2/22/2013  

Single Photons

I  Phase shifters & beam splitters
II  Mach – Zehnder interferometer
III  Non-linear MZ inst.
IV  Quantum algorithms

\[ |0\rangle = \text{no photon} \]

\[ |1\rangle = \text{single photon state} \]

What happens when these states pass through optical components?

I  Phase shifters & beam splitters

\[ |1\rangle \xrightarrow{\text{Time}} e^{i\omega t}|1\rangle \]

\[ \text{Medium} \quad e^{i\omega t_1 + \phi}|1\rangle \rightarrow \text{Phase shift} \]

Phase has to be compared to a reference

\[ \Rightarrow \text{Two modes} \xrightarrow{\text{Phase shift}} \]

\[ |1\rangle \xrightarrow{\text{a}} |1\rangle \]

\[ |1\rangle \xrightarrow{\text{b}} |1\rangle \]

12 in 12 out
Combine modes

\[ H = i \Theta (a b^+ - a^+ b) \]

\[ B = e^{iH} = \exp(i(\Theta (a b^+ - a^+ b))) \]

Baller-ChT Formula

\[ a b^+ = a \cos \theta + b \sin \theta = a' \]

\[ b b^+ = -a \sin \theta + b \cos \theta = b' \]

\[ \Theta = \frac{\pi}{4} \]

\[ b = \frac{b - a}{\sqrt{2}} \]

\[ a' = \frac{b + a}{\sqrt{2}} \]

\[ \frac{a + b}{\sqrt{2}} \]

\[ a' = \frac{a - b}{\sqrt{2}} \]

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Matrix rep,

\[ \begin{bmatrix} a \\ b \end{bmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \]
Single photons

\[ |10 \rangle \quad \text{mod} \quad b \]
\[ |01 \rangle \quad \text{mod} \quad a \]

\[ b^a |10 \rangle = (b + b^*) b |00 \rangle = |100 \rangle \]

\[ |11 \rangle \otimes |0 \rangle \]

\[ |1,0 \rangle = -\sin \theta |01 \rangle + \cos \theta |10 \rangle \]

\[ b |101 \rangle = \cos \theta - \sin \theta \]

\[ \text{B conserves the photon number} \]

\[ b |111 \rangle = -\sqrt{2} \sin \theta \cos \theta |02 \rangle \]

\[ -\sqrt{2} \sin \theta \cos \theta |20 \rangle \]

\[ + (\cos^2 \theta - \sin^2 \theta) |11 \rangle \]

Restrict our attention to \{ |00 \rangle, |10 \rangle, |11 \rangle \} always have \text{ONE photon}

"Dual-rail photon state space

spanned by \{ |01 \rangle \text{ and } |10 \rangle \}

Two-level system

Arbitrary state \( |\psi \rangle = \alpha |01 \rangle + \beta |10 \rangle \)

Theorem: Any \( |\psi \rangle \) can be created from \( |01 \rangle \)

by beamsplitters and phase shifters.
Proof: \( |2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \)

\[
B_\theta |2\rangle = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

**Rotation around \( y \)**

**Phase shifter**

\[
\begin{bmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{bmatrix}
\]

**Irrelevant global phase**

**Rotation around \( z \)**

\[B(\theta) = R_y(-2\theta) \quad P(\tau) = R_z(-\tau)\]

\[U = e^{i\varphi} R_z(\beta) R_y(\gamma) R_z(\delta)\]

**Modern language**

**qubit** \( \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \)

An arbitrary single qubit operation ("gate") can be performed by phase-shifters and beam splitters.

**Ex:** Hadamard gate

\[
H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]
II. Mach-Zehnder Interferometer

Dual-rail photon representation of a qubit allows us to discuss interferometers as gates.

\[
|\text{out}\rangle = B^+ \mathbf{P} \mathbf{B} |\text{in}\rangle
\]

\[
= R_y(\frac{\pi}{2}) \quad R_x(-\xi) \quad R_y(\frac{\pi}{2}) |\text{in}\rangle
\]

\[
= R_x(-\xi) |\text{in}\rangle
\]

- \( \xi = 0 \) balance
- \( \xi = \pi \) swap mode
- \( \xi = \pi \) inverts qubit

Nonlinear MT Interferometer

Linear optics \( \bar{\rho} = \varepsilon_0 \times \bar{E} \)

Nonlinear \( \varepsilon_0 \left( x^1 E + x^2 E^2 + x^3 E^3 \right) \)

\( H_{x \text{ph}} = - x (a^+ + b^+ b) \) cross phase modulation

\( K = e^{i \frac{2 \pi}{L} x L} = e \)

\( \text{OPO} \) \( \text{Ueven} \)
Choose $x L = \pi$

\[ U_{\text{Kerr}} \]

\[ U |00\rangle = |00\rangle \]
\[ U |01\rangle = |01\rangle \]
\[ U |10\rangle = |10\rangle \]
\[ U |11\rangle = e^{i x L} |11\rangle = -|11\rangle \]

\[ l_{\text{out}}) = b^+_{al} K_{bc} B_{al} |11\rangle \]

\[ = e^{i x L} |c+c\rangle \left( \frac{b^+ - a^+}{\sqrt{2}} \right) \left( \frac{b - a}{\sqrt{2}} \right) |11\rangle \]

\[ = e^{i \frac{\sqrt{3}}{2} c^+ c} (a^+ b - b^+ a)/2 \]

Please shift $\bar{a}$ (See Wilt.)

Beam splitter with rotation axis $\bar{a} c^+ c$
Creation of entangled states

\[ |d\rangle \quad |c\rangle \quad |b\rangle \quad |a\rangle \]

\[ |\Phi_0\rangle \quad |\Phi_1\rangle \quad |\Phi_2\rangle \quad |\Phi_3\rangle \]

Then the state after the first two 50/50 beamsplitters is

\[ |\phi_1\rangle = (|01\rangle + |10\rangle)(|01\rangle + |10\rangle) \]
\[ = |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle, \]

up to a normalization factor which we shall suppress for clarity. The Kerr

\[ |\phi_2\rangle = |0101\rangle - |0110\rangle + |1001\rangle + |1010\rangle. \]

Finally, the output state, given by applying \(B^\dagger\) to modes \(a\) and \(b\), is

\[ |\phi_3\rangle = \frac{|1001\rangle + |0110\rangle}{\sqrt{2}} \]

\[ = \frac{(\uparrow\downarrow) + (1\uparrow)}{\sqrt{2}} \]

ENTANGLED STATE