2/27/2013

Single Photons

I Phase shifters & beam splitters
II Mach-Zehnder interferometers
III Non-linear MZ int.
(IV Quantum algorithms)

$|0\rangle$ = no photon

$|1\rangle$ = single photon state

What happens when these states pass through optical components?

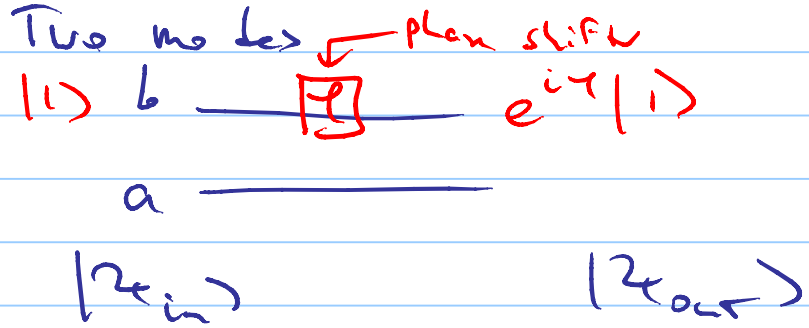
I Phase shifter & beam splitters

$$|1\rangle \xrightarrow[\text{Medium}]{\text{Time}} e^{i\omega_0 t}|1\rangle$$

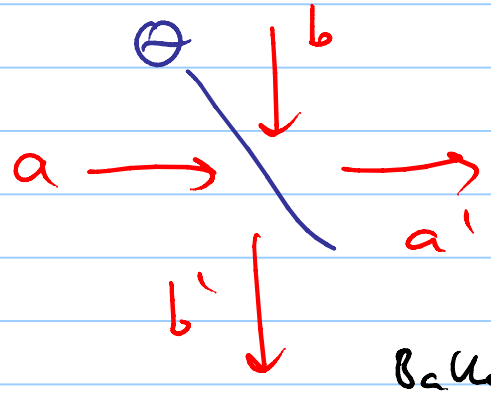
$$e^{i\omega_0 t + i\varphi}|1\rangle \Rightarrow \text{phase shift}$$

Phase has to be compared to a reference

$\Rightarrow$ Two modes



$\Rightarrow$ Combine modes



$$H = i\theta\left(a b^{+} - a^{+} b\right)$$

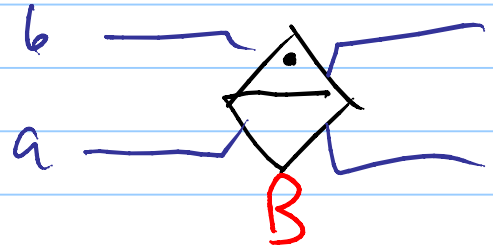
$$B = e^{iH} = \exp\left(\theta\left(a^{+} b - b^{+} a\right)\right)$$

Baker-CH Formula

$$B\, a\, B^{+} = a\cos\theta + b\sin\theta = a'$$

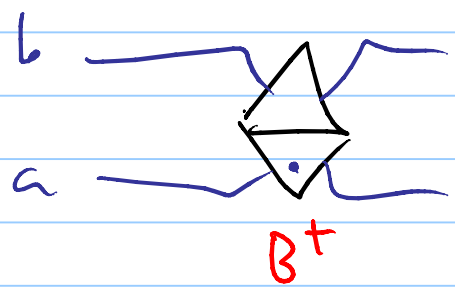
$$B\, b\, B^{+} = -a\sin\theta + b\cos\theta = b'$$

$\theta = \frac{\pi}{4}$



$$b' = \frac{b-a}{\sqrt{2}}$$

$$a' = \frac{b+a}{\sqrt{2}}$$



$$\frac{a+b}{\sqrt{2}}$$

$$\frac{a-b}{\sqrt{2}}$$

3/1/2013

Matrix rep,

$$\begin{bmatrix} a \\ b \end{bmatrix} \longrightarrow \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{“}B\text{”}} \begin{bmatrix} a \\ b \end{bmatrix}$$

Single photons $\overset{b\ a}{|1\ 0\rangle}$ mode b

$|0\ 1\rangle$ a

$$B\underbrace{\overset{b\ a}{|1\ 0\rangle}}_{|1\rangle\otimes|0\rangle} = (B\ b^{+}B^{+})\underbrace{B|0\ 0\rangle}_{|0\ 0\rangle} =$$

$|1,0\rangle$

$$B|0\ 1\rangle = -\sin\theta\ |0\ 1\rangle + \cos\theta\ |1\ 0\rangle$$

$$= \cos\theta \qquad \sin\theta$$

B conserves the photon number

$$B|1\ 1\rangle = -\sqrt{2}\sin\theta\cos\theta\ |0\ 2\rangle$$
$$+\sqrt{2}\sin\theta\cos\theta\ |2\ 0\rangle$$
$$+(\cos^2\theta - \sin^2\theta)\ |1\ 1\rangle$$

Restrict our attention to $\{\cancel{|2,0\rangle}\ \underline{|0,1\rangle\ |1,0\rangle}\}$

always have ONE photon

"Dual-rail photon state space"

spanned by $|0\ 1\rangle$ and $|1\ 0\rangle$

Two-level system

Arbitrary state $|\psi\rangle = \alpha|0\ 1\rangle + \beta\ |1\ 0\rangle$

Theorem: Any $|\psi\rangle$ can be created from $|0\ 1\rangle$ by beamsplitters and phase shifters.

