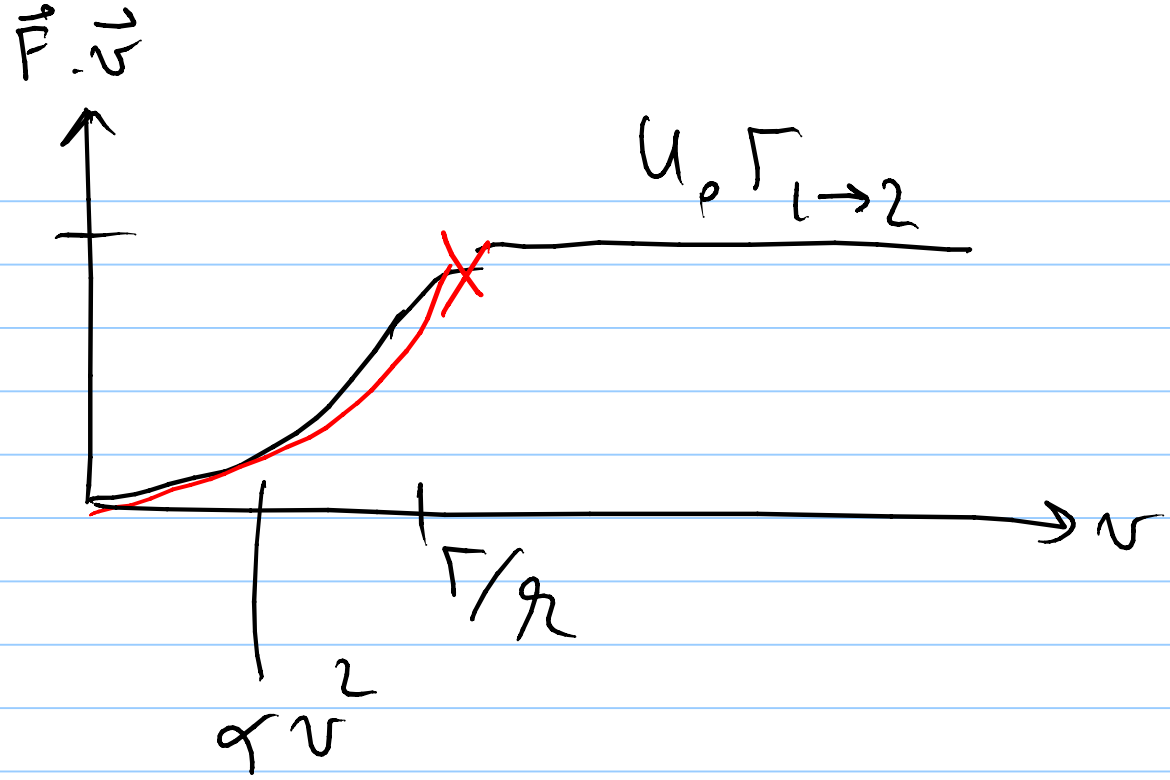
$$\Gamma_{2\to 1} = \Gamma$$

$$U_0 = \frac{\Omega_1^2}{\delta_L}$$

Dressed state potential

$v \geq \Gamma/k$ Sisyphus cooling

Cooling rate $\vec{F}\cdot\vec{v} = U_0\,\Gamma_{1\to 2}$



estimate

$$\alpha = \lim_{v\to 0} \frac{F(v)}{v} \approx \frac{U_0 \Gamma_{1\to 2}}{(\Gamma/k)^2}$$

