

Enhancement and suppression of spontaneous emission and light scattering by quantum degeneracy

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Quantum degeneracy modifies light scattering and spontaneous emission. For fermions, Pauli blocking leads to a suppression of both processes. In contrast, in a weakly interacting Bose-Einstein condensate, we find spontaneous emission to be enhanced, while light scattering is suppressed. This difference is attributed to many-body effects and quantum interference in a Bose-Einstein condensate.

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For a long time, spontaneous emission and light scattering were regarded as intrinsic properties of atoms. However, quantum electrodynamics (QED) revealed the connection between these phenomena and the electromagnetic modes of the vacuum. Spontaneous emission and scattering can only take place when a vacuum mode is available to accommodate the emitted or scattered photon. Cavity-QED experiments [1] exploit the fact that small cavities can be used to modify the vacuum, and thus emission and scattering of light.

Recent breakthroughs in the experimental realization of gaseous quantum degenerate systems of bosons [2–5] and fermions [6] have provided additional means to modify the emission and scattering behavior of atoms. On the one hand, this modification is due to the finite population in the final state of the scattering or emission process. On the other hand, interactions modify the nature of the final states for the recoiling atom, similar to a cavity that alters the mode structure for the photon.

Generally, transition rates between an initial state with population N_1 and a final state with population N_2 are proportional to $N_1(1+N_2)$ for bosons and to $N_1(1-N_2)$ for fermions. This reflects the well-known fact that in a bosonic system the transition into an already occupied state is enhanced by bosonic stimulation, while in fermionic systems, occupation of a state prevents a transition into this state by Pauli blocking.

This simple derivation of transition rates using occupation numbers becomes subtle or even invalid for correlated many-body states such as an interacting Bose-Einstein condensate (BEC) ground state. In this Rapid Communication, we analyze under which circumstances the simple approach can be used to reproduce the correct results for the interaction between light and a BEC. We show theoretically that spontaneous emission in a weakly interacting BEC is enhanced, consistent with the description using occupation numbers, and calculate the enhancement factor. We compare this result to light scattering in a BEC, which is suppressed due to interference effects not included in the simple derivation, as we have shown experimentally and theoretically in previous work [7]. In contrast, in fermionic systems quantum degeneracy leads to a suppression of *both* spontaneous emission and light scattering [8–11].

For simplicity we consider a homogeneous system of atoms at zero temperature, which exhibits all the significant features. The system is assumed to consist of N atoms with

mass m at a density n in a volume V , which is sufficiently large such that the summations over quantum states can be approximated by integrals.

A noninteracting BEC at $T=0$ can be described by a single-particle ground-state wave function with amplitude \sqrt{N} . Thus, the occupation of quantum states with wave vector \mathbf{k} is given by $N(\mathbf{k})=N\delta(\mathbf{k})$. Since the final states for spontaneous emission and light scattering with a finite momentum transfer \mathbf{q} are not occupied, both processes occur at the single-atom rate.

This situation changes drastically for an interacting condensate. Here, two atoms in the zero-momentum state are coupled to states with momenta $+\mathbf{k}$ and $-\mathbf{k}$. This changes the excitation spectrum $\omega(k)$ from the free-particle form, $\hbar\omega(k)=E_r(k)$, to the spectrum of Bogoliubov quasiparticles, $\hbar\omega(k)=\sqrt{E_r(k)(E_r(k)+2\mu)}$. Here, μ is the chemical potential and $E_r(k)=\hbar^2k^2/2m$ the recoil energy associated with the momentum k . The chemical potential is a measure for the interatomic interactions and is related to the s -wave scattering length a by $\mu=4\pi\hbar^2an/m$.

The atom-atom interaction admixes pair correlations into the ground-state wave function $|\text{BEC}, N\rangle$ of a BEC with N atoms, yielding the structure [12]

$$|\text{BEC}, N\rangle = |N, 0, 0\rangle - \alpha|N-2, 1, 1\rangle + \alpha^2|N-4, 2, 2\rangle + \dots, \quad (1)$$

where $\alpha=1-1/u_k^2$. Here $|N_0, N_k, N_{-k}\rangle$ denotes a state with N_0 atoms in the zero-momentum state and $N_{\pm k}$ atoms in states with momentum $\pm k$. In Eq. (1) a summation over all momenta \mathbf{k} is implicitly assumed. The average population of momentum states is given by $N(k)=u_k^2-1=v_k^2$, where $u_k=\cosh\phi_k$, $v_k=\sinh\phi_k$ and $\tanh 2\phi_k=\mu/[E_r(k)+\mu]$.

