MIT 8.422 Atomic Physics II — *Spring, 2017*
Prof. Wolfgang Ketterle — Febuary 27, 2017

Assignment #1

Due: Monday, March 13, 2017
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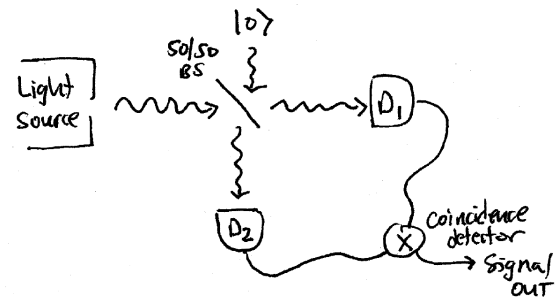
Office Hours: Wednesdays, 2:30-3:30 pm, Thursday 3-4 pm and Friday 3-4 pm, or by email appointment.

1. **The Hanbury-Brown Twiss experiment and $g^{(2)}(\tau)$**

   The second-order coherence function $g^{(2)}(\tau)$ is often measured in the laboratory using an experiment first developed by Hanbury-Brown and Twiss in the 1950's, for studying the light from distant stars. This experiment involves mixing light from the input source with the vacuum, $|0\rangle$, on a 50/50 beamsplitter, and measuring the intensity-intensity correlation function at the output using two detectors and a coincidence circuit:



   This problem examines how this experiment measures $g^{(2)}(\tau)$, and what results are obtained for different input states of light.

   a) Let a, $a^\dagger$, and b, $b^\dagger$ be the raising and lowering operators for the two modes of light input to the beamsplitter, and let the unitary transformation performed by the beamsplitter be defined by

$$a_1 = UaU^\dagger = \frac{a+b}{\sqrt{2}} \qquad (1)$$

$$b_1 = UbU^\dagger = \frac{a-b}{\sqrt{2}} \,. \qquad (2)$$

   For light input in state $|\psi\rangle$, you are given that the output of the coincidence circuit is a voltage

$$V_\psi = V_0 \langle \psi, 0| \, a_1^\dagger a_1 b_1^\dagger b_1 \, |\psi, 0\rangle \,, \qquad (3)$$

   where V_0 is some proportionality constant, and $|\psi, 0\rangle$ denotes a state with $|\psi\rangle$ in mode "a" and $|0\rangle$ in mode "b". In other words, the voltage is the average of the product of the two detected photon signals. Show that V_ψ gives a measure of $g^{(2)}(\tau)$,

$$g^{(2)}(\tau) = \frac{\langle a^\dagger a^\dagger a a \rangle}{\langle a^\dagger a \rangle^2} \qquad (4)$$

   up to a normalization factor. Real detectors will have an additional additive offset (due to dark currents etc.) which we ignore here.

b) The classical expression for $g^{(2)}$ is

$$g^{(2)}_{cl}(\tau) = \frac{\langle \bar{I}(t)\bar{I}(t+\tau)\rangle}{\langle \bar{I}\rangle^2} . \tag{5}$$

Prove that $g^{(2)}_{cl}(0) \geq 1$. It is helpful to use the fact that $\langle (\bar{I}(t) - \langle \bar{I}(t)\rangle)^2\rangle \geq 0$.

c) Compute $g^{(2)}(\tau)$ for the following input light states:

$$|\psi_1\rangle = |\alpha\rangle = e^{-|\alpha|^2/2} \sum_k \frac{\alpha^k}{\sqrt{k!}} |k\rangle \quad \text{(a coherent state)} \tag{6}$$

$$|\psi_2\rangle = |2\rangle \quad \text{(the number state } n = 2) \tag{7}$$

d) Compute $g^{(2)}(\tau)$ for the following input light states, as a function of α:

$$|\psi_3\rangle = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2}} \tag{8}$$

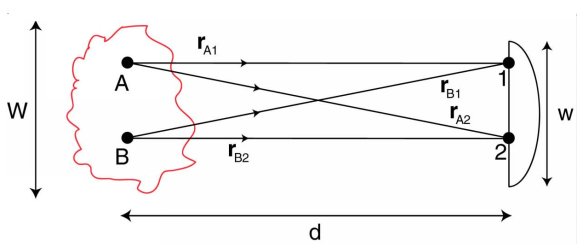
$$|\psi_4\rangle = \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2}} \tag{9}$$

Do either of these two states show non-classical second-orderd coherence? Why (or why not)?

2. **Hanbury Brown and Twiss Experiment with Atoms**

This problem illustrates the coherence and collimation requirements for performing a Hanbury Brown and Twiss (HBT) experiment with atoms. In fact the HBT experiment was done for both bosons (^{4}He) and fermions (^{3}He) by Jeltes and company in 2007 (T. Jeltes et al., Nature **445**, 402 (2007)). (Note: Ignore gravity in this problem.)