$$\approx \frac{\Omega_1^6}{\delta_L^5} \frac{k^2}{\Gamma}$$

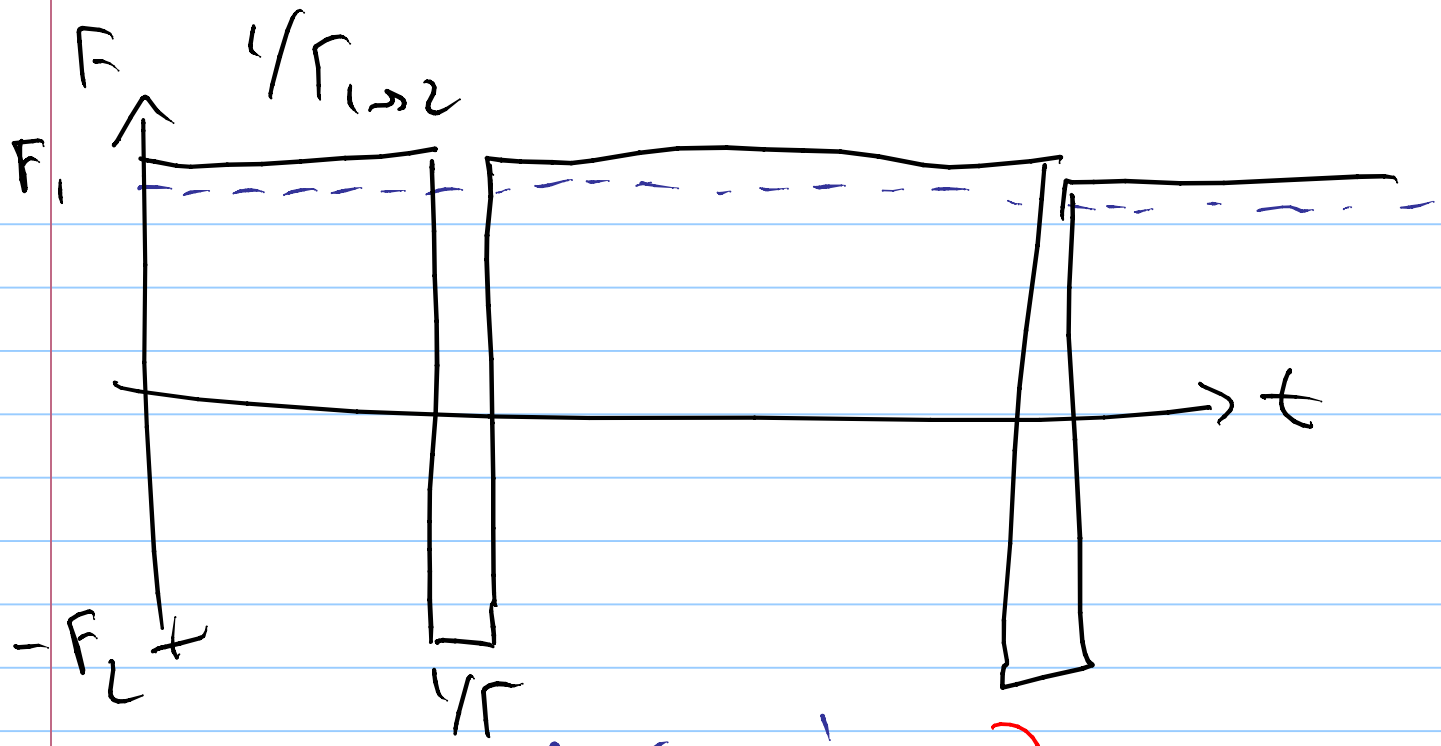
as obtained before

Diffusion

$$D = \int \langle \Delta F(0)\, \Delta F(t) \rangle \, dt$$

atoms are mainly in state $|1\rangle$

$$F_1 = -\nabla U_1(x) \approx k\, U_0$$



$$D = \underbrace{(\hbar U_0)^2}_{\text{Force}^2} \Big/ \Gamma \; \frac{\frac{1}{\Gamma}}{\frac{1}{\Gamma_{1\to 2}}}$$

Prob. that the atom is in state $|2\rangle$

$$= \hbar^2 U_0^2 \, \Gamma_{1\to 2} / \Gamma^2$$

Ultimate temperature

$$k_B T = D/\alpha = U_0$$

Note: For $U_0 < \Gamma$ we have to include heating by atomic recoil

Blue molasses cannot cool below the Doppler limit $\Gamma/2$

Discussion 1

- Dipole traps
- el. and magn forces
- energy conservation

Dipole $\left\{ \begin{array}{l} \text{traps} \\ \text{potential} \end{array} \right.$ $\quad U = \frac{\hbar \delta}{2} \log\left[1 + \frac{I/I_0}{1 + (2\delta/\Gamma)^2}\right]$

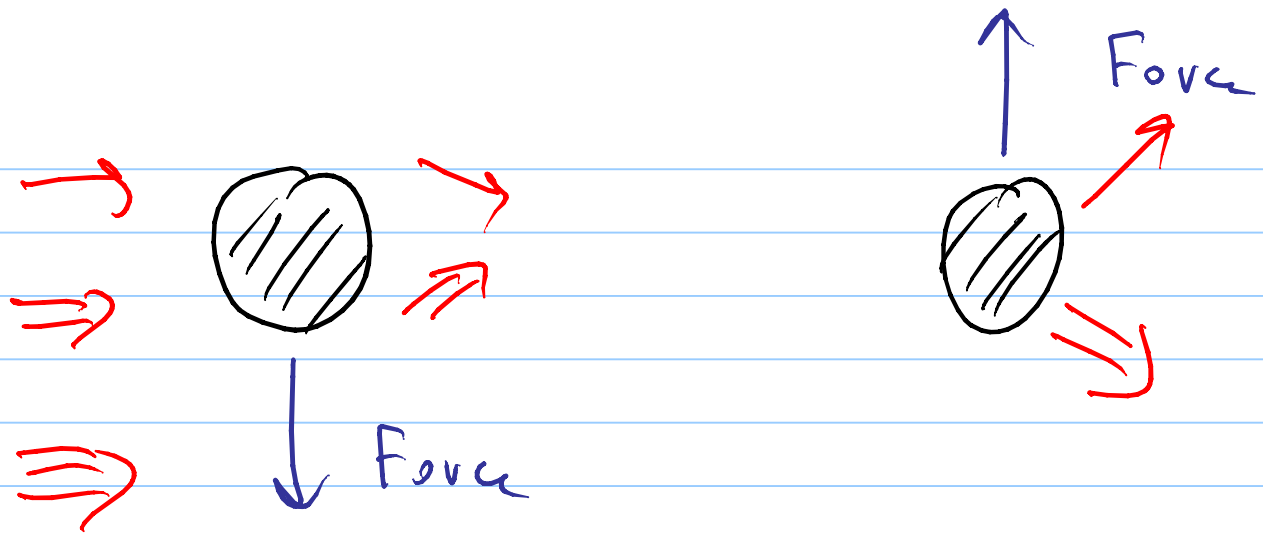
explanations:

