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# Heisenberg-limited spectroscopy with degenerate Bose-Einstein gases

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We propose an experiment that exploits the quantum interference between two noninteracting ensembles of spatially degenerate Bose-Einstein atoms to measure phase shifts of atomic coherences at the Heisenberg limit. [S1050-2947(97)50207-1]

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The recent experiments demonstrating Bose-Einstein condensation (BEC) in alkali-metal vapors [1–3] have opened the possibility of studying the phase properties of degenerate atomic samples. For example, one might look for spatial interference fringes between two freely evolving condensates [4,5] or study the atom-counting statistics for two condensates overlapped at a beam splitter [6]. Given the coherence properties of Bose-Einstein condensed sources, one might hope to develop high-resolution spectroscopic techniques that exploit the quantum degeneracy of these samples to enhance measurement precision.

In this Rapid Communication we propose a scheme that is capable of detecting small phase shifts of atomic coherences with a precision that scales inversely with the number of atoms $N$ used in the measurement. In principle, this Heisenberg-limited method could afford significant gains in precision over standard techniques, where the shot-noise limited precision scales as $1/\sqrt{N}$. Our method uses two degenerate ensembles of Bose-Einstein atoms as inputs to a Ramsey-type atom interferometer [7]. We show that phase shifts introduce fluctuations in the number of atoms detected at the interferometer output ports, and that these fluctuations can then be used as a signature for a phase imbalance between interferometer arms.

The Ramsey technique uses a sequence of two $\pi/2$ pulses separated by an interrogation time $T$ to probe the time evolution of an atomic coherence. This separated oscillatory fields configuration is analogous to a Mach-Zehnder interferometer in optics when the evolution of the ground and excited electronic states is viewed as the two arms of the interferometer. It is used in many experiments as a method to detect small shifts in the relative phase between an atomic coherence and the driving field. These shifts, for example, may arise from a small detuning of the driving field from the atomic resonance or from the presence of an external perturbation.

The proposed method is related to previous work on quantum noise reduction. Optical interferometry with Fock states has been shown to enable Heisenberg-limited phase detection [8]. For atoms, Wineland *et al.* have shown that Ramsey interferometry with maximally entangled spin states can achieve the Heisenberg limit [9].

In order to motivate our method, which requires two condensates at the interferometer inputs ports, we will begin by showing that interferometry with a single condensate is not capable of providing enhanced spectroscopic resolution [10]. Consider a condensate of two-level atoms that is driven with a single resonant excitation pulse. We assume throughout that the atoms are noninteracting, and we neglect spontaneous emission from the excited level. The initial condensate state is taken to be the Fock state

$$|n\rangle=\prod_{i=1}^{N}|\phi_g(\mathbf{x}_i)\rangle, \quad (1)$$

where the subscript $g$ indicates the ground electronic state, $\mathbf{x}_i$ is the spatial coordinate of the $i$th atom, and $N$ is the number of atoms in the condensate. The evolution of this system is described by the Schrödinger equation $\hat{H}|\psi\rangle=i\hbar d|\psi\rangle/dt$, with the Hamiltonian $\hat{H}$ given by a sum over the free particle contributions $\hat{H}_i^{(0)}$ and the coupling terms $\hat{V}_i$, i.e., $\hat{H}=\Sigma_{i=1}^{N}(\hat{H}_i^{(0)}+\hat{V}_i)$. To be explicit, one might consider a magnetic dipole interaction that resonantly couples atomic hyperfine levels of atoms confined in the ground state of a harmonic potential. Substitution of the product state

$$|\psi\rangle=\prod_{i=1}^{N}|\phi(\mathbf{x}_i)\rangle \quad (2)$$

into Schrödinger's equation yields an equation of motion that separates into equations of motion for the individual particles: $\hat{H}_i|\phi(\mathbf{x}_i)\rangle=i\hbar d|\phi(\mathbf{x}_i)\rangle/dt$, with $\hat{H}_i=\hat{H}_i^{(0)}+\hat{V}_i$. Specializing to the Fock state [Eq. (1)], we find that the time evolution of this state is identical to that of $N$ nondegenerate atoms.

We can immediately apply this result to a Ramsey-type experiment. For nondegenerate ensembles of atoms, the phase sensitivity is shot-noise limited. Following the argument of the preceding paragraph, we then conclude that the sensitivity obtained with a Bose-Einstein condensed source must also be shot-noise limited.

One might be led to conclude that all interference with Bose condensed sources would be shot-noise limited; however, we note that only a restricted class of states can be factored into products of single-particle states [Eq. (2)]. For example, the initial state consisting of $n_g^{\mathrm{in}}$ degenerate atoms in the electronic ground state $|g\rangle$ and $n_e^{\mathrm{in}}$ atoms in the electronic excited state $|e\rangle$ cannot be factored in this form.

