Coherent Collisions between Bose-Einstein Condensates

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We study the nondegenerate parametric amplifier for matter waves, implemented by colliding two Bose-Einstein condensates. The coherence of the amplified waves is shown by observing high contrast interference with a reference wave and by reversing the amplification process. Since our experiments also place limits on all known sources of decoherence, we infer that relative number squeezing is most likely present between the amplified modes. Finally, we suggest that reversal of the amplification process may be used to detect relative number squeezing without requiring subshot-noise detection.

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Quantum entanglement has always attracted great interest as it is one of the key differences between quantum and classical physics. Since it can be exploited for various novel applications such as quantum computation and precision measurements, there has been a continuing effort to engineer ever more robust quantum correlated states. Recently, four-wave mixing using Bose-Einstein condensates (BECs) has been proposed as a method for creating two correlated matter waves [1]. In that experiment, two source waves and a third seed wave were used to produce a fourth wave in the conjugate momentum state, resulting in an amplified seed-conjugate wave pair. For large amplification, the relative number fluctuations between the amplified waves can become significantly reduced, and this was recently achieved [2]. In quantum optics, this is known as nondegenerate parametric amplification [3]. Once created, such two-mode number squeezed states could be used for Heisenberg-limited interferometry where phase shifts up to $1/N$ accuracy could be measured [4,5], a significant advantage over the standard shot noise limit of $1/\sqrt{N}$.

Entanglement between the generated matter waves is possible only if the collisional four-wave mixing process is coherent. While the coherence of collisions has been observed before in mean field effects [6,7], it has not been demonstrated for the more common case of momentum changing elastic collisions. Four-wave mixing itself does not require or guarantee coherence since it will still occur for condensates with short coherence lengths [8], and previous experiments did not explicitly show that coherence was maintained throughout the collisional process.

In this paper, we demonstrate the coherence of elastic collisions by observing high contrast interference between the conjugate wave and a reference wave. Because of the strong coherence retained, we also observe a reversal of the amplification process. The success of these two experiments places an upper bound on the decoherence effects in our experiments, providing evidence that a significant amount of relative number squeezing is present in the pair-correlated beams. The reversibility of the amplification process further suggests a possible way of directly detecting the two-mode number squeezing and entanglement between the two correlated states, without requiring subshot-noise detection.

In the Heisenberg picture, the evolution of the annihilation operators a_3, a_4 of a mode pair that is parametrically amplified by the source modes a_1, a_2 is given by

$$\begin{aligned}\frac{da_3}{dt} &= -\frac{8\pi\hbar a}{Vm} i a_4^\dagger a_1 a_2 - i\frac{1}{2\hbar}\Delta\varepsilon a_3,\\ \frac{da_4}{dt} &= -\frac{8\pi\hbar a}{Vm} i a_3^\dagger a_1 a_2 - i\frac{1}{2\hbar}\Delta\varepsilon a_4,\end{aligned} \tag{1}$$

where V is the volume, m is the mass, a is the s-wave scattering length, and $\Delta\varepsilon$ is the energy mismatch between the source modes and the amplified modes [2]. The solution to these equations for highly occupied source modes is an exponential growth with a rate constant $\eta = \sqrt{[(2\overline{\mu})/\hbar]^2 - [(\Delta\varepsilon)/(2\hbar)]^2}$, where $\overline{\mu}$ is the geometric mean of the chemical potentials of the two source waves. In our experiment, the multiple waves used were generated from a single BEC using two-photon stimulated optical transitions [9]. The evolution of annihilation operators b_1, b_2 for modes which are coupled by such a Bragg transition is given by

$$\frac{db_1}{dt} = -\Omega i b_2, \qquad \frac{db_2}{dt} = -\Omega^* i b_1, \tag{2}$$

where Ω is the coupling strength. In both the Bragg and amplification processes, the generated wave incurs an overall phase shift of 90° relative to its source.

The experimental setup was almost identical to the one used in [2]. Sodium condensates of about 13×10^6 atoms each were created in a cylindrical magnetic trap with an axial frequency of 20 Hz and a radial frequency of 40 Hz. They had a Thomas-Fermi radius of 30 μm radially and 60 μm axially, a chemical potential of $h \times 1.8$ kHz, and a speed of sound of 6 mm/s. From an initial condensate $\mathbf{k}_1$, a small seed $\mathbf{k}_3$ with a velocity of 14 mm/s was generated using a 20 μs Bragg (seed) pulse (see Fig. 1). In the next 40 μs, a second source wave $\mathbf{k}_2$ with a velocity of