- OBE
- dressed atom picture
- $U = -\vec{d}\,\vec{E}$

$\left.\begin{array}{l} \text{below} \\ \text{above} \end{array}\right\}$ resonance $\Rightarrow$ $\vec{d}$ $\left.\begin{array}{l} \text{in} \\ \text{out of} \end{array}\right\}$ phase with $\vec{E}$

$\Rightarrow \left\{ \begin{array}{l} \text{attractive} \\ \text{repulsive} \end{array} \right.$ forces

- macroscopic dielectric objects



$n > 1$ $\qquad\qquad$ $n < 1$

optical tweezers $\Rightarrow$ manipulate objects within a cell

$\Rightarrow$ revolution in biology (A. Ashkin, Chu)

5/8/2013

Electric vs magnetic forces

Fundamental forces:

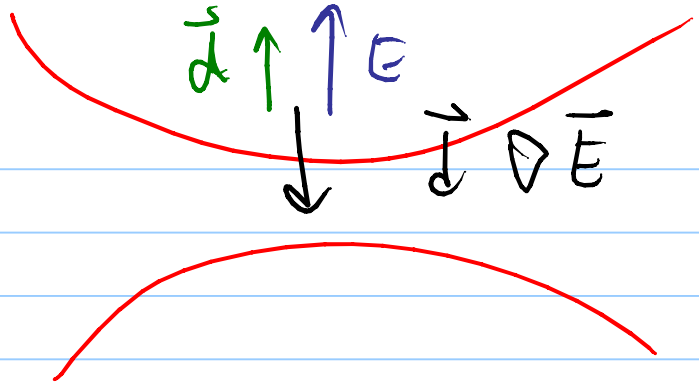
electric forces $\qquad q E$

$q_1 E_1 - q_2 E_2 \rightarrow$

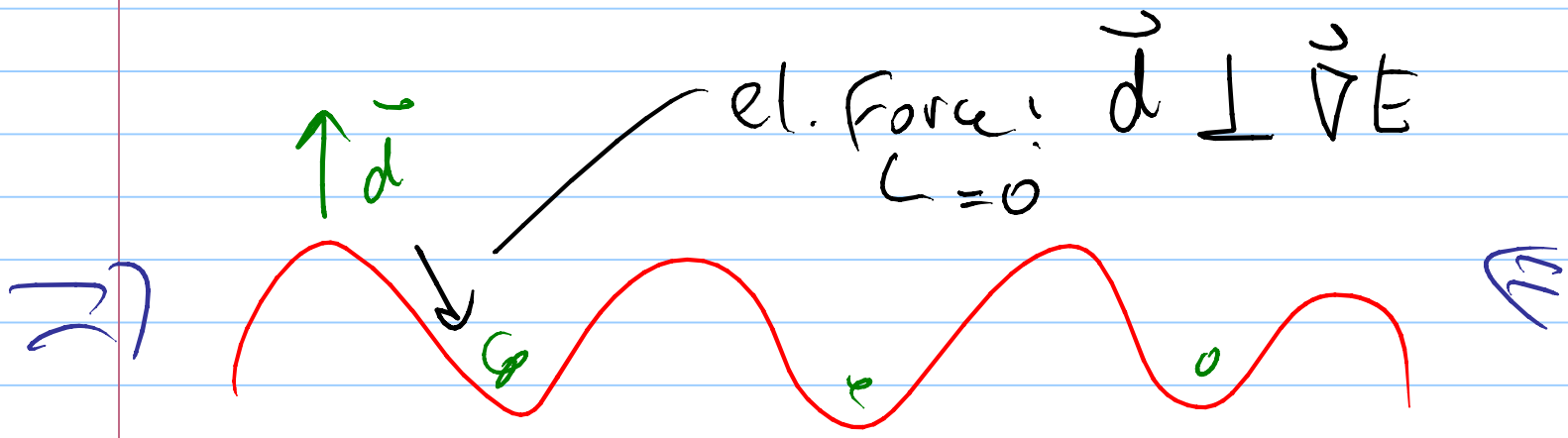
$d \nabla E$

magnetic forces $\qquad$ Lorentz Force

$\vec{v} \times \vec{B}$



electric force



$$F_{Lorentz} = \frac{d}{dz}\left(\frac{1}{2}\alpha E^2\right)$$

$v \times B$

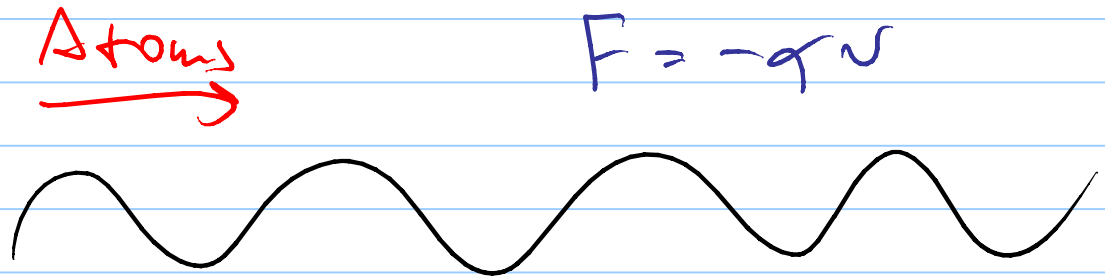
$\nabla \times A \sim kA \sim \frac{k}{\omega} E$