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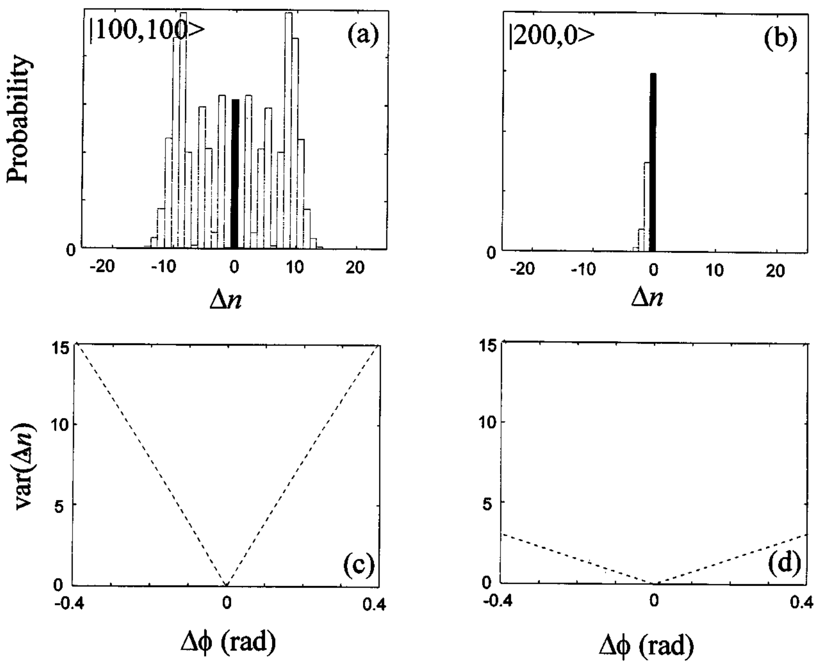


FIG. 1. (a) Probability distribution for $\Delta n \equiv n_e^{\text{out}} - n_g^{\text{in}}$ obtained by numerical diagonalization the matrix defined by Eqs. (3) and (4). The input is a dual Fock state with 100 atoms in each condensate. Open bars indicate the distribution resulting from a phase shift of $\Delta\phi = 0.05$ rad during the free evolution interval. For comparison, the solid bar indicates the distribution of $\Delta n$ for $\Delta\phi = 0$. (b) Probability distribution for a single Fock state input with 200 atoms, with phase shifts $\Delta\phi = 0.05$ rad (open bars) and $\Delta\phi = 0$ (solid bar). (c) Variance of the distributions for $\Delta n$ as a function of the phase shift $\Delta\phi$ for the input state $|100,100\rangle$. (d) Variance of the distribution for $\Delta n$ for the input state $|200,0\rangle$.

We next consider interferometry with this nonfactorizable state consisting of two condensates. The initial system is taken as the dual Fock state $|\psi\rangle = |n_g^{\text{in}}, n_e^{\text{in}}\rangle$. All atoms are assumed to occupy identical center-of-mass states, and for simplicity we neglect recoil effects in driving the transitions. In the interaction picture, after making the rotating-wave approximation, the Hamiltonian matrix elements are

$$\langle n_g, n_e | \hat{H} | n_g, n_e \rangle = -\hbar n_e \delta \qquad (3)$$

and

$$\langle n_g, n_e | \hat{H} | n_g - 1, n_e + 1 \rangle = \frac{\hbar \Omega_{eg}}{2} \sqrt{n_g (n_e + 1)}, \qquad (4)$$

where $\delta$ is the laser detuning and $\Omega_{eg}$ is the single-atom Rabi frequency associated with the coupling $\hat{V}$. The scaling of the matrix elements with the number of ground-state atoms $n_g$ and excited-state atoms $n_e$ follows directly from the properties of the condensate wave functions [11]. For an interaction time $\tau$ such that $\Omega_{eg}\tau \equiv \pi/2$ ($\pi/2$ pulse), the excitation pulse creates a coherent mixing of the ground state $|g\rangle$ and the excited state $|e\rangle$. For $\delta = 0$ (resonant excitation), we note that the evolution of the internal states can be described by a unitary operator that is equivalent to the operator describing a beam splitter in quantum optics.