To study the effect of the presence of a BEC on spontaneous emission, we consider an excited atom at rest added to a BEC of N ground-state atoms. This system is described by an initial state $|i\rangle=\hat{a}_{e,0}^\dagger|\text{BEC}, N\rangle$, where $\hat{a}_{e,0}^\dagger$ creates an electronically excited atom at rest. We use Fermi's golden rule to obtain the rate for spontaneous emission. The only difference to the single-atom spontaneous decay rate Γ comes from the overlap matrix elements to the final momentum state $|f\rangle$, $\langle f|\hat{a}_{k_L}^\dagger\hat{a}_{e,0}|i\rangle$ where $\hat{a}_{k_L}^\dagger$ is the creation operator for a free ground-state atom with momentum k_L . Summing over all final states one arrives at

$$\gamma_{\text{BEC}} = \Gamma \langle \text{BEC}, N | \hat{a}_{k_L} \hat{a}_{k_L}^\dagger | \text{BEC}, N \rangle. \quad (2)$$

Thus, the spontaneous emission rate is proportional to the square of the norm of the state vector $|e^+\rangle = \hat{a}_{k_L}^\dagger | \text{BEC}, N \rangle$.

To calculate the norm of $|e^+\rangle$ explicitly, we transform to Bogoliubov operators by substituting $\hat{a}_k = u_k \hat{b}_k - v_k \hat{b}_{-k}^\dagger$. The operators \hat{b}_k^\dagger and \hat{b}_k are the creation and annihilation operators for the microscopic quasiparticle excitations of a weakly interacting condensate. Hence, the many-body ground-state wave function of the condensate $| \text{BEC}, N \rangle$ corresponds to the quasiparticle vacuum defined by the relation $\hat{b}_k | \text{BEC}, N \rangle = 0, \forall k$. Thus, we obtain

$$F_{\text{Bose}}^{\text{spont}} = \langle e^+ | e^+ \rangle = u_{k_L}^2 = 1 + N(k_L) = \left\{ \cosh \left[\frac{1}{2} \tanh^{-1} \left(\frac{k_s^2}{k_L^2/2 + k_s^2} \right) \right] \right\}^2, \quad (3)$$

where $\hbar k_s = mc$ is the momentum of an atom moving at the speed of sound, which is related to the chemical potential by $\mu = mc^2$. Enhancement of spontaneous emission in a BEC is significant if k_s becomes comparable to the wave vector k_L of the emitted photon, since for small momentum transfer $u_{k_L}^2 = k_s^2/k_L^2$. Equation (3) and its interpretation are the major results of this paper. It should be noted that this enhancement of spontaneous emission in a BEC is different from the phenomenon of superradiance as discussed by Dicke [13]. Superradiance occurs in a noninteracting gas of sufficient column density, whereas the enhancement of spontaneous emission described here requires interactions that scale with density.

The result of Eq. (3) that spontaneous emission in a weakly interacting BEC is enhanced, is striking in view of our earlier finding [7] that light scattering in a BEC is suppressed. The operator describing a light-scattering event with momentum transfer \mathbf{q} is the Fourier transform of the atomic density operator $\hat{\rho}(\mathbf{q}) = \sum_m \hat{a}_{m+\mathbf{q}}^\dagger \hat{a}_m$. If $\hat{\rho}(\mathbf{q})$ acts on $| \text{BEC}, N \rangle$, only terms involving the zero-momentum state $m=0$ yield significant contributions. By applying Fermi's golden rule we found that the scattering rate is proportional to the norm of the state vector

$$|e\rangle \approx \frac{(\hat{a}_q^\dagger \hat{a}_0 + \hat{a}_0^\dagger \hat{a}_{-q}) | \text{BEC}, N \rangle}{\sqrt{N}} \approx (\hat{a}_q^\dagger + \hat{a}_{-q}) | \text{BEC}, N \rangle = |e^+\rangle + |e^-\rangle, \quad (4)$$

where we have replaced \hat{a}_0^\dagger and \hat{a}_0 by \sqrt{N} following the usual Bogoliubov formalism [14]. After transforming to Bogoliubov operators we obtain a suppression factor of

$$S_{\text{Bose}}(q) = \langle e | e \rangle = (u_q - v_q)^2, \quad (5)$$

which is the static structure factor for a BEC. Generally, the static structure factor is the normalized response of a system to a perturbation with wave vector \mathbf{q} . For small \mathbf{q} , corresponding to phononlike quasiparticle excitations, $S_{\text{Bose}}(q)$