$\dot{d} = \alpha \dot{E} \sim \alpha \omega E$

$k \propto E^2$

Energy conservation & dipole force

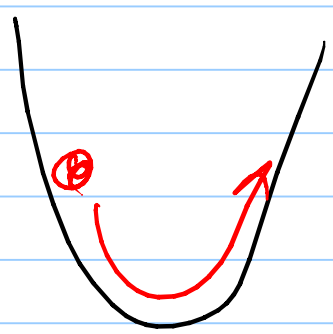
- CW Experiment



$F = -\alpha v$

upper sideband of Mollow triplet is emitted at larger Ω than the lower sideband $\Rightarrow$ energy is radiated away

- transient experiment



atom is dielectric medium

$\Rightarrow$ phase modulation of transmitted laser beam

$\Rightarrow$ frequency modulation

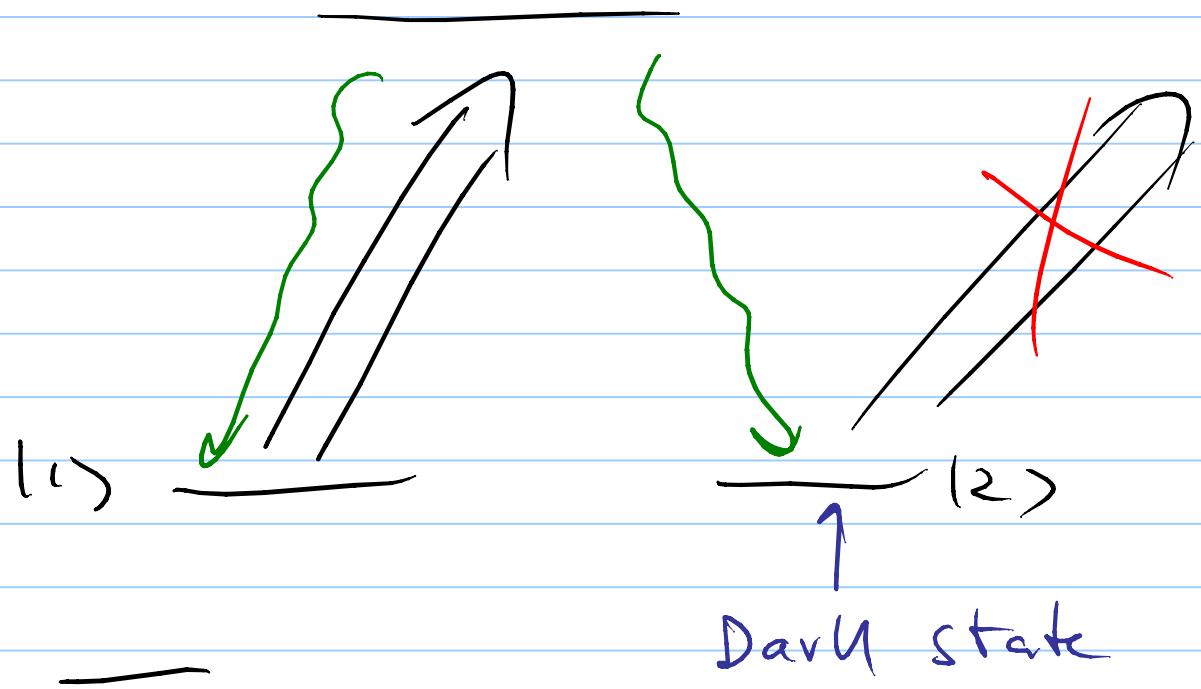
$\Rightarrow$ see HW

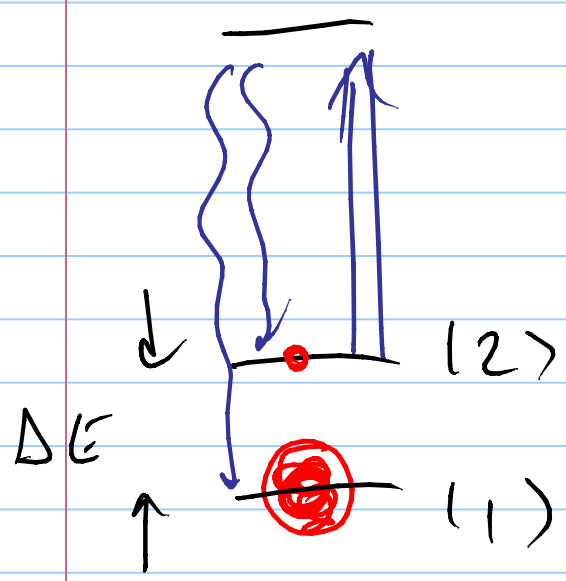
L 18

