Photon-number-resolved detection of photon-subtracted thermal light

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We examine the photon statistics of photon-subtracted thermal light using photon-number-resolved detection. We demonstrate experimentally that the photon number distribution transforms from a Bose–Einstein distribution to a Poisson distribution as the number of subtracted photons increases. We also show that second- and higher-order photon correlation functions can be directly determined from the photon-number-resolved detection measurements of a single optical beam.

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Tools to characterize the photon statistics of an optical radiation field are becoming increasingly important for applications ranging from fundamental quantum optics [1] to quantum information science and quantum metrology. For example, the inherent quantum noise of light determines the ultimate sensitivity in light-based gravitational-wave measurements [2]. Information of the photon statistics of a radiation field is contained in correlations of the field [3]. Measuring these correlations is typically done by splitting the light into multiple beams, sending them to multiple detectors, and implementing coincidence counting [4], and it has never been a trivial task to experimentally access the third- or higher-order correlation functions of a light field [5,6]. Taking advantage of the recently developed high-efficiency, photon-number-resolving detectors [7–9], we show that the high-order correlations of an optical radiation field can be determined in a straightforward manner with photon-number-resolved detection of a single optical beam. We use this method to examine the photon statistics of photon-subtracted thermal light and show that our experimental measurements match well with theoretical predictions, thus validating the method.

A thermal light with mean photon number μ and variance $\Delta = \langle(\hat{a}^+\hat{a})^2\rangle - \langle\hat{a}^+\hat{a}\rangle^2 = \mu + \mu^2$ is diagonal in the Fock representation, $\hat{\rho} = \sum_{n=0}^{\infty} \frac{\mu^n}{(1+\mu)^{n+1}} |n\rangle\langle n|$, and is super-Poissonian as defined by $\Delta > \mu$. Conditioning on k-photon subtraction, the photon-subtracted thermal state is also diagonal in the Fock representation,

$$\hat{\rho}_k = \frac{(\hat{a})^k \hat{\rho} (\hat{a}^+)^k}{\mathrm{Tr}((\hat{a})^k \hat{\rho} (\hat{a}^+)^k)} = \sum_{n=0}^{\infty} \frac{(n+k)!}{n!k!} \frac{\mu^n}{(1+\mu)^{k+n+1}} |n\rangle\langle n| \tag{1}$$

with mean photon number

$$\mu_k = (k+1)\mu \tag{2}$$

and variance $\Delta_k = (1+\mu)\mu_k$, where $\hat{a}^+$ and $\hat{a}$ are photon creation and annihilation operators. The Mandel Q parameter remains a constant, $(\Delta_k/\mu_k - 1) = \mu$ [1]. The nth-order correlation function of the k-photon-subtracted thermal light is given as

$$g_k^{(n)}(0) = \frac{\langle(\hat{a}^+)^n(\hat{a})^n\rangle}{\langle\hat{a}^+\hat{a}\rangle^n} = \frac{(k+n)!}{k!(k+1)^n}. \tag{3}$$

Equation (2) shows that the mean photon number of a thermal light increases after photon subtraction. This counterintuitive result demonstrates the quantum nature of the photon-subtraction operation [10–12]. The statistics of the photon-subtracted thermal light remain super-Poissonian, albeit less so as the number of subtracted photons increases.

As the number of subtracted photons becomes larger, the intensity noise of the photon-subtracted thermal light is substantially reduced, with the state approximately expressed as $\hat{\rho}_k \sim \sum_{i=0}^{\infty} (\lambda^i/i!) |i\rangle\langle i|$ and $g_k^{(n)}(0) \to 1$, where $\lambda = k\mu/(1+\mu)$. Thus the photon-subtracted thermal state approaches a mixture of phase-randomized coherent states, which is described by

$$\hat{\rho}_{\mathrm{ph}} = \int_0^{2\pi} \frac{d\varphi}{2\pi} ||\alpha|e^{i\varphi}\rangle\langle|\alpha|e^{-i\varphi}| \propto \sum_{i=0}^{\infty} \frac{|\alpha|^2}{i!} |i\rangle\langle i|$$

with $|\alpha|^2 = \mu_k$.

In the following, we analyze how to access photon statistics directly by using photon-number-resolved detection in an experiment. The photon-subtraction measurement (Fig. 1) is implemented by distributing the optical power (μ_{AB}) between a transmitting port (arm 1) and a reflecting port (arm 2) of a polarizing beam splitter (PBS). By attributing all loss to photon detection, the state of the light after passing through the PBS is written as

$$\hat{\rho} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \sum_{j=0}^{n} \sqrt{\binom{n}{k}\binom{n}{j} \tau^k (1-\tau)^{n-k} \tau^j (1-\tau)^{n-j}} \times \frac{\mu_{AB}^n}{(1+\mu_{AB})^{n+1}} |n-k\rangle|k\rangle\langle n-j|\langle j| \tag{4}$$