$= \hbar q/2mc$ approaches zero. Light with wave vector k_L scattered at an angle θ imparts a momentum $\hbar q = 2\hbar k_L \sin(\theta/2)$ to the atomic system. By integrating Eq. (5) over all possible scattering angles θ and accounting for the dipolar emission pattern, we find that Rayleigh scattering from a BEC is suppressed by a factor [7]

$$F_{\text{Bose}}^{\text{scatt}} = \frac{k_s}{\sqrt{k_s^2 + k_L^2}} \left(\frac{15 k_s^5}{8 k_L^5} + \frac{23 k_s^3}{8 k_L^3} + 2 \frac{k_s}{k_L} + \frac{k_L}{k_s} \right) - \left(\frac{15 k_s^6}{8 k_L^6} + \frac{9 k_s^4}{4 k_L^4} + \frac{3 k_s^2}{2 k_L^2} \right) \tanh^{-1} \left(\frac{k_L}{\sqrt{k_s^2 + k_L^2}} \right). \quad (6)$$

For comparison, we briefly summarize the suppression of spontaneous emission and light scattering for a fermionic system. A Fermi gas at $T=0$ with Fermi momentum $\hbar k_F$ is characterized by $N(\mathbf{k}) = \theta(k_F - k)$, i.e., all momentum states with $k < k_F = (6\pi n)^{1/3}$ are occupied. If we add an electronically excited atom at rest to the Fermi sea, its spontaneous decay rate is suppressed by a factor

$$F_{\text{Fermi}}^{\text{spont}} = 1 - N(k_L) = \theta(k_L - k_F). \quad (7)$$

When off-resonant light with initial wave vector \mathbf{k}_L is scattered from a filled Fermi sphere into an outgoing wave with final wave vector $\mathbf{k}_L + \mathbf{q}$, the scattering rate is suppressed by [15]

$$S_{\text{fermi}}(\mathbf{q}) = \int d\mathbf{k} N(\mathbf{k}) [1 - N(\mathbf{k} + \mathbf{q})] = \begin{cases} \frac{3q}{4k_F} - \frac{q^3}{16k_F^3} & \text{if } 0 < q < 2k_F, \\ 1 & \text{if } q > 2k_F. \end{cases} \quad (8)$$

Equation (8) is the static structure factor for a Fermi gas at zero temperature. Integrating over all possible scattering angles θ and accounting for the dipolar emission pattern, we find that the total suppression factor for Rayleigh scattering from a Fermi sea is given by

$$F_{\text{Fermi}}^{\text{scatt}} = \begin{cases} \frac{69}{70} \frac{k_L}{k_F} - \frac{43}{210} \frac{k_L^3}{k_F^3} & \text{if } k_L < k_F \\ 1 - \frac{3}{10} \frac{k_F^2}{k_L^2} + \frac{9}{70} \frac{k_F^4}{k_L^4} - \frac{1}{21} \frac{k_F^6}{k_L^6} & \text{if } k_L > k_F. \end{cases} \quad (9)$$

Figure 1 shows the influence of quantum degeneracy on the atom-light interaction. Using Eqs. (3), (6), (7), and (9) we have plotted the rates for spontaneous emission (solid lines) and light scattering (dashed lines), normalized by the single-atom rates, for a weakly interacting BEC [Fig. 1(a)] and a degenerate Fermi gas [Fig. 1(b)]. A significant deviation from the free-particle rate is clearly observable if the photon-momentum is comparable to k_s for bosons and k_F for fermions.

Let us now discuss the intrinsic difference between atom-light interaction in a BEC and in a degenerate Fermi gas. In

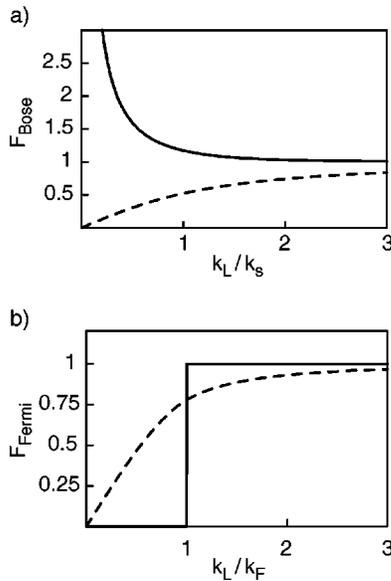


FIG. 1. Modification of spontaneous emission (solid line) and light scattering (dashed line) due to quantum degeneracy. In (a) we have plotted the enhancement factor for spontaneous emission and the suppression factor for light scattering for a weakly interacting Bose-Einstein condensate as a function of the light wave vector k_L in units of k_s , the wave vector of an atom moving at the speed of sound. In (b) the suppression factors for spontaneous emission and light scattering in a Fermi gas at $T=0$ are plotted as a function of k_L in units of the Fermi wave vector k_F .