Techniques for ultralow temperatures

- Sub-Doppler, Sub-Recoil cooling
- Magnetic trapping
- Evaporative cooling

Cooling by optical pumping





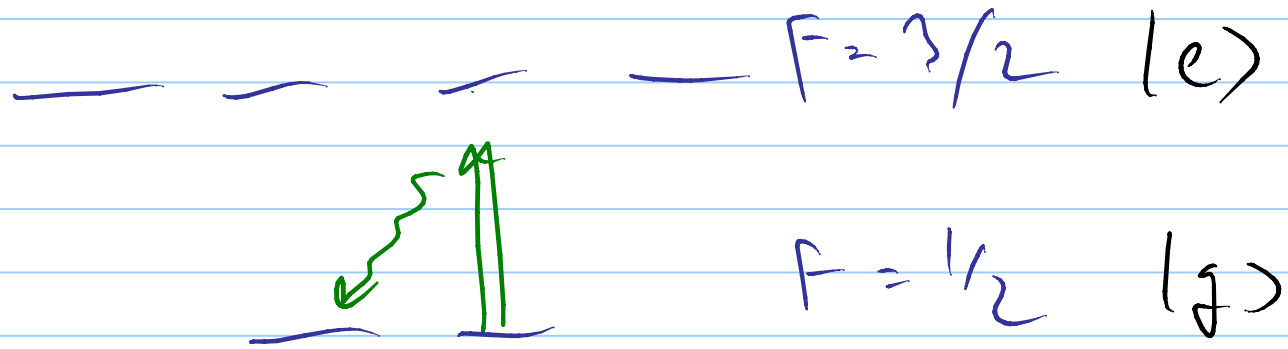
$$\frac{n_2}{n_1} = e^{-\beta \Delta E} \gg 1$$

$$n_2 \to 0 \Rightarrow T \to 0$$

Laser cooling $\hat{=}$ optical pumping in translation space

Sub-Doppler Cooling

New effects in multilevel atoms

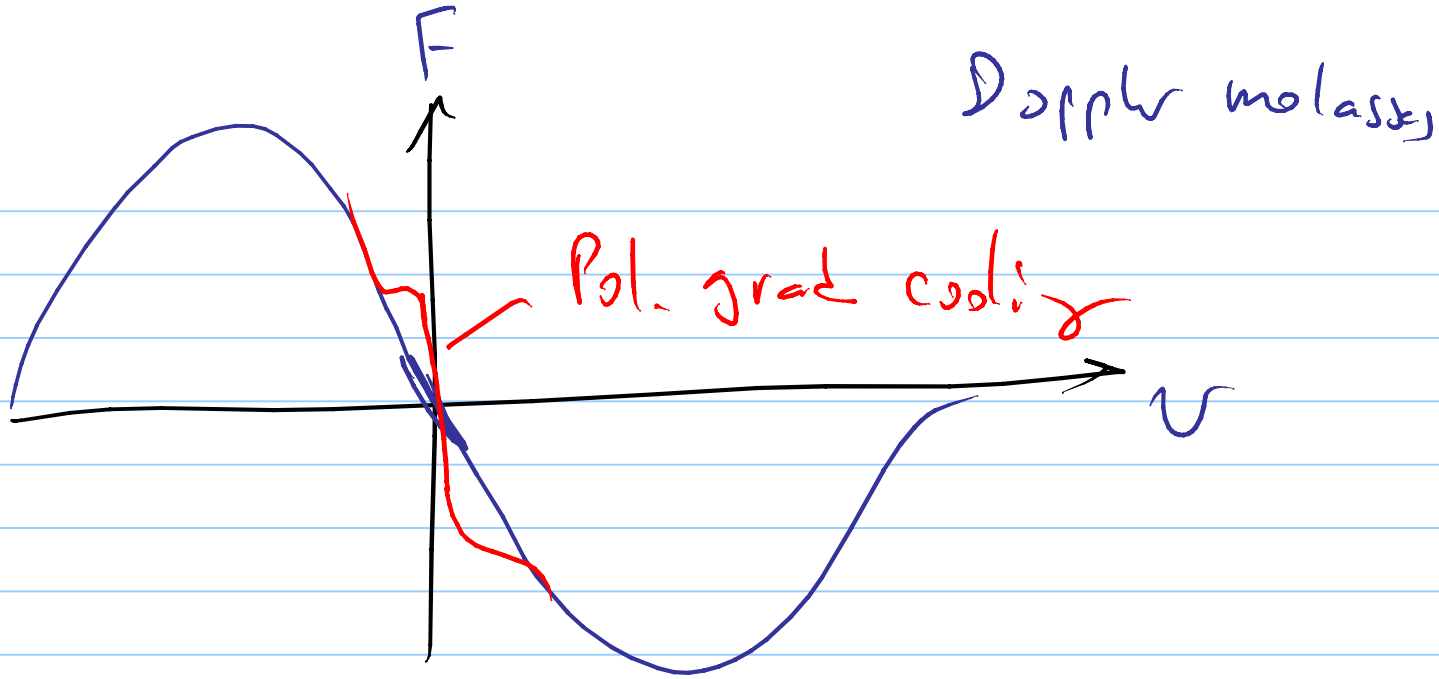


resonances between ground states

width $\Gamma' = \frac{1}{\tau_p} \sim \left(\frac{\Omega}{\delta}\right)^2 \Gamma$