<u>Proof</u> $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$B_\theta |\psi\rangle = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Rotation around $\hat{y}$

Phase shifter $\underbrace{e^{-i\varphi/2}}_{\text{irrelevant global phase}} \underbrace{\begin{bmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2} \end{bmatrix}}_{\text{Rotation around } \hat{z}} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$B(\theta) = R_y(-2\theta) \qquad P(\varphi) = R_z(-\varphi)$$

$$U = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

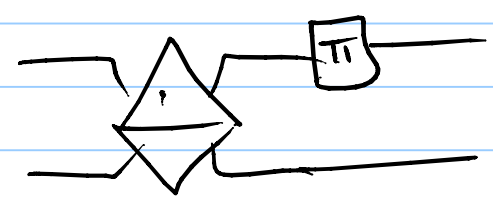
Modern language

qubit $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

arbitrary single qubit operation ("gates") can be performed by phase-shifters and beam splitters.

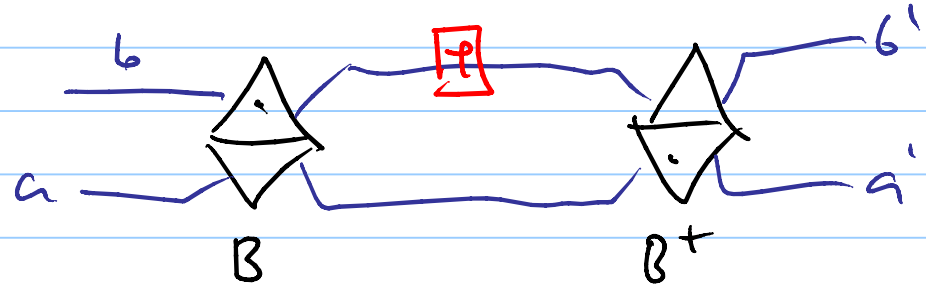
<u>ex:</u> Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



II. Mach-Zehnder Interferometer

Dual-rail photon representation of a qubit allows us to discuss interferometers as gates

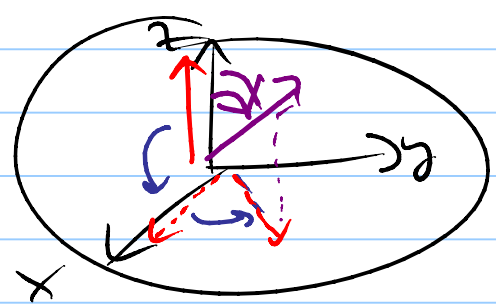


$B^+ B = I$

$$|\text{out}\rangle = B^+ \; P \; B \; |\text{in}\rangle$$

$$= R_y\left(\tfrac{\pi}{2}\right) \; R_z(-\varphi) \; R_y\left(-\tfrac{\pi}{2}\right) |\text{in}\rangle$$

$$= R_x(-\varphi)\,|\text{in}\rangle$$



$\varphi = 0$ balanced

$\varphi = \pi$ swap modes inverts qubit

Nonlinear MZ Interferometer

Linear optics $\vec{P} = \varepsilon_0 \chi \vec{E}$

pol. tensor

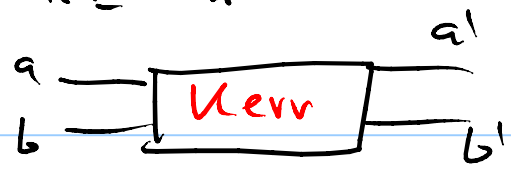
Non linear $\varepsilon_0 \left(\chi^1 E + \underbrace{\chi^2 E^2}_{\text{OPO}} + \underbrace{\chi^3 E^3}_{\text{Kerr}} \ldots \right)$

$H_{XPM} = -\chi \, a^+ a \, b^+ b$ cross-phase modulation Hamiltonian

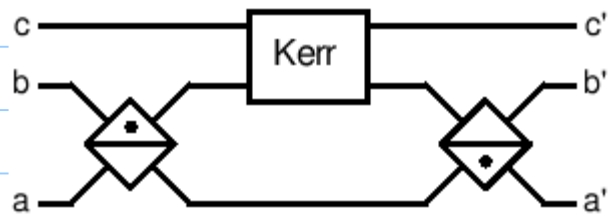
Crystal of length L

$K = e^{-iHL} = e^{i\chi L a^+ a b^+ b}$

choose $\chi L = \pi$



$$K|00\rangle = |00\rangle$$
$$K|01\rangle = |01\rangle$$
$$K|10\rangle = |10\rangle$$
$$K|11\rangle = e^{i\chi L}|11\rangle = -|11\rangle$$