a Fermi sea, the suppression of both light scattering and spontaneous emission is a *single-particle* effect caused by the nonavailability of final states due to the Pauli exclusion principle. Indeed, one would obtain the same result for spontaneous emission as for light scattering [Eq.(9)] if the initial momentum of the excited atom were randomly distributed over the Fermi sphere.

In an interacting BEC the situation is significantly different because pair correlations in the ground state, i.e., *many-body* effects, are responsible for both the enhancement of spontaneous emission and suppression of light scattering. The finite population in states with $k \neq 0$ due to quantum depletion lends a very intuitive explanation for the enhancement of spontaneous emission. If the interactions in the condensate are sufficiently strong such that momentum states with $k=k_L$ have non-negligible occupation, spontaneous emission of an atom at rest is enhanced by bosonic stimulation. This intuitive argument is correct, but it would incorrectly predict that light scattering is also enhanced.

The suppression of light scattering occurs due to the correlation between the admixtures of states with momentum k and $-k$. This leads to a destructive quantum interference between the two processes $|N,0,0\rangle + \hbar q \rightarrow |N,1,0\rangle$ and $|N-2,1,1\rangle + \hbar q \rightarrow |N,1,0\rangle$, in which either an excitation with momentum $-\mathbf{q}$ is annihilated or an excitation with momentum \mathbf{q} is created. Both processes transfer momentum \mathbf{q} to the condensate and are individually enhanced by bosonic stimulation. Therefore, a simple rate equation model would predict enhanced light scattering. However, since the initial states are correlated the two processes leading to the same final

state interfere destructively. Thus for a BEC with repulsive interactions light scattering is suppressed.

The results presented above for suppressed light scattering in both bosonic and fermionic systems also apply to the scattering of massive impurities. This has been studied theoretically for degenerate Bose [16] and Fermi [17,18] gases, and was experimentally observed in a BEC [19].

How strong would the enhancement of spontaneous emission in currently realized Bose-Einstein condensates be? Condensates of ^{23}Na atoms confined in an optical trap have reached a density of $3 \times 10^{15} \text{ cm}^{-3}$ [20]. For this density the speed of sound $\hbar k_s/m = 2.8 \text{ cm/s}$ and the recoil velocity $\hbar k_L/m = 2.9 \text{ cm/s}$ are approximately equal and we find $N(k_L) \approx 0.15$. Thus, the observation of enhanced spontaneous emission in a BEC is within experimental reach. Excited atoms at rest could be produced by injecting ground-state atoms with momentum $\hbar k_L$ into a condensate and using a counterpropagating laser beam to excite them and bring them to rest. The enhancement of spontaneous emission could then be observed as frequency broadening of the absorption line.

The fact that light scattering is suppressed, but spontaneous emission is enhanced, could be exploited for studies of decoherence in a BEC. When a photon is absorbed by a BEC (the first step of light scattering), it creates a (virtual) excited state that has an external wave function that includes pair correlations. Any decoherence of this coherent superposition state, for example by interaction with the thermal cloud, could destroy the interference effect discussed above and turn the suppression of light scattering into an enhancement. Another possibility of creating an excited state atom in a BEC is using Doppler-free two-photon excitation, a scheme already used to probe condensates of atomic hydrogen on the $1s \rightarrow 2s$ transition [5]. In this case, enhancement of spontaneous emission could be observed if the excited-state lifetime is longer than the coherence time.

In conclusion, we have discussed suppression and enhancement of light scattering and spontaneous emission in quantum degenerate systems, and shown that in a weakly interacting BEC, the quantum depletion can enhance spontaneous emission by bosonic stimulation. This contrasts earlier results on suppressed light scattering in a BEC. As we have shown, both the reduced light scattering and the enhanced spontaneous emission in a BEC are related to quantum depletion of the condensate. However, the enhanced spontaneous emission appears to be physics beyond the Gross-Pitaevskii equation, while the static structure factor $S(q)$ and the reduced light scattering can be obtained from the Gross-Pitaevskii equation [21].

Note added. Recently we found a theoretical paper [22] that predicts an enhancement of light scattering in a weakly interacting Bose-Einstein condensate contrary to our findings.

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