- Often $k_B T_{Final} \propto \hbar \Gamma'$ e.g. Doppler cooling
  (but here $\Gamma' \frac{\Gamma}{\delta}$)

- τ_p is long $\Rightarrow$ long lag times
  $\propto \tau_{Fricr} \propto \tau_{Lag}$


F
Doppler molasses
Pol. grad cooling
v

Laser cooling below the Doppler limit by polarization gradients: simple theoretical models

J. Dalibard and C. Cohen-Tannoudji

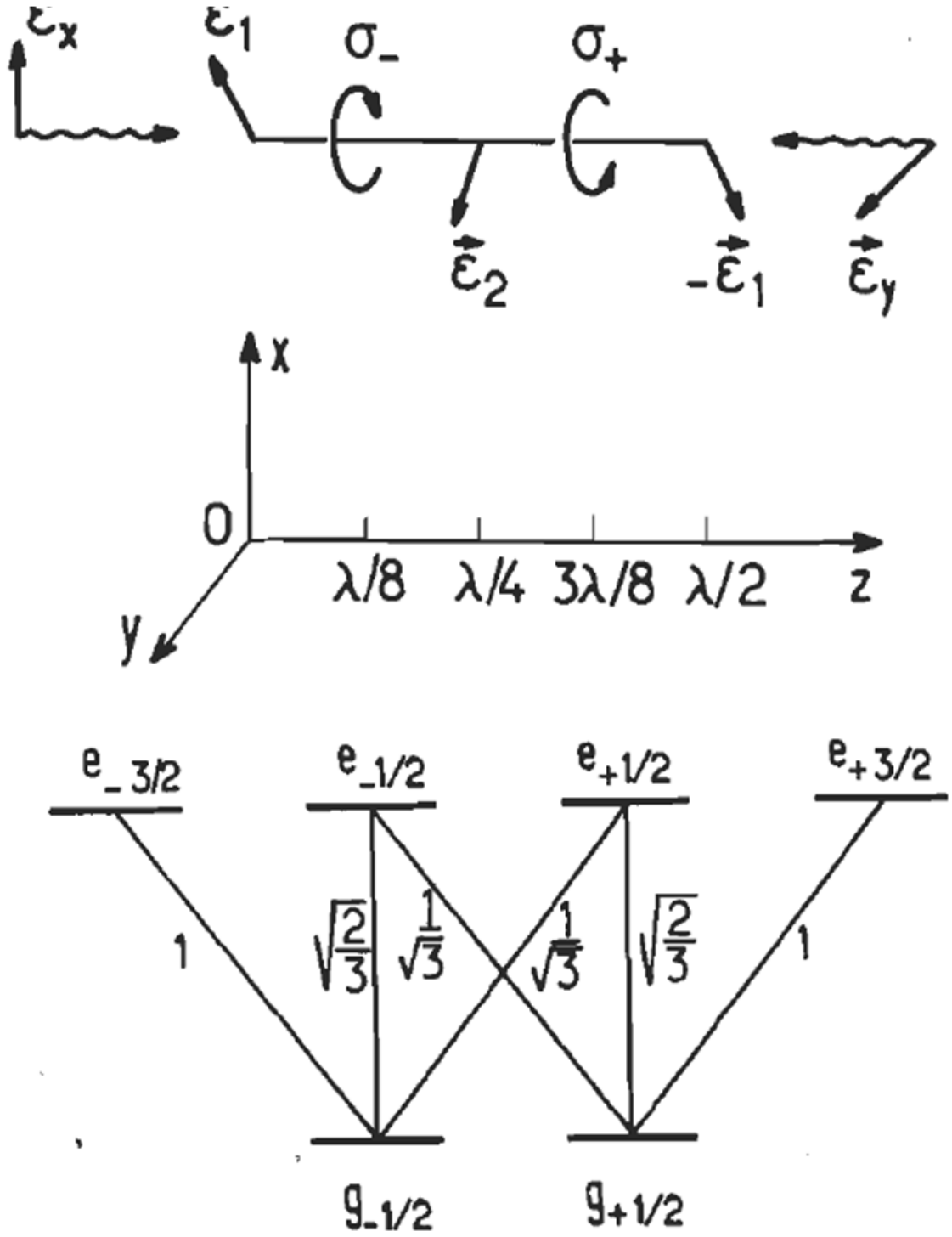


Fig. 2. Atomic level scheme and Clebsh–Gordan coefficients for a $J_g = 1/2 \leftrightarrow J_e = 3/2$ transition.