$|c\rangle = |0\rangle \qquad a', b' \to a, b$

$\quad\;\; = |1\rangle \qquad a', b' \to b, a$

SWAP

$$|\text{OUT}\rangle = B^{+}_{ab}\, \underbrace{K_{bc}}_{e^{i\chi L b^{+}b c^{+}c}}\, B_{ab}\, |\text{IN}\rangle$$

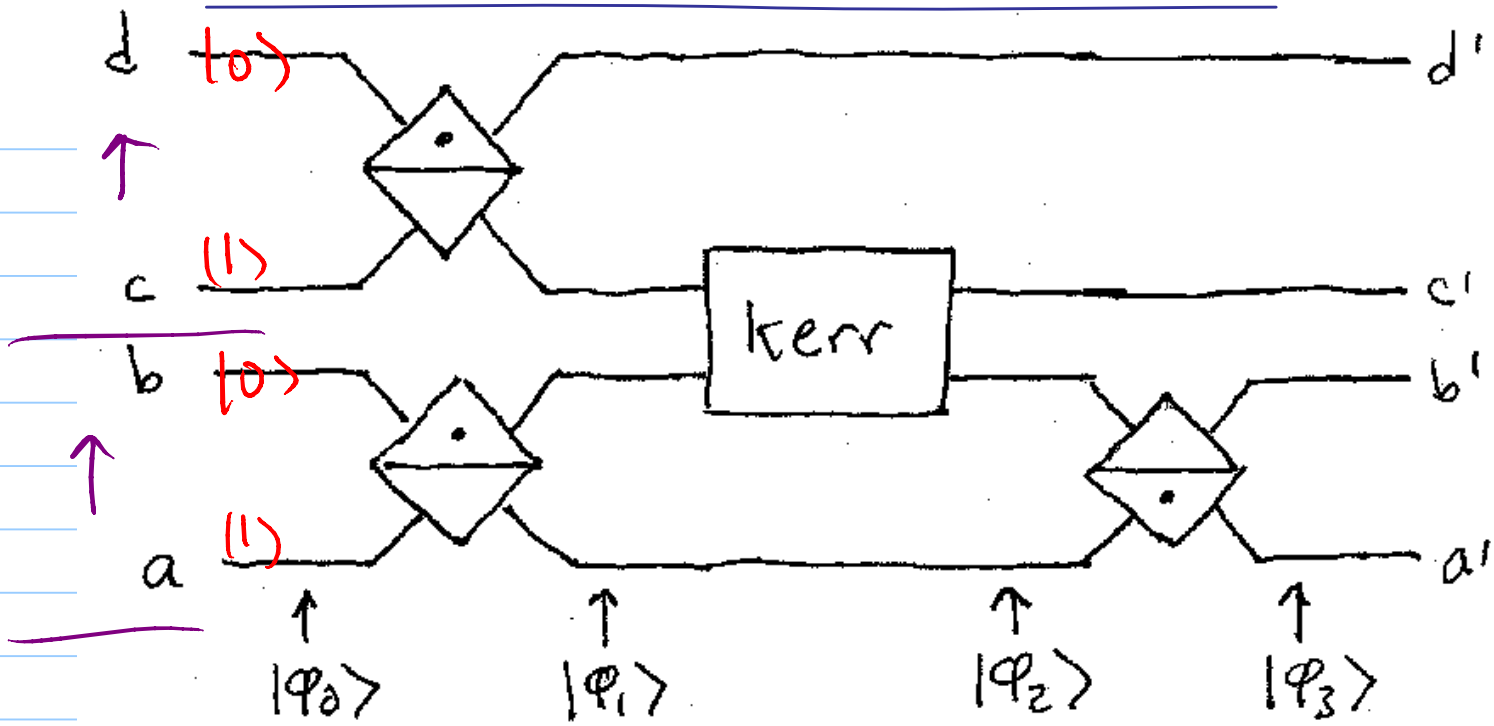
$$e^{i\chi L c^{+}c\left(\frac{b^{+}-a^{+}}{\sqrt{2}}\right)\left(\frac{b-a}{\sqrt{2}}\right)}|\text{IN}\rangle$$

$$= \ldots\; e^{i\underbrace{\xi c^{+}c}_{\text{CONTROL}}\,(a^{+}b - b^{+}a)/2}\; \ldots$$

Phase shifts ($\to$ see Wikis)

beam splitter with rotation angle $\xi c^{+}c$

Creation of entangled states



$|dcba\rangle$, then the state after the first two 50/50 beamsplitters is

$$\begin{aligned} |\phi_1\rangle &= (|01\rangle + |10\rangle)(|01\rangle + |10\rangle) \\ &= |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle \,, \end{aligned}$$

up to a normalization factor which we shall suppress for clarity. The Ker

$$|\phi_2\rangle = |0101\rangle - |0110\rangle + |1001\rangle + |1010\rangle \,.$$

Finally, the output state, given by applying $B^\dagger$ to modes a and b, is

$$|\phi_3\rangle = \frac{|1001\rangle + |0110\rangle}{\sqrt{2}} \,. = \frac{(\uparrow\downarrow) + (\downarrow\uparrow)}{\sqrt{2}}$$

ENTANGLED STATE !