We are now in a position to investigate the state of the system following the two $\pi/2$ pulse sequence. First, we note that if the detuning $\delta$ is zero, there is no phase shift during the free propagation interval between pulses, and the number of atoms exiting in the excited state, $n_e^{\text{out}}$, is equal to the number of atoms initially in the ground state, $n_g^{\text{in}}$. A phase shift inserted during the free propagation interval manifests itself as fluctuations at the output ports. We characterize these fluctuations by calculating the probability distribution for the quantity $\Delta n \equiv n_e^{\text{out}} - n_g^{\text{in}}$. This is illustrated for the dual Fock state $|n_g^{\text{in}} = 100,\ n_e^{\text{in}} = 100\rangle$ [Fig. 1(a)] and for the single Fock state $|n_g^{\text{in}} = 200,\ n_e^{\text{in}} = 0\rangle$ [Fig. 1(b)], and a phaseshift $\Delta\phi = 0.05$ rad. We parametrize these distributions by their variance $\text{var}(n) \equiv \sqrt{\langle n^2\rangle - \langle n\rangle^2}$. Figures 1(c) and 1(d) show the scaling of var($\Delta n$) with the phase shift $\Delta\phi$ for both the dual Fock state and single Fock state inputs.

We now show how observation of these fluctuations can lead to Heisenberg-limited sensitivity. To do this, we exploit the result that var($\Delta n$) is proportional to the phase shift (for small $\Delta\phi$) to estimate the phase shift $\Delta\phi_{\min}$ that results in a variance of one atom. The idea is that the presence of a phase imbalance of $\Delta\phi_{\min}$ in principle is likely to lead to a nonzero value for the measured number difference $\Delta n$ for any given measurement. On the other hand, the presence of a shift $\Delta\phi < \Delta\phi_{\min}$ is not likely to result in a fluctuation away from $\Delta n = 0$ after a single measurement, and is thus considered unresolvable. The scaling of $\Delta\phi_{\min}$ for different atom numbers $N$ is shown in Fig. 2. For the dual Fock state, $\Delta\phi_{\min} \sim 1/N$ when $N \gg 1$, while for the single Fock state $\Delta\phi_{\min}$

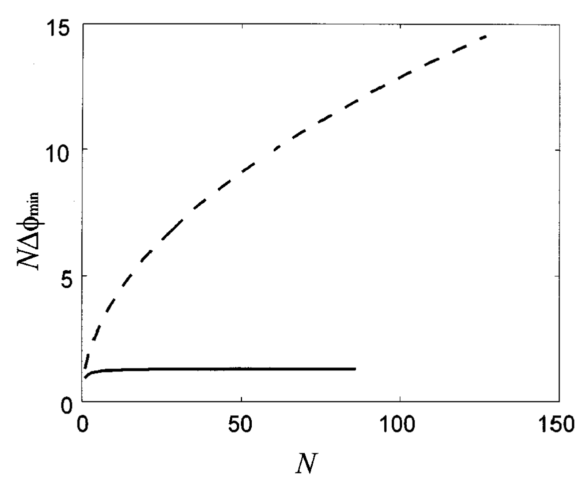


FIG. 2. Variation of the minimum detectable phase shift $\Delta\phi_{\min}$ with the total number $N$ of atoms for a dual Fock state $|N/2, N/2\rangle$ (solid) and a single Fock state $|N,0\rangle$ (dashed) at the input.

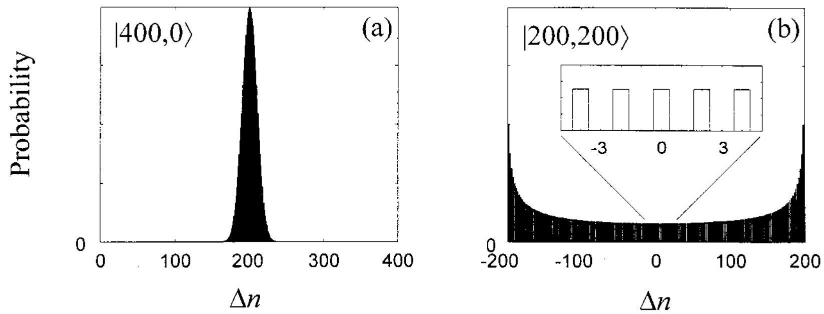


FIG. 3. Probability distributions for $\Delta n$ following a single $\pi/2$ pulse. (a) Distribution for the single Fock state $|400,0\rangle$. (b) Distribution for the dual Fock state $|200,200\rangle$.

$\sim 1/\sqrt{N}$. The $1/N$ scaling for the dual Fock state is the signature of a Heisenberg-limited phase measurement.

Insight into this result can be gained by analyzing the state of the system after the first beam splitter ($\pi/2$ pulse), as shown in Fig. 3. After the beam splitter, the system is in a state characterized by a large uncertainty in the number of atoms in each arm of the interferometer. One might expect that this large uncertainty in number, then, would allow precise definition of the phase [12]. In fact, it can be shown that a simultaneous measurement of the phase of each output port results in a Heisenberg-limited correlation between them [8]. The Ramsey (or Mach-Zehnder) interferometer geometry exploits these correlations to measure small phase shifts without requiring an explicit measurement of the absolute phase of each interfering beam. For comparison, we have also shown the much narrower output distribution for a single Fock state input.