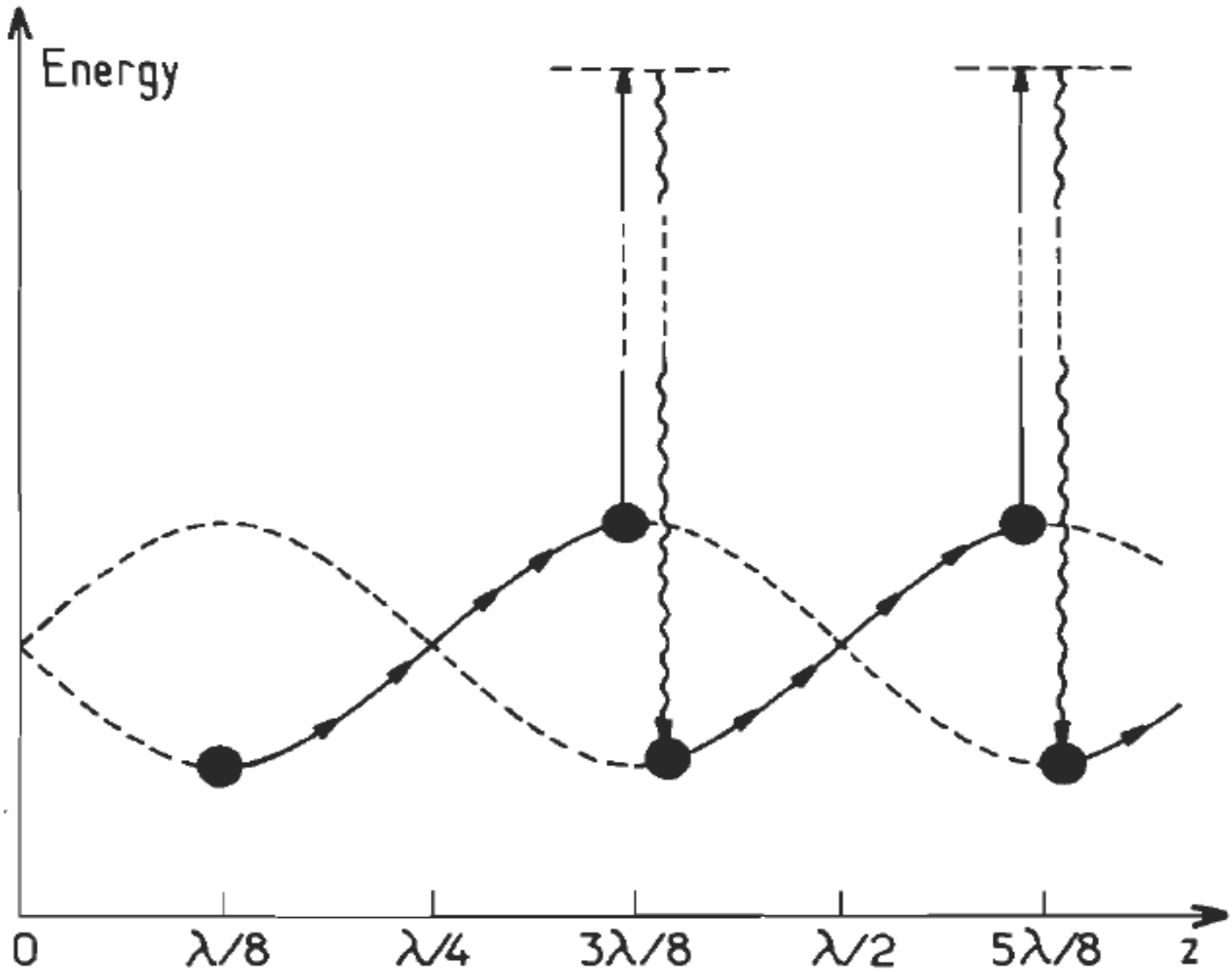


Fig. 4. Atomic Sisyphus effect in the lin ⊥ lin configuration. Because of the time lag τ_p due to optical pumping, the atom sees on the average more uphill parts than downhill ones. The velocity of the atom represented here is such that $v\tau_p \sim \lambda$, in which case the atom travels overs λ in a relaxation time τ_p. The cooling force is then close to its maximal value.

