Assignment #2

Due: Friday May 5, 2017
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Office Hours: Wednesdays, 2:30-3:30 pm, Thursday 3-4 pm and Friday 3-4 pm, or by email appointment.

1. **Density Limit in a MOT**. In a 3-D magneto-optical trap, the density of the trapped atoms is limited by a net outward radiation pressure which opposes the trapping force. We can divide the density-dependent photon-pressure force into two parts. First there is a repulsive 'radiation trapping force' due to atoms absorbing photons scattered from other atoms in the trap. Also, there is an attractive 'attenuation force' which is caused by atoms at the side of the cloud attenuating the laser beams, thus creating an intensity imbalance which leads to an inward force.

   (a) Show that the radiation trapping force obeys the equation

   $$\nabla \cdot \mathbf{F}_R = \frac{6\sigma_L \sigma_R I n}{c},$$

   where $\mathbf{I}$ is the intensity of one of the trapping laser beams, n is the number density of atoms in the cloud, σ_L is the cross-section for absorption of the laser beam, and σ_R is the cross-section for absorption of the scattered light. (**Hint**: Find the magnitude of the force between two atoms separated by a distance d, where one atom re-radiates a laser photon and the second atom absorbs it. Now, since this is an inverse-square force, you can use Gauss' law to find $\nabla \cdot \mathbf{F}_R$. Assume that photons are only scattered twice.)

   (b) The attenuation force may be obtained simply by replacing σ_R with $-\sigma_L$, so that

   $$\nabla \cdot \mathbf{F}_A = -\frac{6\sigma_L^2 I n}{c}.$$

   Explain why this is so.

   (c) The total force is the sum of $\mathbf{F}_R, \mathbf{F}_A$, and the trapping force $\mathbf{F}_T = -\kappa \mathbf{r}$, where κ is the spring constant of the trap. For stability we require that the total force is attractive, $\nabla \cdot \mathbf{F}_{total} < 0$. Find the maximum density of the trapped cloud at a given κ from the condition $\nabla \cdot \mathbf{F}_{total} = 0$.

   (d) Give a qualitative argument for whether we expect $\sigma_R = \sigma_L, \sigma_R > \sigma_L$, or $\sigma_R < \sigma_L$. (**Hint**: sketch the absorption and emission spectra for an atom in a strong laser field with a red detuning.)

(e) Suppose that some of the atoms can be put into a 'dark state', so that only a fraction f of the atoms absorb the trapping light. How do F_R, F_A and F_T vary with f? What happens to the limiting density n_{max}? This is the concept of the Dark SPOT trap (PRL **70**, 2253 (1993)).

2. **Microscopic nature of the dipole force.**

So far in 8.422, we have covered different types of forces on an atom due to its interaction with light. We will take a closer look into the microscopic nature of these forces, and identify which forces come from electric or from magnetic fields.

Let's review what happens when an atom is illuminated with laser light; the electric field $\vec{E}$ induces an atomic dipole moment $\vec{d}$ which oscillates with a frequency equal to its driving frequency ω. The amplitude d_o is related to the electric field amplitude E_o by the relation, $d_o = \alpha E_o$ where α is the *complex polarizability*, which depends on the driving frequency ω.

(a) In classical physics, you have learned that moving charges lead to two types of forces:

$$F_C = \left(\vec{d} \cdot \vec{\nabla}\right) \vec{E} \qquad , \qquad F_M = q\vec{v} \times \vec{B}$$

where F_C and F_M correspond to the Coulomb or electric force and magnetic part of the Lorentz force respectively. What are these forces for the oscillating dipole?(leave your answer in terms of $\alpha, \vec{E}$ and $\vec{B}$)

(b) Now, let's illuminate an atom at $z = 0$ with a traveling EM wave defined as (in S.I. units),

$$\vec{E} = E_o \cos(\omega t - kz)\, \hat{e}_x \qquad , \qquad \vec{B} = \frac{E_o}{c} \cos(\omega t - kz)\, \hat{e}_y.$$

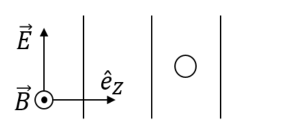

FIG. 1: Traveling wave illuminating an atom

Assume that the dipole oscillates in phase with $\vec{E}$ (α is real). What is the average magnitude of F_C and F_M in all directions (e_x, e_y, e_z)?

