Dipole Forces Within the Dressed-Atom Picture

Ref. : J. Dalibard, C. C.-T., JOSA B 1985
• API

Provides insight for
• Dipole trans
• Blue molasses

Remember:
• Splitting between dressed states
  \[ \Delta \Omega(\tau) = \hbar \sqrt{\Omega_1^2(\tau) + \delta_\Omega^2} \]

• Steady-state population of dressed states
  \[ \Pi_1^{st} = \frac{\sin^4 \omega}{\cos^4 \theta + \sin^4 \omega} \quad (\tan 2\theta = -\Omega_1/\delta_\Omega) \]
  \[ \Pi_2^{st} = \frac{\cos^4 \omega}{\sin^4 \theta + \cos^4 \omega} \]

• Relaxation rate for \( \Pi_{12} \) to reach steady state
  \[ \Gamma_{pop} = \Gamma \left( \cos^4 \theta + \sin^4 \omega \right) \]
Mean dipole force, $\nu = 0$

\[ F_i = -\frac{h}{2} \nabla \Omega (\nu) \quad \text{Force} = \text{derivative of position-dependent energy levels} \]

\[ F_2 = +\frac{5}{2} \nabla \Omega (\nu) \]

\[ \langle F_{\text{dip}} \rangle = F_1 \pi_1^s + F_2 \pi_2^s = -\frac{\delta c}{2} \frac{\Omega_1^2}{\Omega_1^2 + \delta_c^2} \]

\[ \begin{align*}
\delta_c > 0 & \quad \Rightarrow \quad \langle F_{\text{dip}} \rangle > 0 \\
\delta_c = 0 & \quad \Rightarrow \quad \langle F_{\text{dip}} \rangle = 0 \\
\delta_c < 0 & \quad \Rightarrow \quad \langle F_{\text{dip}} \rangle < 0
\end{align*} \]

\[ \langle F_{\text{dip}} \rangle = F_1 \pi_1^s + F_2 \pi_2^s \quad (1) \]

Mean dipole force for a slowly moving atom

The mean dipole force is given by

\[ \tau_{\text{par}} = \frac{1}{\Gamma_{\text{par}}} \]
\[ \Pi_i = \Pi_{i}^{\text{st}}(\vec{v} - \bar{v} T_{\text{op}}) \]

Extra force opposite to motion

\[ F_{\text{dip}}(\vec{r}, \vec{v}) = \]

\[ F_{\text{dip}}^{\text{st}} = \frac{2 \times 3}{\Gamma} \frac{\delta_{l}}{(\Omega^2_{l}(\vec{r}) - 2 \delta_{l}^2)^3} \]

\[ \Theta_i(x) = 2 \pi \cos \theta_x \quad \delta_l > 0 \]

\[ \lambda - \text{average} \implies \langle F_{\text{dip}}^{\text{st}} \rangle = 0 \]

\[ \langle F_{\text{dip}} \rangle = -\alpha \bar{v} \]

For \( \delta_l \gg \Lambda \)

\[ \alpha \approx \bar{a} \]

\[ \alpha = \frac{4 \delta_l \Omega_{l}^2}{\Gamma} \delta_{l}^6 \]

\[ \text{Atomic momentum diffusion} \]

\[ 2 D_{p} = \frac{d}{dt} \left( \langle p^2 \rangle - \langle p \rangle^2 \right) \]

\[ = 2 \left( \langle \vec{p} \cdot \vec{F}(0) \rangle - \langle \vec{p} \rangle \langle \vec{F}(0) \rangle \right) = \]
\[
2 \int_0^\infty (\vec{F}(t) \cdot \dot{\vec{F}}(0) - \langle \vec{F}(t) \rangle \langle \dot{\vec{F}}(0) \rangle) \, dt + D_p - \int_0^\infty dt \left[ \langle F(0) F(t) \rangle - \langle F(0) \rangle^2 \right] \]

\[
\langle \dot{E}_{\text{heat}} \rangle = D_p / M \\
\hbar \omega \bar{T} = D_p / \alpha
\]