In the examples above we have assumed an equal number of atoms in each input port. In practice it is unlikely that such a state could be prepared. It is important, therefore, to explore the phase sensitivity to the initial balance of ground- and excited-state atoms. Figure 4 shows that in fact the phase sensitivity is only weakly sensitive to an imbalance over a broad range. As in the balanced input case above, the phase sensitivity $\Delta\phi_{\min}$ is inferred from the phase shift that produces a variance of one atom in the distribution for $\Delta n$.

The method presented above provides an algorithm for determining the presence of a small phase shift. In practice one may want to measure a larger phase shift $\phi_{\rm sig}$. This can be accomplished by application of a compensating phase shift $\phi_c$. The shift $\phi_c$ would then be adjusted to minimize the observed fluctuations at the output ports, so that $\phi_c + \phi_{\rm sig} \sim 0$. This method requires single-atom detection resolution at the output port. If some technical noise occurs during the detection process, the precision of the measurement is reduced to $\Delta\phi_{\min} = \delta n/N$, where $\delta n$ is the atom number resolution of the detection process.

In order to implement this scheme, it is necessary to measure the initial number of atoms in one of the condensates as well as the number of atoms at the output port (e.g., $n_g^{\rm in}$ and $n_e^{\rm out}$). The output atom number can be measured with standard fluorescence techniques. On the other hand, knowledge of the number of atoms at the input port requires a nondestructive number measurement just before the interferometer sequence. Such a measurement might be accomplished with dispersive light scattering [13].

It is interesting to consider the experimental sensitivities that might be achieved using these methods. For example, if a dual Fock state $|n,n\rangle$ ($n = 10^6$) is used in a Cs atomic fountain clock with interrogation time $T = 1$ s, the quantum-noise-limited short-term stability is of the order of $10^{-16}/\sqrt{\rm Hz}$. For spectroscopy in traps, interrogation times greater than 100 s might be achieved. This could lead to a resolution better than 10 nHz in a single shot. Such gains could be significant in, for example, searches for a permanent electric dipole moment (EDM experiments). We note that recently Myatt *et al.* [14] have experimentally realized a dual condensate source.

To conclude, we have shown that under certain conditions the use of degenerate Bose-Einstein ensembles as input states to a Ramsey-type interferometer can lead to Heisenberg-limited phase measurements. Although we studied the case of nuclear magnetic resonance spectroscopy using the Ramsey method of separated oscillatory fields, these arguments apply to any system of coupled Fock states, including, for example, tunneling between wells in a double-well potential.

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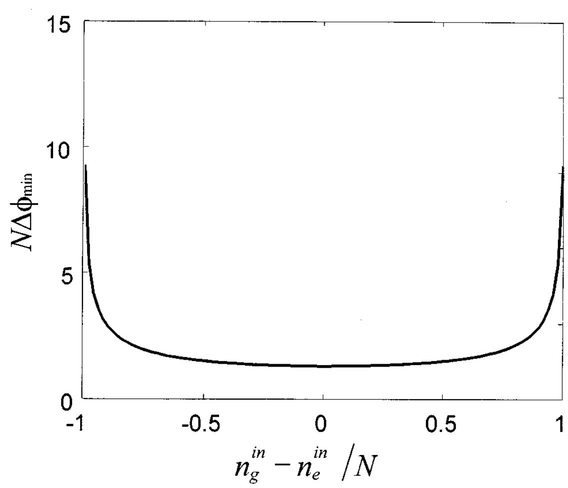


FIG. 4. Variation of the minimum detectable phase shift $\Delta\phi_{\min}$ with the initial balance of ground- and excited-state atoms $(n_g^{\rm in} - n_e^{\rm in})/N$ for a total number of atoms $N = 50$.

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 193 (1995).

[2] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).

[3] K. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).

[4] M. Naraschewski, H. Wallis, A. Schenzle, J. I. Cirac, and P. Zoller (unpublished).

[5] J. Javanainen and S. M. You, Phys. Rev. Lett. **76**, 161 (1996).

[6] Y. Castin and J. Dalibard (unpublished).

[7] N. F. Ramsey, *Molecular Beams* (Oxford, London, 1963), p. 124.

[8] M. J. Holland and K. Burnett, Phys. Rev. Lett. **71**, 1355 (1993).

[9] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, Phys. Rev. A **46**, R6797 (1992).

[10] We will use the word condensate here, and in subsequent text, to refer to an ensemble containing a definite number of degenerate, but noninteracting, bosons.

[11] M. A. Kasevich and S. Harris, Opt. Lett. **21**, 677 (1996).

[12] Z. Y. Ou, Phys. Rev. Lett. **77**, 2352 (1996).

[13] M. R. Andrews, M.-O. Mewes, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **273**, 84 (1996).

[14] C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. **78**, 586 (1997).