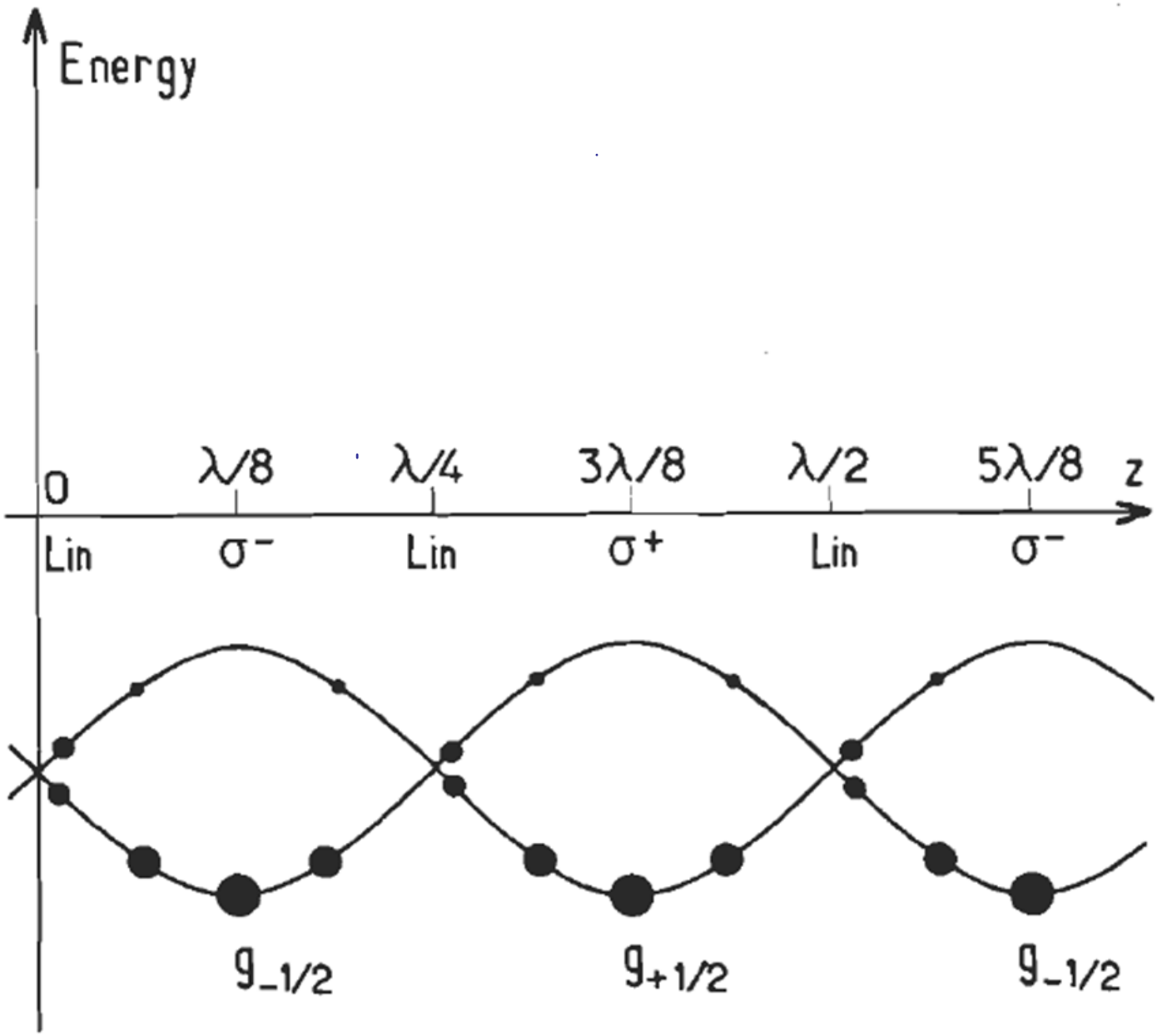


Fig. 3. Light-shifted energies and steady-state populations (represented by filled circles) for a $J_g = 1/2$ ground state in the lin $\perp$ lin configuration and for negative detuning. The lowest sublevel, having the largest negative light shift, is also the most populated one.

Sub-Recoil Cooling

Proof of impossibility

× initial energy $\frac{1}{2} m \vec{v}^2$

after abs. $\frac{1}{2}\left(m\vec{v} + \hbar \vec{k}_L\right)^2$

× after em $\frac{1}{2}\left(m\vec{v} + \hbar\vec{k}_L - \hbar\vec{k}_e\right)^2$

$\Delta E = 2E_{rec} + \hbar\left(\vec{k}_L - \vec{k}_e\right)\vec{v}$

$\overline{\Delta E} = 2E_{rec} + \hbar \vec{k}_L \cdot \vec{v} < 0$ random

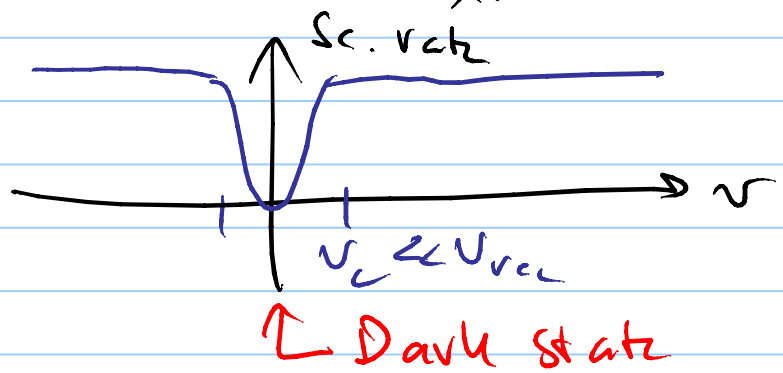
$\frac{(\hbar k)^2}{2m}$

for cooling

Cooling requires $\vec{k}$ antiparallel to $\vec{v}$

$\Rightarrow \overline{\Delta E} > 0$ when $v < v_{rec} = \frac{\hbar k}{m}$

But: If last scattered photon is not random



"Velocity space"

optical pumping

Practical implementation:

- Raman cooling
- VSCPT