**Radiative cascade**

For a switch between \( F_1 \) and \( F_2 = -F_1 \)

\[
|F_1| = |F_2| = \frac{\hbar}{2} \Delta \omega
\]

on resonance: coherency time \( \tau = 2\Gamma \)

\[
D_{\text{dip}} \geq \frac{\hbar^2 (\Delta \omega)}{2 \Gamma}
\]

result for arbitrary detuning \( \delta \)

\[
D_{\text{dip}} \geq \frac{\hbar^2 (\Delta \omega)}{2 \Gamma} \left( \frac{\Delta \omega}{\Delta \omega^2 + 2 \delta^2} \right)
\]
Atoms moving in a standing wave

$s \nu \geq \Gamma$ atoms move several $\lambda$ per lifetime

$| \psi_{(N+1)} \rangle$

Both $|1\rangle$ and $|2\rangle$ preferential spots at the top of the hill

$SISYPHUS$ cooling

Cooling rate $\approx U_0 \Gamma_{1\rightarrow 2}$
Cooling Atoms with Stimulated Emission


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FIG. 3. Detector current vs position of the hot-wire detector. The corresponding transverse atomic velocities are given in m/s. Peak current is $2.2 \times 10^9$ atoms/s. The full lines are intended merely as visual aids. Curve a, laser beam off (HWHM 2 m/s); curve b, laser beam on with a positive detuning ($\delta/2\pi = +30$ MHz); curve c, laser beam on with a negative detuning ($\delta/2\pi = -30$ MHz).
Cooling in an intense Standing Wave

Summary of concepts in the simplest limit \( \delta_L \gg \lambda \gg \Gamma \)

neglect Factors \( \ll 1 \)

\[ \hbar = 1 \]

\[ N_1(x) = 2 N_1 \cos \pi x \quad \text{SU} = b \]

\[ |1\rangle = |a\rangle + \frac{N_1(x)}{\delta_L} |b\rangle \]

\[ |2\rangle = |b\rangle - \frac{N_1(x)}{\delta_L} |a\rangle \]

\[ \Gamma_{1 \rightarrow 2} = \Gamma \left( \frac{N_1(x)}{\delta_L} \right)^4 \]

\[ \Gamma_{2 \rightarrow 1} = \Gamma \]

Dressed state potential

\[ \nu \geq \frac{\Gamma}{\delta} \quad \text{Sisyphus cooling} \]

Cooling rate \( \vec{F} \cdot \vec{v} = U_0 \Gamma_{1 \rightarrow 2} \)
\[ \alpha = \lim_{\nu \to 0} \frac{F(\nu)}{\nu} \approx \frac{U_0 \Gamma_{1 \to 2}}{(\Gamma/\chi)^2} \]

\[ \approx \frac{R_s^6 \, k}{8 \varepsilon^5 \, \gamma} \]

as obtained before

**Diffusion**

\[ D = \int \nabla \Delta F(0) \, \Delta F(t) \, dt \]

atoms are mainly in state 1(1)

\[ F_1 = -\nabla U_1(x) \approx k \, U_0 \]
\[ D = \frac{(kU_0)^2}{\Gamma} \left\{ \frac{1}{\Gamma_{1\rightarrow 2}} \right\} \]

\[ = \hbar^2 \frac{U_0^2}{\Gamma_{1\rightarrow 2}} / \Gamma^2 \]

**Ultimate temperature**

\[ k_B T = D/\pi = U_0 \]

**Note:** For \( U_0 < \Gamma \) we have to include heating by atomic recoil. 

*Blue molasses cannot cool below this limit.*
Discussion

- Dipole traps
- El. and mag. forces
- Energy conservation

\[ U = \frac{k}{2} \log \left[ 1 + \frac{\hat{I}/I_0^2}{1 + 8R/A} \right] \]

Explanations:

- OBE
- Dressed atom picture
  \[ U = -\frac{d}{dE} \] (below resonance) \( \rightarrow \) \( \frac{d}{dE} \) (above resonance)
  \[ \text{\text{phase with } } \vec{E} \text{\text{ in }} \rightarrow \text{\text{out of }} \]
  \[ \rightarrow \{ \text{attractive forces} \} \]
  \[ \{ \text{repulsive forces} \} \]

- Macroscopic dielectric objects
optical tweezers ⇒ manipulate objects within a cell ⇒ revolution in biology (A. Ashkin, Chu)