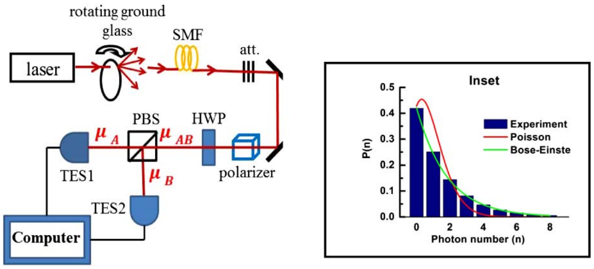

Fig. 1. Schematic of experiment with photon-subtracted thermal light and photon-number-resolved detection. SMF, single-mode fiber; att., optical attenuators; HWP, half-wave plate; PBS, polarizing beam splitter; TES1(2), transition-edge-sensors. Inset: measured photon number distribution **P(n)** of pseudo-thermal light (blue bars), calculated Poisson distribution (red curve), and Bose–Einstein distribution with $\mu_B = 1.31$ (green curve).

with τ being the transmittance of PBS for the horizontally polarized light.

A k-photon detection is represented by the positive-operator-valued measure,

$$\hat{\Pi}_k = \sum_{i=k}^{\infty} \binom{i}{k} \eta_{1(2)}^k (1-\eta_{1(2)})^{i-k} |i\rangle\langle i|, \tag{5}$$

where η_1 (η_2) is the photon-detection efficiency of arm 1 (2).

The probability of coincident detection of a k-photon in arm 1 and a j-photon in arm 2 is given by

$$P(j,k) = \mathrm{Tr}(\hat{\Pi}_j \hat{\rho} \hat{\Pi}_k) = \frac{(j+k)!}{j!k!} \frac{\mu_A^k \mu_B^j}{(1+\mu_A+\mu_B)^{j+k+1}}, \tag{6}$$

where $\mu_A = \mu_{AB}\tau\eta_1$ and $\mu_B = \mu_{AB}(1-\tau)\eta_2$ are the measured mean photon numbers of arm 1 and arm 2. They can be directly obtained from the photon-number-resolved measurements, for example, with $\mu_A = P(1,0)/[P(0,0)]^2$ and $\mu_B = P(0,1)/[P(0,0)]^2$. Conditioning on the detection of k-photons in arm 1, the measured mean photon number of arm 2 is

$$\mu_{2|k} = \mu_{2|0}(k+1), \tag{7}$$

where $\mu_{2|0} = \mu_B/(1+\mu_A)$ is the mean photon number conditioned on zero-photon subtraction.

Equation (7) is identical to Eq. (2) when the light directed to arm 1 (μ_1) is very small. For $k \gg j$, the conditional photon number distribution of arm 2 can be approximated by a Poisson distribution:

$$P(j|k) = \frac{P(j,k)}{\sum_{j=0}^{\alpha} P(j,k)} \alpha \frac{[k\mu_B/(1+\mu_A+\mu_B)]^j}{j!}. \tag{8}$$

Measuring correlations, particularly high-order correlations, is essential to the study of photon statistics of optical radiation. With photon-number-resolving detectors, the correlation functions of an optical field can be accessed with photon-number-resolved detection of a single optical beam, which are given as

$$g^{(n)}(0) = \frac{\langle(\hat{a}^+)^n(\hat{a})^n\rangle}{\langle\hat{a}^+\hat{a}\rangle^n} = \frac{\sum_{j=0}^{\infty} \frac{(j+n)!}{j!} \mathrm{Tr}(\hat{\rho}\hat{\Pi}_{j+n})}{(\sum_{j=0}^{\infty}(j+1)\mathrm{Tr}(\hat{\rho}\hat{\Pi}_{j+1}))^n}. \tag{9}$$

For the photon-subtracted thermal light studied here, the correlation functions are determined from the experimentally measured photon counting statistics by using

$$g^{(n)}(0) = \frac{\sum_{j=0}^{\infty} \frac{(j+n)!}{j!} P(j+n|k)}{(\sum_{j=0}^{\infty}(j+1)P(j+1|k))^n}. \tag{10}$$

We generated pseudo-thermal light by passing laser pulses (850 nm, 50 kHz, with a pulse duration of 4 ps) through a rotating ground glass. A single-spatial optical mode is selected from the scattered light by using a single-mode optical fiber. Its polarization is prepared by a Glan–Taylor polarizer. We used a half-wave plate and a PBS to distribute the optical power between the transmitting and the reflecting ports of the PBS, which are sent to photon-number-resolving transition-edge sensors [7–9], TES1 and TES2. The measured photon number distribution of the light in arm 2 is seen (Fig. 1) to be Bose–Einstein rather than Poissonian, with $\mu_B = 1.31$ and $g^{(2)}(0) = 1.94$ obtained from the measured photon-counting statistics using Eq. (10). This direct measurement in Fock space confirms that the pseudo-thermal light source created by passing the coherent laser light through a rotating ground glass with a selected single-spatial mode exhibits Bose–Einstein statistics.