If a free particle starts at point A at time $t = 0$ with an amplitude (wavefunction) ψ_A, then the amplitude at another point 1 and time $t = \tau$ is proportional to $\psi_A e^{i(\mathbf{k}\cdot\mathbf{r}_{A1} - \omega\tau)}$, where $\mathbf{r}_{A1}$ is the vector from A to 1, $\mathbf{k}$ is the particle's wavevector, and $\hbar\omega$ is its total energy. This can be regarded as Huygen's principle for matter waves, and is a special case of the Feynman path integral formulation of quantum mechanics.



(Based on figure 19-5, in G. Baym, *Lectures on Quantum Mechanics*)

(a) Correlation function

Assume we have a particle at A with amplitude ψ_A and one at B with amplitude ψ_B. The joint probability P of finding one particle at 1 and one at 2 is

$$P = \left|\psi_A e^{i\phi_{A1}} \psi_B e^{i\phi_{B2}} \pm \psi_A e^{i\phi_{A2}} \psi_B e^{i\phi_{B1}}\right|^2 \tag{10}$$

and is proportional to the second-order coherence function $g^{(2)}(1,2)$. The $\pm$ is for bosons/fermions and makes the two-particle wavefunction symmetric/antisymmetric under the exchange of particles. Here, $\phi_{A1} = \mathbf{k}_A \cdot \mathbf{r}_{A1} - \omega\tau$ is the phase factor for the path from point A to detector 1, etc. Calculate P as a function of $\mathbf{r}_{21}$, the vector from point 2 to point 1 on the detector.

(b) Transverse Collimation

Assume you are given a source (e.g. a ball of trapped atoms) with transverse dimension W and detector with transverse dimension w where $|\mathbf{r}_{21}| \leq w$. The distance between source and detector d is much greater than all other distances. The transverse component of the phase factor in part (a) can be written: $\phi_t = (\mathbf{k}_A - \mathbf{k}_B)_t \cdot (\mathbf{r}_{21})_t$. Assume that the signal at the detector is mainly due to atoms with wavevectors distributed around $\mathbf{k}_0$. Argue that the transverse collimation required to see second order correlation effects can be expressed as $Ww \ll d\lambda_{dB}$, where λ_{dB} is the deBroglie wavelength corresponding to $\mathbf{k}_0$. (Hint: How does ϕ_t vary for atoms originating at different points in the source and being detected at different points on the detector?)

Consider a ^{6}Li MOT at 500 μK. Calculate the deBroglie wavelength. Assuming a MOT and detector of approximately equal size ($W \approx w$), estimate an upper bound on the MOT and detector size using $d = 10$ cm.

(c) Longitudinal Collimation

(i) The longitudinal component of the phase factor in part (a) can be written: $\phi_l = (\mathbf{k}_A - \mathbf{k}_B)_l \cdot (\mathbf{r}_{21})_l$. Assume a Gaussian distribution of wavevector differences $p(\mathbf{k}_A - \mathbf{k}_B) = e^{-|\mathbf{k}_A - \mathbf{k}_B|^2\gamma^2}$ where the width γ is related to the temperature of the atoms. Calculate $\langle P \rangle$ using this distribution and your result from part (a). Sketch $\langle P \rangle$ for both fermions and bosons, indicating the extent of $(\mathbf{r}_{21})_l$ over which the second order correlation effect can be seen. (Hint: Use the fact that $\phi_t \ll 2\pi$ from part (b) to simplify the integral.)

(ii) Now assume you have a pulsed source of atoms with longitudinal dimension L. Atoms are released at time $t = 0$ and detected at some later time $t = \tau$. Give geometric arguments to show that the wavevectors of detected atoms must obey $|\,(\mathbf{k}_A - \mathbf{k}_B)_l\,| \leq \frac{mvL}{\hbar d}$, where the velocity $v = \frac{d}{\tau}$. This implies that the different velocity groups separate during the expansion, narrowing (by a factor $\frac{L}{d}$) the velocity distribution of atoms detected at any particular time.

Consider again the ^{6}Li MOT from part (b). Assuming $\tau = 0.1$s and $L \approx W$, estimate the necessary timing resolution of the detector in order to see second order correlation effects?

(d) Phase-Space Volume Enhancement

We now pull all the pieces together. The peak in $g^{(2)}(1,2)$ is visible for $(\mathbf{k}_A - \mathbf{k}_B)\cdot\mathbf{r}_{21} \leq 2\pi$. This is equivalent to saying that we must detect atoms from within a single phase space cell, defined by $\delta p_x \delta x \leq h$ (and likewise for y and z). In our trapped atom sample, the 3D volume of a phase space cell is $\delta x \delta y \delta z = (\lambda_{dB})^3$. Liouville's theorem says that as our ball of atoms expands, the number of phase space cells remains constant. Verify that, by using this pulsed source, the volume of a coherent phase space cell is increased by a factor d^3/W^2L by the time atoms reach the detector. What is the order of magnitude of this increase (assuming $L \approx W$)?

Estimate the average occupation of a cell of phase space for the ^{6}Li MOT from parts (b) and (c). Use the following numbers for the ^{6}Li MOT: 10^{10} atoms in 1 cm^3. How does this compare with the average occupation of a BEC or a degenerate Fermi cloud?