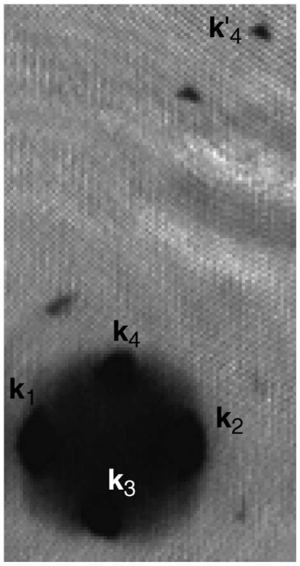

FIG. 1. The two source waves $\mathbf{k}_1$ and $\mathbf{k}_2$, the seed $\mathbf{k}_3$, the conjugate wave $\mathbf{k}_4$, and the detection state $\mathbf{k}_4'$ after 20 ms ballistic expansion. Also seen is the collisional halo around all four waves as well as several small unlabeled waves, generated by off-resonant scattering. The field of view is 1.0×1.9 mm.

20 mm/s was split off from $\mathbf{k}_1$ using a $\pi/2$ pulse. All Bragg beams in our experiment were detuned from the sodium D_2 line by 40 GHz to avoid spontaneous Rayleigh scattering. Subsequent collisions between $\mathbf{k}_1$ and $\mathbf{k}_2$ acted as a parametric amplifier for the seed wave $\mathbf{k}_3$, creating a fourth conjugate wave $\mathbf{k}_4$ in the momentum conserving state. The number occupation of $\mathbf{k}_3$ and $\mathbf{k}_4$ modes then grew exponentially with an expected rate of $\eta = 17$/ms on average across the condensate. At the same time, the parametric amplifier also enhanced spontaneous scattering into initially unseeded modes, giving rise to the collisional halo seen in Fig. 1 [10]. The resulting loss of atoms from the source waves limited the four-wave mixing process to 400 μs.

After 80 μs of amplification, we probed the phase coherence of the conjugate wave $\mathbf{k}_4$ by interfering it with a reference wave. For optimal interference, the reference wave needed to have a well-defined relative phase and be identical in size and momentum to $\mathbf{k}_4$. The reference wave was outcoupled from $\mathbf{k}_2$ using another 20 μs Bragg (seed) pulse. By changing the phase of one of the Bragg beams, the phase of the reference wave relative to $\mathbf{k}_4$ was varied. The resulting occupation of the conjugate momentum state $\mathbf{k}_4$ was monitored by selectively transferring a fixed fraction of atoms in $\mathbf{k}_4$ to a detection state $\mathbf{k}_4'$ (see Fig. 1). For this, a 40 μs Bragg pulse with large momentum transfer ($2\hbar k$) was used. This ensured that the $\mathbf{k}_4'$ atoms were well separated during ballistic expansion and their number was easily determined. Direct number measurement of the mode $\mathbf{k}_4$ was not possible since the amplification process continued during ballistic expansion.

The interference fringe obtained is plotted in Fig. 2. By fitting a sinusoidal curve to the raw data, we obtained a fringe contrast of $68\% \pm 11\%$. This proves that the conjugate wave maintains the phase relation established during amplification and is in a single quantum state. In addition, we expected destructive interference to take place at 90° due to the accumulation of all the 90° phase shifts introduced by the four-wave mixing and Bragg processes [see Eqs. (1) and (2)]. Our value of $77° \pm 9°$ agrees fairly well with this, and the small discrepancy could be due to the Bragg beams not being fully resonant. It is also interesting to note that the position of the fringe minimum is a direct signature of the positive sign of the scattering length a, since a negative scattering length would have resulted in a minimum at 270°. Usually, only the cross section, which is proportional to a^2, is measured using these momentum changing collisions.

A direct implication of the sustained coherence is the reversibility of the amplification. If $\Delta\varepsilon$ is small, the second term in Eq. (1) becomes negligible and the overall evolution is solely determined by the unitary four-wave mixing term. By changing the sign of one of the four waves, we can apply the inverse operator and the amplified populations will subsequently deamplify to zero due to the entanglement between the coupled modes. This is true even for the modes which were not deliberately seeded, since spontaneous emission into these modes also occurs by the same pairwise collision process. A similar experiment using photons has already been performed [11].

After 80 μs of amplification, we initiated deamplification by inducing a 180° phase shift in $\mathbf{k}_2$. The sign reversal was achieved by applying a 120 μs $-\pi$ pulse resonant for the momentum difference between the source waves $\mathbf{k}_1$ and $\mathbf{k}_2$. To check that the amplification process was not permanently disrupted by the Bragg pulse, we performed a control experiment where the $-\pi$ pulse was