Electric vs magnetic Forces

Fundamental Forces:

electric forces

\[ q \mathbf{E} \]

\[ q_1 \mathbf{E}_1 - q_2 \mathbf{E}_2 \Rightarrow \] Lorentz Force

\[ \mathbf{F} = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \]

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\[ F_{\text{Lorentz}} = \frac{1}{\beta c^2} \left( \frac{1}{2} \alpha E^2 \right) \]

\[ U \approx \begin{cases} \dot{\sigma} \times A \sim \hbar A \sim \frac{\hbar}{\omega} E \\ i = \dot{\alpha} E \sim \alpha \omega E \end{cases} \] \[ \beta \propto E^2 \]
Energy conservation & dipole force

- CW Experiments

\[ \text{Atoms} \rightarrow F = -\mathbf{qV} \]

upper sideband of H-atom triplet is emitted at larger \( \Delta \) than the lower sideband \( \Rightarrow \) energy is radiated away

- Transient experiments

atom is dielectric medium
\( \Rightarrow \) phase modulation of transmitted laser beam
\( \Rightarrow \) frequency modulation
\( \Rightarrow \) see HW
Techniques for ultralow temperatures

- Sub-Doppler, Sub-Recoil cooling
- Magnetic trapping
- Evaporative cooling
Cooling by optical pumping

Laser cooling ≠ optical pumping in translation space
Sub-Doppler Cooling

New effects in multilevel atoms

\[ \left| \mathcal{F} = \frac{3}{2} \right> \]

\[ \mathcal{F} = \frac{1}{2} \left| \mathcal{L} > \right> \]

Resonances between ground states

Width \[ \Gamma' = \frac{1}{T_p} \sim \left( \frac{\Omega}{\delta} \right)^2 \Gamma \]

\[ \Omega_{\sqrt{3}} \Gamma' \ll \Gamma \] is Doppler cooling

\[ \text{but here } \Gamma' \ll \frac{\delta}{\hbar} \]

\[ T_p \text{ is long } \Rightarrow \text{ long laser times} \]

Opt. & T

Friar & Clay
Doppler molasses

\[ F \]

Polarization cooling

\[ \theta \]
Laser cooling below the Doppler limit by polarization gradients: simple theoretical models

J. Dalibard and C. Cohen-Tannoudji

Fig. 2. Atomic level scheme and Clebsch–Gordan coefficients for a $J_g = 1/2 \leftrightarrow J_e = 3/2$ transition.
Fig. 4. Atomic Sisyphus effect in the lin- lin configuration. Because of the time lag $\tau_p$ due to optical pumping, the atom sees on the average more uphill parts than downhill ones. The velocity of the atom represented here is such that $v \tau_p \sim \lambda$, in which case the atom travels overs $\lambda$ in a relaxation time $\tau_p$. The cooling force is then close to its maximal value.
Fig. 3. Light-shifted energies and steady-state populations (represented by filled circles) for a $J_g = 1/2$ ground state in the lin $\perp$ lin configuration and for negative detuning. The lowest sublevel, having the largest negative light shift, is also the most populated one.
Sub-Relativistic Cooling

**Proof of impossibility**

- **Initial energy** \( \frac{1}{2} m \vec{v}^2 \)

- After absorption,
  \( \frac{1}{2} \left( m \vec{v} + e \vec{A}_e \right)^2 \)

- After emission,
  \( \frac{1}{2} \left( m \vec{v} + e \vec{A}_e - e \vec{A}_e \right)^2 \)

\[ \Delta E = 2 E_{\text{rec}} + \frac{(e \vec{A}_e \cdot \vec{v})^2}{2m} \]

For cooling, \( \vec{A}_e \) is parallel to \( \vec{v} \)

\[ \Rightarrow \Delta E > 0 \text{ when } v < v_{\text{rec}} \approx \frac{e \vec{A}_e}{m} \]
But: if last photon is not random

\[ v < \nu_{\text{sec}} \]


\[ \downarrow \text{Dark state} \]

\[ \uparrow \text{Sc. vch} \]

\[ \text{Velocity space} \]

\[ \text{optical pumping} \]

Practical implementation:
- Raman cooling
- VS CPT