(c) Let's delay the dipole response to the driving frequency. The dipole now oscillates at a phase ϕ relative to $\vec{E}$. Write the resulting dipole in terms of the in-phase and out-of-phase quadratures.

$$\vec{d} \propto [\alpha' \cos(\omega t - kz) + \alpha'' \sin(\omega t - kz)]\, \hat{e}_x \tag{1}$$

What is α', and α'' in terms of ϕ and the magnitude of α? What is the average magnitude of F_C and F_M in all directions (e_x, e_y, e_z)? Explain what kind of light force is the F_M that you have calculated.

(d) On resonance and strong saturation, a two level atom's atomic dipole moment has a steady state solution that is

$$< \hat{d} >= -2d_{ab}\nu_{st}\sin(\omega t) \tag{2}$$

where $\nu_{st} = \Gamma/2\Omega$, when $\Omega \gg \Gamma$, and $\Omega = d_{ab}E_o$. What is now the Lorentz force for the saturated dipole?

(e) Let's now understand the forces in optical lattices. Back to the in-phase model, add a second light source propagating in the opposite direction to create a standing wave at the location of the atom. The EM-fields are now

$$\vec{E} = E_o \cos(kz)\sin(\omega t)\,\hat{e}_x \qquad , \qquad \vec{B} = -\frac{E_o}{c}\sin(kz)\cos(\omega t)\,\hat{e}_y.$$

What are now the forces F_M and F_C in all directions (e_x, e_y, e_z)? We can write the force as the gradient of an energy potential, what is this potential?

(f) Using the fields defined in (b), expand the definition of the illuminating light such that its amplitude has a Gaussian transverse beam profile $f(x,y,z)$, where $f(x,y,z) = \frac{w_0}{w(z)}e^{-r^2/w(z)^2}$ and $w(z) = w_0\sqrt{1+(z/z_R)^2}$, and $z_R = kw_0^2/2$ is the Rayleigh range. What are the average magnitudes of F_C and F_M in all directions?

(g) Calculate the forces the two level atom receives due to the reactive force (i.e. dipole optical force)? Compare your results with those found in (f)
Note: The dipole potential captures correctly the force, whether it is the electric of magnetic part of the Lorentz force.

(h) The discrepancy between your results in f and g are resolved by going beyond the paraxial approximation. When a light beam with a transverse profile is focused through a lens, it acquires a longitudinal component to its polarization. Take a single Gaussian beam focused by a lens to a waist w_0 at $z = 0$. The $\vec{B}$ acquires a longitudinal component which leads to fields at the waist (in the plane $z = 0$)

$$E_x = E_o \sin(\omega t)\, f(x,y,z) \qquad , \qquad B_z = 2\frac{E_o}{c}\frac{y}{kw_0^2}\cos(\omega t)\, f(x,y,z=0).$$

Compared to part f, what additional force do you now get from the field B_z? Compare to the force due to the dipole potential.

(i) We are still missing the microscopic force along the z direction. For this, we have to consider the longitudinal component of the electric field in a Gaussian focus. Neglecting quadratic terms in z, the longitudinal field for small z is

$$E_z = 2E_o\frac{x}{kw_0^2}\left(\cos(\omega t) + \frac{z}{z_R}\sin(\omega t)\right) f(x,y,z) \tag{3}$$

What additional force do you now get from the longitudinal field E_z?
Now summarize your findings by stating which components (x,y, z) of the electric and/or magnetic fields are responsible for the trapping forces in x, y, z.
More details about the electric and magnetic fields in a Gaussian beam can be found in **PRL 90, 053002 (2003)**)