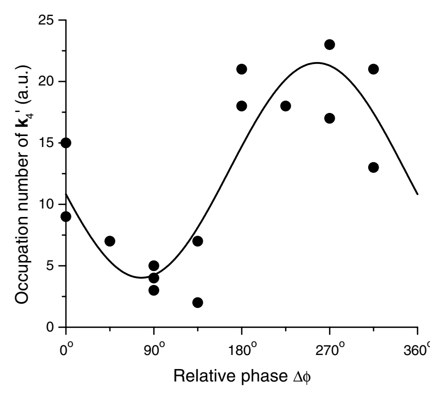

FIG. 2. Coherence of the conjugate wave. Shown is the interference of the conjugate wave with a reference wave. The relative phase shift was introduced by shifting the phase of the rf wave which drives the lower-frequency acousto-optic modulator in the Bragg setup.

replaced by a $-\pi/2$ and $+\pi/2$ pulse applied in quick succession. Like the first half of the $-\pi$ pulse, the $-\pi/2$ pulse transferred all the atoms from $\mathbf{k}_2$ back into $\mathbf{k}_1$ but, unlike the second half of the $-\pi$ pulse, the $+\pi/2$ pulse built up mode occupation in $\mathbf{k}_2$ with the original phase. If the Bragg pulses did not otherwise disturb the phase relation of the system, amplification should then resume. To minimize undesirable off-resonant scattering in these two experiments, the duration of the Bragg pulses was increased by a factor of 1.5 to allow for higher momentum selectivity. A third experiment was also performed where amplification was allowed to proceed unchecked. In all three cases, we monitored the evolution of $\mathbf{k}_4$ by applying the detection pulse and counting the number of atoms in $\mathbf{k}_4'$ after variable intervals.

Figure 3 shows the results obtained and clearly demonstrates the reversal of the amplification process. As a quantitative measure of the deamplification process, we defined the deamplification factor as the ratio of the maximum mode occupation to the remnant left at the end of deamplification, and obtained a factor of $\sim$5. In addition, we infer that this reversal is phase dependent, since amplification continues if we do not reverse the phase of the second source wave [Fig. 3 (○, ×)]. This demonstrates that the four-wave mixing process does not suffer significant decoherence during the observation time.

In Fig. 3, we also plot numerical simulations of all three cases and find good qualitative agreement with the experiment. For these simulations, the energy mismatch $\Delta\varepsilon$ was the main adjustable parameter and a_1 and a_2 were taken to be c-numbers since they are highly

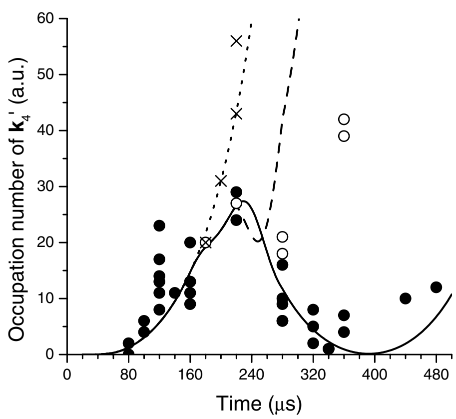

FIG. 3. Phase coherent amplification and deamplification: (•) Deamplification of the conjugate wave. (○) Continued amplification due to a π pulse with a 180° phase shift applied in midpulse (see text). (×) Uninterrupted amplification. The region below the horizontal axis gives the sequence of events for deamplification: The shaded regions correspond to Bragg pulses and clear regions correspond to free system evolution. The solid, dashed, and dotted lines are independent numerical simulations of the respective processes.

occupied relative to the other modes. Apart from $\Delta\varepsilon$, the only other adjustable parameters in the simulations were an overall normalization constant and the size of the initial seed, which was assumed to be small. Numerically, optimal deamplification was achieved for $\Delta\varepsilon = h \times 1.0$ kHz, and adjustments to this value by $h \times 0.6$ kHz either way still yielded a fit consistent with the experimental data. The simulations also took into account the off-resonant scattering of the seed and conjugate wave during the $-\pi$ pulse, which caused a 20% loss of atoms from the amplified waves and accounted for the dip in mode occupation seen in Fig. 3 (○). Finally, beyond 300 μs, amplification slows down in Fig. 3 (○) and departs from the numerical simulation (dashed line) due to depletion of the source waves as a result of scattering and subsequent amplification of the unseeded modes [2].

Theoretically, maximum deamplification should occur for $\Delta\varepsilon = 0$ in the absence of any extraneous phase shifts. Our experiment deviates from this ideal situation as it suffered from off-resonant scattering during the $-\pi$ pulse resulting in a calculated phase shift of 0.6 rad in the amplified modes. This was due to the finite duration of the $-\pi$ pulse, which off resonantly coupled out atoms from the amplified waves. The observed value of $\Delta\varepsilon = h \times 1.0$ kHz therefore might have compensated for this phase shift as the amplified modes evolved over time.