Figure 2 shows that the measured photon number distributions of the photon-subtracted thermal light with subtracted photon numbers $m = 0$, 2, 4, 6 or 8 are consistent with the predicted conditional probability distribution using Eq. (8). The envelopes of the Bose–Einstein and Poisson distributions are also shown. [The mean photon numbers used in calculations were experimentally measured μ_B and/or μ_A, and values obtained

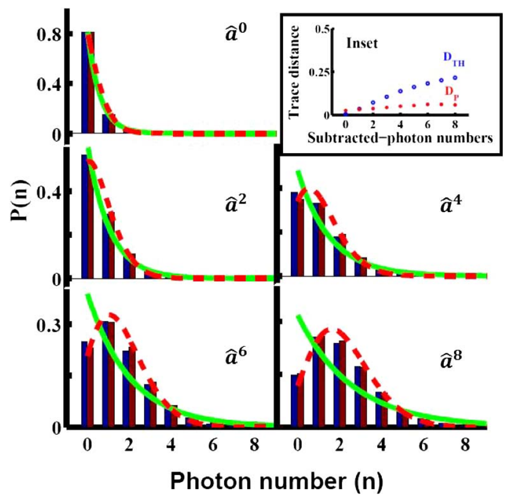

Fig. 2. Photon number distribution $P(n)$ of photon-subtracted thermal light (arm 2 with $\mu_B = 0.43$ conditioned on arm 1 with $\mu_A = 0.96$). Experimental data (blue bars), calculations (red bar) using Eq. (8). Poisson (red dashed curve) and Bose–Einstein (green curve) distributions are shown as lines as a guide to the eye. Inset: calculated trace distances $D_{\mathrm{TH}(P)} = \mathrm{Tr}|\hat{\rho} - \hat{\rho}_{\mathrm{TH}(P)}|/2$.

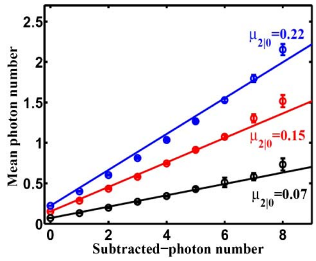

Fig. 3. Mean photon numbers of photon-subtracted thermal light (arm 2) versus subtracted-photon number (arm 1). Experimental data (points); predictions (lines) using Eq. (8) with experimentally determined values of (μ_A, μ_B), which are, from top to bottom, (0.96, 0.43), (0.51, 0.23), and (0.59, 0.11).

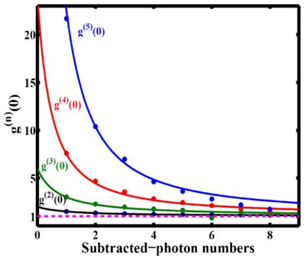

Fig. 4. Correlations of photon-subtracted thermal light (arm 2 with $\mu_B = 0.43$) versus subtracted-photon numbers (arm 1 with $\mu_A = 0.96$), with a lower bound of 1 (dashed line). Experimental data (dots); predictions (lines) using Eq. (3) (see Media 1).

through Eq. (7).] The observed transformation of the photon-subtracted thermal light from a Bose–Einstein to a Poisson distribution is apparent as the number of subtracted photons increases. This transformation is quantified with a trace distance defined as $D_{\mathrm{TH}(P)} = \mathrm{Tr}|\hat{\rho} - \hat{\rho}_{\mathrm{TH}(P)}|/2$, where $\hat{\rho}_{\mathrm{TH}}$ ($\hat{\rho}_P$) is the state of ideal thermal (coherent) light having mean photon numbers equal to that of the photon-subtracted thermal light $\hat{\rho}$. $D_{\mathrm{TH}(P)}$ varies from zero (for two identical density matrices) to one (for density matrices with maximal difference). As shown in Fig. 2 (inset), $D_{\mathrm{TH}(P)} \approx 0$ for $m = 0$; and as m increases, D_P stays near 0, while D_{TH} increases monotonically to ≈ 0.25, indicating that the photon number distribution of the photon-subtracted thermal light deviates from the Bose–Einstein distribution and is better described by a Poisson distribution.

Figure 3 shows that the measured mean photon number of photon-subtracted thermal light increases linearly with the number of subtracted photons. The experimental data are consistent with the theoretical predictions using Eq. (7), which has the experimentally measured mean photon numbers of the photon-subtracted thermal light as the only input.

Figure 4 shows experimentally measured correlations of the photon-subtracted thermal light with mean photon numbers ranging from 0.22 to ≈ 2, which are consistent with the theoretical predictions using Eq. (3). We note that, in both Figs. 3 and 4, the experimental data, particularly the measurement results of $g^{(5)}(0)$, start to deviate from theoretical predictions as the number of detected photons increases beyond seven. This is likely because the TESs used in this experiment were able to detect photon number states with photons up to nine, but with significant overlap between signals of adjacent high-photon-number states. The possible misidentification of high-photon-number states can lead to the deviation of those experimental measurements, where high-photon-number states are important, from theoretical predictions. Improving the photon number resolution of TES for high-photon-number states is under investigation [13].

In conclusion, we have shown how correlation functions of photon-subtracted thermal light can be determined by using photon-number-resolved detection of a single optical beam. We demonstrated that the photon statistics transform from a Bose–Einstein distribution to a Poisson distribution as the number of subtracted photons increases. We expect the method demonstrated in this study to be broadly useful to the field of quantum optics and quantum information science.

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