We also found that the deamplification process is extremely phase sensitive. Compared to the tolerance in $\Delta\varepsilon$ for amplification [2], the $h \times 0.6$ kHz tolerance for deamplification is relatively small. While amplification is inherently a stable process which enforces a certain phase relationship between the four waves, deamplification is unstable with respect to the phase. As soon as the phase relation is lost, deamplification ends and amplification resumes. Fortunately, deamplification remains possible since its tolerance is still larger than the estimated $h \times 0.4$ kHz half-width of the inhomogeneous mean field shift. However, in order to remain within the narrow range of $\Delta\varepsilon$ allowed, we had to align the Bragg beams to within milliradians of the optimal value.

In principle, the pairwise collisions of atoms during deamplification could be used to demonstrate relative number squeezing and entanglement between the two amplified modes. Without losses, the mode occupations after deamplification will reflect the relative number uncertainty in the initial waves. For coherent or classical waves, the remnant will follow Poissonian statistics and be of order $\sqrt{N}$, where N is the initial mode occupation. In contrast, for perfect two-mode number squeezing, the remnant will always be exactly zero. Moreover, complete deamplification of the seed and conjugate waves can occur only if the sum of their individual phases remain well defined throughout the process. These two conditions cannot be met for unentangled waves, since their individual number and phase are canonically conjugate variables and cannot simultaneously be well defined.

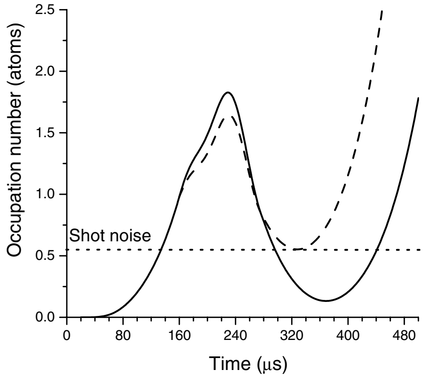

FIG. 4. Deamplification of perfectly squeezed modes with off-resonant scattering (solid line). Identical process but with coherent waves (dashed line). The vertical scale is the expectation value of the mode occupation.

To illustrate this, we simulated the evolution of the conjugate wave for our experimental conditions, this time without an initial seed (Fig. 4, solid line). Here, initial mode occupation is provided by spontaneous scattering, which will nevertheless still occur pairwise. While complete deamplification does not occur as the squeezing has been degraded by off-resonant scattering, it still dips below the shot noise level given by the minimum of the control graph (Fig. 4, dashed line), which assumes coherent waves. The control simulation is started at 160 μs, immediately preceding the $-\pi$ pulse, using nonentangled coherent waves with the same unity mode occupation. To detect relative number squeezing, we need to obtain deamplification factors of greater than $\sqrt{N}$.

Experimentally, direct detection of relative number squeezing in our strongly occupied beams is not possible since it would require a much larger deamplification factor than what we currently observe. In this, we are limited by the inhomogeneous mean field and the depletion of the source waves. However, for initially unseeded modes, a deamplification factor of 5 is already evidence for subshot-noise correlations (Fig. 4). This shows that, by monitoring a collection of modes with a similar energy mismatch, detection of the entanglement is possible with current experimental parameters. Practically, however, the occupation in these modes was masked by surrounding modes with a different energy mismatch. Further complicating detection, the momenta of the states were blurred out during the acceleration of the outcoupled atoms in the gradient of the mean field of the condensate.

Finally, we briefly consider other possible sources of decoherence in our amplifier and discuss how their effects may be minimized. Collisions between the source and amplified waves can compromise squeezing as atoms are scattered into or out of the amplified waves in a random way. As shown in [2], at low density, no stimulated scattering occurs and the spontaneous scattering rate was estimated to be significant only after the source waves are about to be depleted. By further manipulating the size of the condensate such that the amplified waves separate out before the source waves are depleted, we can ensure that the squeezing will be preserved.

A further study of the nondegenerate matter wave amplifier should explore the issue of the "beam quality" of the squeezed waves. The phase errors incurred due to the inhomogeneous mean field induce spatially dependent phase shifts in the wave function of the squeezed waves, rendering them hard to isolate. Other sources of phase errors are not a concern as they appear to have little effect on our experiment. In particular, phase fluctuations across the condensate (first observed by [8]) are absent for our parameters [12,13]. Moreover, since the atoms travel at a maximum velocity of 20 mm/s for a duration of 300 μs, the required coherence length of 6 μm is easily satisfied.

In conclusion, we have demonstrated the coherence of a nondegenerate matter wave amplifier by measuring the phase of the conjugate wave and by reversing the amplification process. While we do not yet directly detect number squeezing in our amplified beams, we have evaluated all known decoherence processes and concluded that they are well controlled in our experiment. Therefore the atomic nondegenerate parametric amplifier studied here is a promising candidate for generating entangled and number squeezed beams.

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